QUANTUM STATE:

REALITY OR MERE PROBABILITY?

How PBR theorem elevated this question

to a higher level

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Outline:

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2. (Not only scientific) history of the problem

3. PBR theorem: Main ideas

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1. INTRODUCTION

What is the meaning of the quantum state $|\psi\rangle$?
- Is it an objective property of a *single* system? (ψ-ontology)
- Or is it only a tool to calculate *probability*? (ψ-epistemology)
- Or is it both?
- What does it even *mean* that a property is "objective"? (precise definition)

- Many brilliant physicists (Bohr, Einstein, ...) tried to answer, but a generally accepted answer has not been produced.

- Discussions of such questions have a bad reputation (on the borderline between science and philosophy).

- We need a *clever reformulation* of the problem, to make it more scientific and less philosophical.

- The PBR theorem (2012) is an important step in that direction.

What is PBR theorem?

The theorem proved by M.F. Pusey, J. Barrett, T. Rudolph, in Nature Phys. **8**, 476 (2012); arXiv:1111.3328 (v3).

Consists of

1. Mathematical definition of the difference between "ontological" and "epistemological".
2. Technical proof (with the aid some auxiliary assumptions) QM $\Rightarrow$ $|\psi\rangle$ is ontological (objectively real)!

Conclusion also confirmed by a recent experiment:
D. Nigg *et al*, arXiv:1211.0942.
- confirms QM, and thus reality of $|\psi\rangle$.

2. (NOT ONLY SCIENTIFIC) HISTORY OF THE PROBLEM

- Einstein 1926: "God does not play dice."
- But later (contrary to wide misbelief) Einstein did *not* insist on determinism.
- He insisted on *reality*:

Physical systems *have* properties even when we don't measure them.

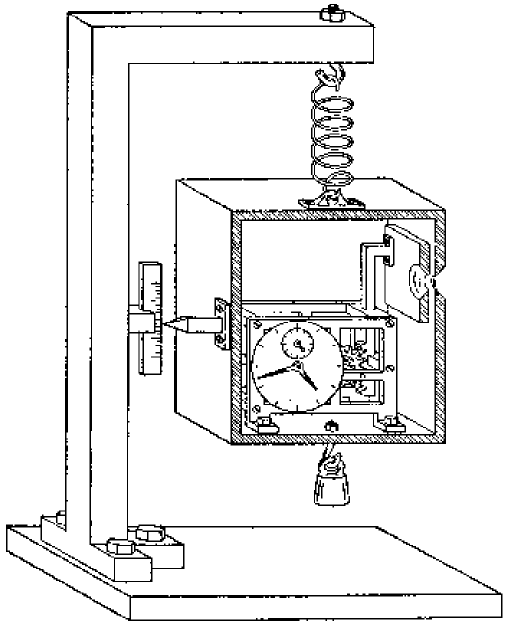
This is in contrast with Copenhagen view of uncertainty relations:
In momentum-eigenstate $|p\rangle$, the position x is not merely *unknown*; instead, the particle does not even *have* position x.

If it looks too philosophical ...

... to argue that he is right, Einstein proposed many thought experiments.

Two most famous ones:

- Einstein photon-in-a-box paradox (1930)



- Einstein-Podolsky-Rosen paradox (1935)

(Recently realized that they are equivalent: Dieks and Lam 2008, Nikolić 2012.)

Breakthrough by Bell theorem (1964):
If reality λ exists (whatever it is), it must necessarily be *non-local*.

- This elevated the problem to a higher scientific level.
- However, it does not tell whether λ exists, nor what it is.

Different interpretations of QM suggest different answers:

1. Copenhagen-collapse interpretation (von Neumann 1932):
- $\lambda = \psi$
- measurement causes collapse of ψ (not explained why?)
- the collapse is non-local

2. Copenhagen-information interpretation
- λ does not exist
- ψ is not real, only a tool to calculate probability
- collapse is just our update of knowledge

3. Copenhagen-Bohr interpretation
- ψ is real, but only on the microscopic level
- there is a fundamental border between micro and macro
- not specified where that border is?

4. Copenhagen-pragmatic interpretation
- shut up and calculate
- most popular among practical physicists

5. Bohm interpretation (1952)
- $\lambda = (\psi, x)$ (both ψ and particle position x separately exist)
- trajectory $x(t)$ guided by ψ
- ψ does not collapse
- $x(t)$ satisfy non-local equations of motion

6. Many-world interpretation (Everett, 1957)
- $\lambda = \psi$
- ψ does not collapse
- all branches of ψ are real

7. Statistical ensemble interpretation (Ballentine, 1970):
- ψ is not real, i.e., not a property of an individual system
- ψ is a property of a statistical ensemble of many
  similarly prepared systems
- λ might exist, but does not tell what it is

How to know who is right?

