Quantum Foundations IV: Quantum theory of measurement and Bohmian mechanics

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Part 1.

MOTIVATION

- Interpretations of QM are attempts
to resolve the measurement problem of QM.
- We don't know which interpretation (if any) is true.
- So what's the use of interpretations?

- Useful if they help to make QM **intuitive**
(without contradicting any of the experimentally established facts).

- Intuition is subjective, what's intuitive to me
may not be intuitive to you.
- For me, the most intuitive is the **Bohmian** interpretation.
- My main goal today is to try to convey this intuition to you.

- It will include some quantum-measurement theory
that does not depend on the interpretation.

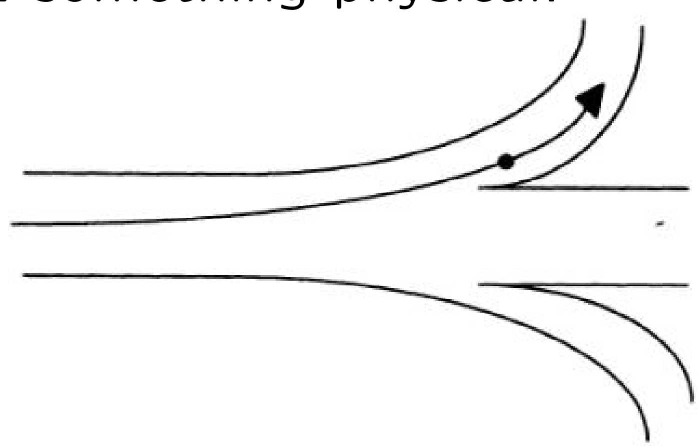
Quantum measurements and Bohmian mechanics in a nutshell:

- Schrödinger equation (micro system + macro measuring apparatus)
$\Rightarrow$ the full wave function splits into several (macroscopically different) branches

$$|\Psi\rangle = |\text{apparatus shows result 1}\rangle + |\text{apparatus shows result 2}\rangle + \ldots$$

Why do we observe only one of those?

Bohmian interpretation:
- Because what we observe is not $|\Psi\rangle$.
- We observe something which **always** has well defined position, so it cannot enter more than one branch.
$\Rightarrow$ Particular branch becomes physical because it becomes **filled with** something physical:



Why is this intuitive (to me)?

- One has a mental picture of what "actually" happens
(lacking in shut-up-and-calculate).
- One can think/speak of properties of the system
even when they are not measured
(lacking in Copenhagen).
- No need for a mysterious collapse.
- No need for many worlds.

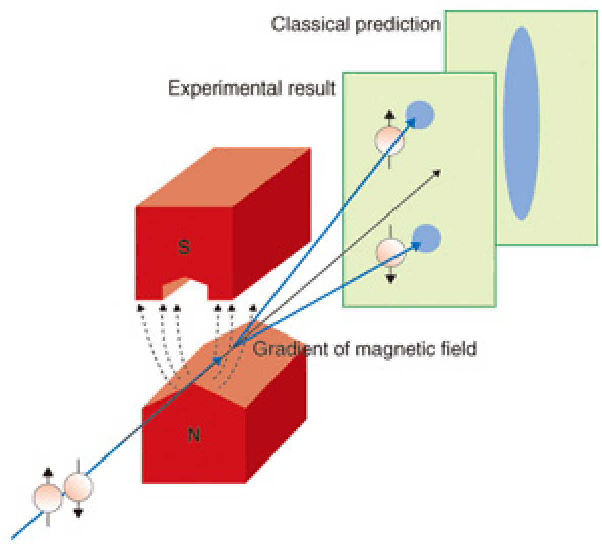
An extra bonus:
- The equation for the positions may be deterministic.
$\Rightarrow$ Quantum probability may not be fundamental.
However:
(it has to be emphasized because many outsiders get it wrong)

Determinism is **not the main** motivation for the Bohmian interpretation.

- The main motivation is ontology
(properties existing even without measurements).

Example - measurement of spin:

- Spin is measured by Stern-Gerlach apparatus.
- The wave function is split by the magnet.
- What we observe is not the spin as such.
- We observe the **position** (up or down) of the spot on the screen.



The (simplest version of) Bohmian interpretation:
- We observe only one spot because the particle travels only along one of the branches.

Part 2.

WHAT IS MEASUREMENT?

The perceptible

Observable:

- In QM it is a noun (in normal English it is an adjective).
- Hermitian operator in the Hilbert space.
- Related to (but not identical with) a measurable quantity.
- *"Quantum phenomena do not occur in a Hilbert space, they occur in a laboratory."* - Asher Peres

Perceptible:

- In physics it is a noun (in normal English it is an adjective).

(coined in H.N., Int. J. Quantum Inf. (to appear); arXiv:1811.11643)

- A thing or phenomenon amenable to **direct** human perception.
- Perceptibles: tables, chairs, Moon, macroscopic instrument, click in detector, picture of atom produced by electron microscope, ...
- Non-perceptibles: wave function, electron, photon, atom, ...

- The distinction between perceptibles and non-perceptibles is similar to the distinction between macroscopic and microscopic.
- All perceptibles are macroscopic.
- But the converse is not true. Some macroscopic entities are not perceptibles (e.g. gravitational field, radio wave).

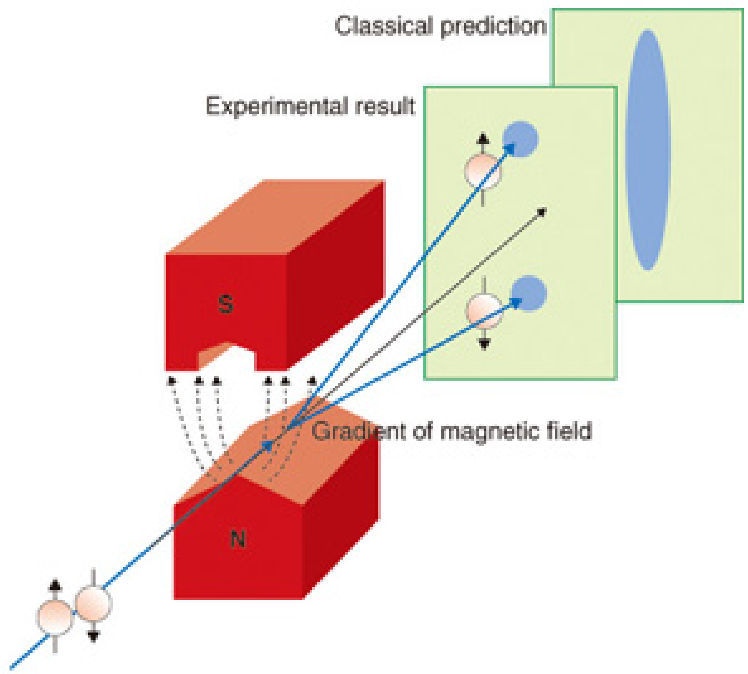
- **To make a measurable prediction means to predict a property of a perceptible.**

- Like with macroscopic/microscopic, there is no strict border between perceptible/non-perceptible.
- Is a one cell microorganism macroscopic or microscopic?
- Is perception of one cell microorganism by optical microscope direct or indirect? (It must be direct to be called perceptible.)
- Even though there is no strict border, the concepts are useful.

All observations can be reduced to macroscopic positions

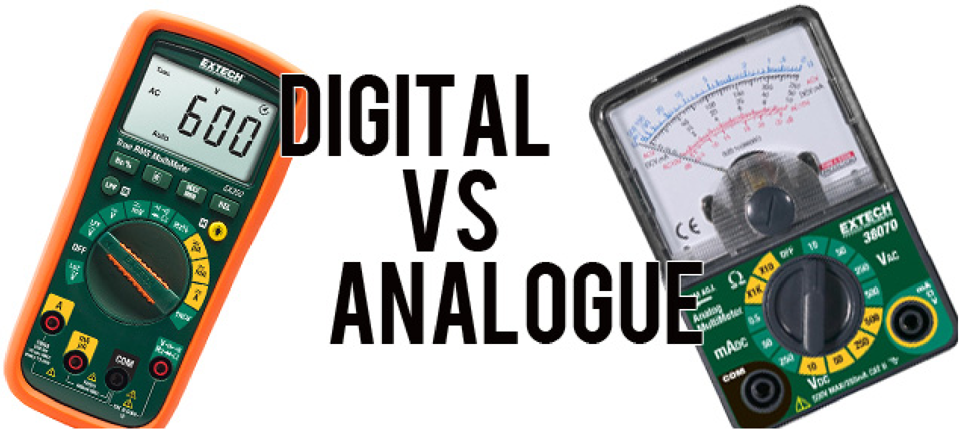
- All perceptibles are macroscopic, which means big in position space.
$\Rightarrow$ When 2 perceptibles can be distinguished, it means that they can be distinguished by macroscopic positions of something.

Measurement of spin:
- Spin is an observable, but not a perceptible.
- Spin is measured by Stern-Gerlach apparatus.
- Perceptible is a big dark spot on the screen.