3. PBR THEOREM: MAIN IDEAS

New breakthrough (similar to the Bell theorem):

PBR theorem (2012)

- The most serious attempt so far to actually *prove* that

ψ is real

(Note, however, that the theorem does *not* say that
ψ is fundamental, or that ψ is the only reality.)

- But what does it even *mean* to prove that "something is real"?
- We need to *define* the meaning of the words

"something is real"

in a *precise* way, such that a rigorous theorem is possible.

Example from classical probability - coin flipping:

To make it non-trivial, assume *unfair* coin flipping

$$p(\text{head}) \neq p(\text{tail})$$

- Are $p(\text{head})$ and $p(\text{tail})$ *intrinsic* properties of a *single* coin?
- If they are, we shall say that $p(\text{head})$ and $p(\text{tail})$ are *real* (objective) properties of the coin.

Two possibilities:

1. Unfair *coin*
- $p(\text{head}) \neq p(\text{tail})$ because the distribution of the coin-mass is not uniform
- this is a property of the *coin itself*
- from the knowledge of $\lambda = \text{mass distribution}$
  $\Rightarrow p(\text{head}), p(\text{tail})$ can be determined *uniquely*

2. Unfair *flipping*
- $p(\text{head}) \neq p(\text{tail})$ because the act of *flipping is unfair*
- this is *not* a property of the coin
- from the knowledge of $\lambda = \text{mass distribution}$
  $\Rightarrow p(\text{head}), p(\text{tail})$ can *not* be determined uniquely

This motivates the general *definition*:

A probability distribution $\mu(\lambda)$ is *ontic*
(i.e., corresponds to something real)
iff it can be determined uniquely from the fundamental λ.

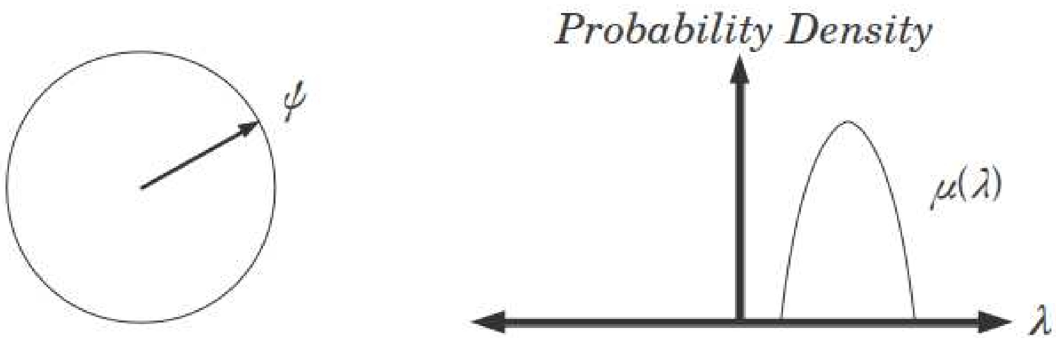
Otherwise, $\mu(\lambda)$ is called *epistemic*.

Now apply to QM:
- QM is an unfair game (not all probabilities are equal).
- Is QM an unfair "coin" or an unfair "flipping"?

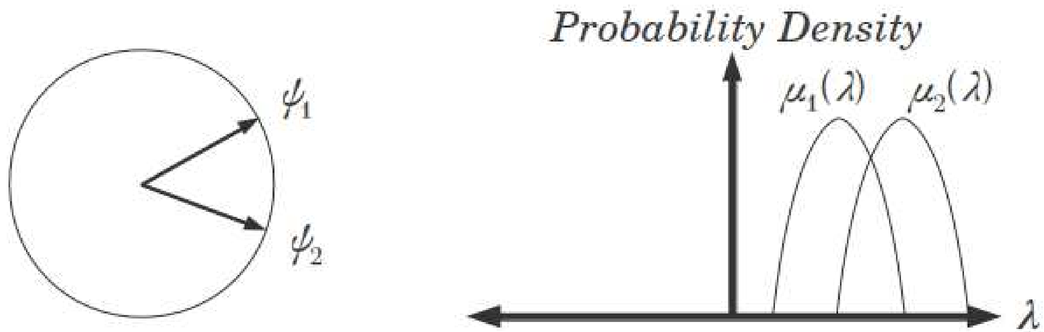
Analogy: coin flipping $\leftrightarrow$ QM
set $\{\text{head}, \text{tail}\}$ $\leftrightarrow$ set of all different states in the Hilbert space $\{|\psi\rangle\}$

($|\psi\rangle$ is different from $|\psi'\rangle$ iff $|\psi'\rangle \neq c|\psi\rangle$.)