More sophisticated instruments:

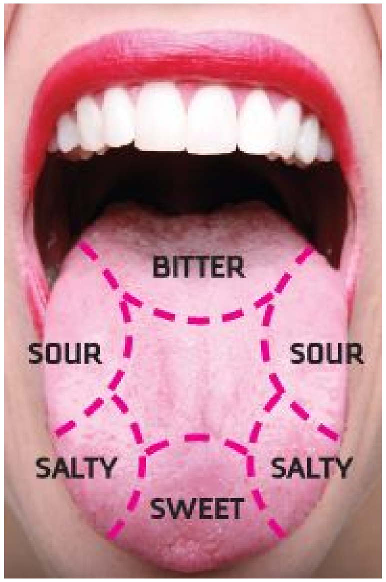
- Analog: position of macro pointer.
- Digital: positions of lines that make a digit.



Click in the detector:

- Sound determined by macroscopic oscillations
(e.g. membrane of the speaker).
- This oscillation is a macro position as a function of time.

What about senses such as color, taste and smell?
- Created in the eye, tongue or nose (and interpreted by brain).
- Determined by **which** nerve is stimulated.
- One can fool the brain: e.g. electro-stimulation
of the sweet nerve creates the illusion of sweetness.
- Different nerves have different macro positions
(most pronounced in the tongue).



$\Rightarrow$ Senses are perceptibles too, determined by macro positions of nerves.

(If you think it's weird to talk about biology and senses in mathematical physics, check out von Neumann's "Mathematical Foundations of QM"!)

Part 3.

THE QUANTUM THEORY MEASUREMENT

The origin of Born rule in QM

- In any basis $\{|k\rangle\}$, the Born rule **postulates** probability

$$p_k = |\langle k|\psi\rangle|^2$$

- However it is not necessary to postulate it.
- We **derive** it from the Born rule in the position-space only.

- We need probabilities of perceptibles
(e.g. probability that detector will click).
$\Rightarrow$ Probabilities of perceptibles must be computed in the position space.
- However, there is no strict border between
perceptible and non-perceptible.
$\Rightarrow$ Compute **all** probabilities in the position space.

- Measure observable $\widehat{K}$ with eigenstates $|k\rangle$.
- Macroscopic apparatus (the perceptible) can be described by its quantum microscopic state $|\Phi\rangle$.
- Initial microscopic state of the apparatus $|\Phi_0\rangle$.
- Interaction $\Rightarrow$ unitary transition

$$|k\rangle|\Phi_0\rangle \to |k\rangle|\Phi_k\rangle$$

$\Rightarrow$ Wave functions have a negligible overlap in multi-position space

$$\Phi_{k_1}(\vec{x})\Phi_{k_2}(\vec{x}) \simeq 0 \quad \text{for} \quad k_1 \neq k_2$$

where $\Phi_k(\vec{x}) \equiv \langle\vec{x}|\Phi_k\rangle$,

$$\vec{x} \equiv (\mathbf{x}_1, \ldots, \mathbf{x}_n)$$

n = number of particles constituting the apparatus

$$\int d\vec{x}\, |\Phi_k(\vec{x})|^2 = 1, \quad d\vec{x} \equiv d^{3n}x$$

- For a superposition $|\psi\rangle = \sum_k c_k|k\rangle$:

$$|\psi\rangle|\Phi_0\rangle \to \sum_k c_k|k\rangle|\Phi_k\rangle$$

- A more realistic analysis includes also environment

$$|\psi\rangle|\Phi_0\rangle|E_0\rangle \to \sum_k c_k|k\rangle|\Phi_k\rangle|E_k\rangle \equiv |\Psi\rangle$$

$$\Rightarrow \boxed{|\Psi\rangle = \sum_k c_k|\Phi_k\rangle|R_k\rangle,} \quad |R_k\rangle \equiv |k\rangle|E_k\rangle$$

- $|\Phi_k\rangle$ describes the perceptible, $|R_k\rangle$ all the rest.

Multi-position representation

$$\Psi(\vec{x},\vec{y}) = \sum_k c_k \Phi_k(\vec{x}) R_k(\vec{y})$$

- Born rule in the multi-position space

$$\rho(\vec{x},\vec{y}) = |\Psi(\vec{x},\vec{y})|^2 \simeq \sum_k |c_k|^2 |\Phi_k(\vec{x})|^2 |R_k(\vec{y})|^2$$

$$\Rightarrow \rho^{(\text{appar})}(\vec{x}) = \int d\vec{y}\, \rho(\vec{x},\vec{y}) \simeq \sum_k |c_k|^2 |\Phi_k(\vec{x})|^2$$

$\Rightarrow$ Probability to find the apparatus particles in the support of $\Phi_k(\vec{x})$:

$$\boxed{p_k^{(\text{appar})} = \int_{\text{supp } \Phi_k} d\vec{x}\, \rho^{(\text{appar})}(\vec{x}) \simeq |c_k|^2}$$

- This coincides with the Born rule in arbitrary k-space. *Q.E.D.*

Part 4.
Bohmian mechanics

Motivation for BM

The main assumption that motivates BM:

All perceptibles are ontic.

- E.g. the Moon is there even if nobody observes it.
- Motivated by common sense.
- The opposite would be that the Moon is only in our mind.
- Impossible to prove or disprove by scientific method.
- It's only a thinking tool (hard to think the opposite).

Most of the motivation for BM arises from this common sense axiom!

Bell theorem expressed in the language of perceptibles:

If perceptibles are ontic, then perceptibles are non-local.

- If the correlated, yet spatially separated, measurement outcomes are there even before a single local observer detects the correlation, then measurement outcomes are governed by non-local laws.
- Avoids talk about "hidden variables".
- Not depend on determinism.

- Perceptible is determined by microscopic positions $\vec{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_n)$ of apparatus particles.
$\Rightarrow$ The **simplest** possibility is that all $\vec{x}$ are ontic.
- But there is no strict border between perceptible/non-perceptible.
$\Rightarrow$ The **simplest** possibility is that positions $\vec{y}$ of all the rest are also ontic.

We have derived the QM Born rule in arbitrary k-space from the Born rule in position space.
$\Rightarrow$ **Any** theory for which

$$\rho(\vec{x}, \vec{y}; t) = |\Psi(\vec{x}, \vec{y}, t)|^2$$

has the same measurable predictions as QM.

So far we found motivation for two requirements:
1) perceptibles are ontic (common sense)
2) $\rho(\vec{x},\vec{y};t) = |\Psi(\vec{x},\vec{y},t)|^2$ (QM)

- A simple theory that satisfies both requirements is:
All positions $\vec{q} = (\vec{x},\vec{y})$ are **ontic** and **random**.
- Almost like standard QM, except that $\vec{q}$ are ontic.
- However, such theory does **not explain** Born rule for $\vec{q}$.
- The Born rule for $\vec{q}$ is **postulated**.

Can we **explain** the Born rule for $\vec{q}$?
- $\vec{q}$ is ontic $\Rightarrow$ it has a value $\vec{Q}(t)$ at each time t.
- In principle $\vec{Q}(t)$ could be stochastic (not deterministic).
- However, $\vec{Q}(t)$ must be compatible with $\rho(\vec{q};t) = |\Psi(\vec{q},t)|^2$, which is a deterministic function of t.
$\Rightarrow$ Suggests (not proves) that $\vec{Q}(t)$ could be deterministic too.

Construction of BM

- How can a deterministic law for $\vec{Q}(t)$ be compatible with probability $\rho(\vec{q};t) = |\Psi(\vec{q},t)|^2$?
- The condition is that $\vec{Q}(t)$ is determined by a law of the form

$$\frac{d\vec{Q}(t)}{dt} = \vec{v}(\vec{Q}(t),t)$$

where $\vec{v}(\vec{q},t)$ is a function that satisfies the continuity equation

$$\boxed{\frac{\partial|\Psi|^2}{\partial t} + \vec{\nabla}(|\Psi|^2\vec{v}) = 0}$$

- If $\rho(\vec{q};t_0) = |\Psi(\vec{q},t_0)|^2$ for initial t_0,
then continuity equation $\Rightarrow$ $\rho(\vec{q};t) = |\Psi(\vec{q},t)|^2$ for $\forall t$.

- Continuity equation analogous to Liouville equation
in classical statistical mechanics.
$\Rightarrow$ $\rho(\vec{q};t) = |\Psi(\vec{q},t)|^2$ is quantum equilibrium, can be explained
even without assuming initial $\rho(\vec{q};t_0) = |\Psi(\vec{q},t_0)|^2$.

- Is there such $\vec{v} = (\mathbf{v}_1, \ldots, \mathbf{v}_N)$? ($N$ = number of particles)

- In non-relativistic QM it is well-known that Schrödinger equation itself implies a continuity equation of that form, with

$$\mathbf{v}_a = \frac{-i\hbar}{2m_a} \frac{\Psi^* \overleftrightarrow{\boldsymbol{\nabla}}_a \Psi}{\Psi^* \Psi} = \frac{\mathrm{Re}(\Psi^* \hat{\mathbf{v}}_a \Psi)}{\Psi^* \Psi}$$

$\hat{\mathbf{v}}_a = \hat{\mathbf{p}}_a / m_a$ = velocity operator
$\hat{\mathbf{p}}_a = -i\hbar \boldsymbol{\nabla}_a$ = momentum operator

Spin: $\Psi^* \cdots \Psi \to \Psi^\dagger \cdots \Psi = \sum_\alpha \Psi^*_\alpha \cdots \Psi_\alpha$
(sum over all spin indices)

$\Rightarrow$ BM works for non-relativistic QM.

Non-locality:
- Velocity $\mathbf{v}_a(\mathbf{x}_1, \cdots, \mathbf{x}_n)$ of one particle depends on positions of all other particles, at the same time.
- Consequence of entanglement, in agreement with the Bell theorem.
- Not Lorentz covariant.

Wave function of the measured subsystem

Full wave function: $\Psi(\vec{x},\vec{y},t)$ - satisfies Schrödinger equation
$\vec{x}$ = positions of the apparatus & environment particles
$\vec{y}$ = positions of the measured microscopic subsystem

Conditional wave function of the subsystem:

$$\psi_c(\vec{y},t) = \frac{\Psi(\vec{X}(t),\vec{y},t)}{N(t)}$$

where $N(t) = \sqrt{\int d\vec{y}\,|\Psi(\vec{X}(t),\vec{y},t)|^2}$.
- can be defined only in Bohmian mechanics
- does not always satisfy Schrödinger equation

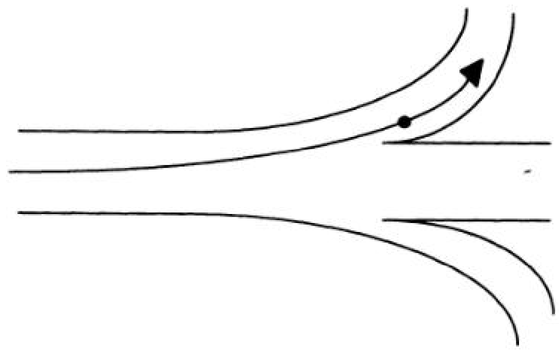
$\psi_c(\vec{y}, t)$ explains the **effective** wave-function collapse:

- After measurement

$$\Psi(\vec{x}, \vec{y}, t) = c_1 \psi_1(\vec{y}, t) \Phi_1(\vec{x}, t) + c_2 \psi_2(\vec{y}, t) \Phi_2(\vec{x}, t) + \ldots$$

where $\Phi_1(\vec{x}, t) \Phi_2(\vec{x}, t) \simeq 0, \ldots$

- Suppose that apparatus particles $\vec{X}(t)$ entered $\Phi_1(\vec{x}, t)$:



$\Rightarrow$ $\Phi_2(\vec{X}(t), t) \simeq 0$, $\Phi_3(\vec{X}(t), t) \simeq 0$, $\ldots$ so

$$\psi_c(\vec{y}, t) \simeq e^{i\varphi_1(t)} \psi_1(\vec{y}, t), \quad e^{i\varphi_1(t)} = \frac{\Phi_1(\vec{X}(t), t)}{|\Phi_1(\vec{X}(t), t)|}$$

$\Rightarrow$ The effective wave function is $\psi_1(\vec{y}, t)$
(wave function that determines trajectories $\vec{Y}(t)$)

- This is the effective wave function collapse (without a true collapse).

That's how BM solves the measurement problem!

The origin of uncertainty

- BM is deterministic, so why can't it make deterministic predictions of measurement outcomes?
- Because of quantum equilibrium.

Analogy with thermal equilibrium:
- In full thermal equilibrium, macroscopic changes can only happen due to rare statistical fluctuations.
- Thermodynamics makes deterministic predictions of macroscopic changes only when the full system is **not** in thermal equilibrium.

- Equilibrium does not need an explanation.
- It's the **absence** of equilibrium that needs explanation (still not clear why is Universe not in thermal equilibrium).

- Why can't BM trajectories be directly observed?
- Because there are no local interactions between BM particles.

Analogy with gravity:
- That's like trying to observe Moon's trajectory by watching tides.
- Gravity is a long range interaction $\Rightarrow$ observation of effect on B caused by A does not directly reveal the position of A.
- That's why there is no direct evidence for astrophysical dark matter (hypothetic matter with negligible interactions, except gravitational).

$\Rightarrow$ The absence of direct evidence for BM trajectories analogous to the absence of direct evidence for dark matter.

Forthcoming talks:

Quantum Foundations V:
Relativistic QFT from a Bohmian perspective: A proof of concept

Quantum Foundations VI:
Suggestions welcome