- In general, experimentalists do *not* have a full control over all fundamental degrees of freedom λ.
$\Rightarrow$ When they prepare $|\psi\rangle$ in the laboratory, this actually means that they have prepared some *probability distribution* $\mu(\lambda)$:



Assume that for states $|\psi_1\rangle$, $|\psi_2\rangle$ their λ-distributions overlap:



- For $\lambda \in$ overlap, one cannot know whether λ belongs to $\mu_1(\lambda)$ or $\mu_2(\lambda)$

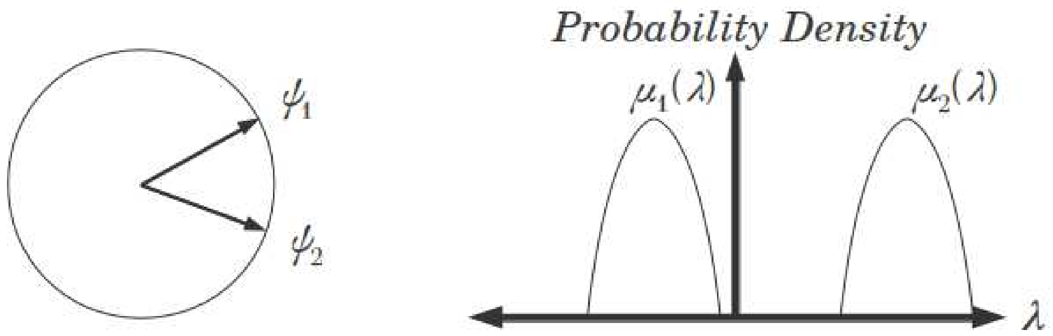
$\Rightarrow$ λ does not uniquely determine $\mu(\lambda)$

$\Rightarrow$ (by definition) $\mu(\lambda)$ is not ontic

$\Rightarrow$ $|\psi\rangle$ is not ontic

$\Rightarrow$ To prove that $|\psi\rangle$ is *not* ontic, it is sufficient to prove that there is *at least one pair* $|\psi_1\rangle$, $|\psi_2\rangle$ ($|\psi_1\rangle \neq c|\psi_2\rangle$) for which $\mu_1(\lambda)$ and $\mu_2(\lambda)$ do overlap.

$\Rightarrow$ The converse (that $|\psi\rangle$ is ontic)
is much more difficult to prove:
- One needs to prove that for *any pair* $|\psi_1\rangle$, $|\psi_2\rangle$ ($|\psi_1\rangle \neq c|\psi_2\rangle$)
  the overlap does not exist:



- Yet, the PBR theorem proves exactly this!

This is not only difficult to prove (sketch in the next section), but also very surprising:

- The absence of overlap $\mu_1(\lambda)\mu_2(\lambda) = 0 \quad \forall\, \lambda$
is not surprising when $\langle\psi_1|\psi_2\rangle = 0$.

- What is surprising is that $\mu_1(\lambda)\mu_2(\lambda) = 0 \quad \forall\, \lambda$
even when $\langle\psi_1|\psi_2\rangle \neq 0$.

- Why is that surprising? Because

$$\begin{aligned}
& && \langle\psi_1|\psi_2\rangle \neq 0 \\
\left[1 = \int da|a\rangle\langle a|\right] &\;\Rightarrow && \int da\langle\psi_1|a\rangle\langle a|\psi_2\rangle \neq 0 \\
&\;\Rightarrow && \langle\psi_1|a\rangle\langle a|\psi_2\rangle \neq 0 \;\text{ for some }\; a \\
\left[\rho_i(a) = |\langle a|\psi_i\rangle|^2\right] &\;\Rightarrow && \rho_1(a)\rho_2(a) \neq 0 \;\text{ for some }\; a
\end{aligned}$$

and yet $\mu_1(\lambda)\mu_2(\lambda) = 0\ \forall\lambda$.

- In other words, QM-distributions overlap,
but corresponding λ-distributions do not overlap!

4. PBR THEOREM:
SKETCH OF THE PROOF

- We prove the absence of overlap
for a simple example of a pair of non-orthogonal states.
(PBR generalize it to an arbitrary pair.)

2-dimensional Hilbert space with orthogonal basis $|0\rangle$, $|1\rangle$.
Another orthogonal basis $|+\rangle$, $|-\rangle$

$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

Non-orthogonal pair $|0\rangle$, $|+\rangle$:

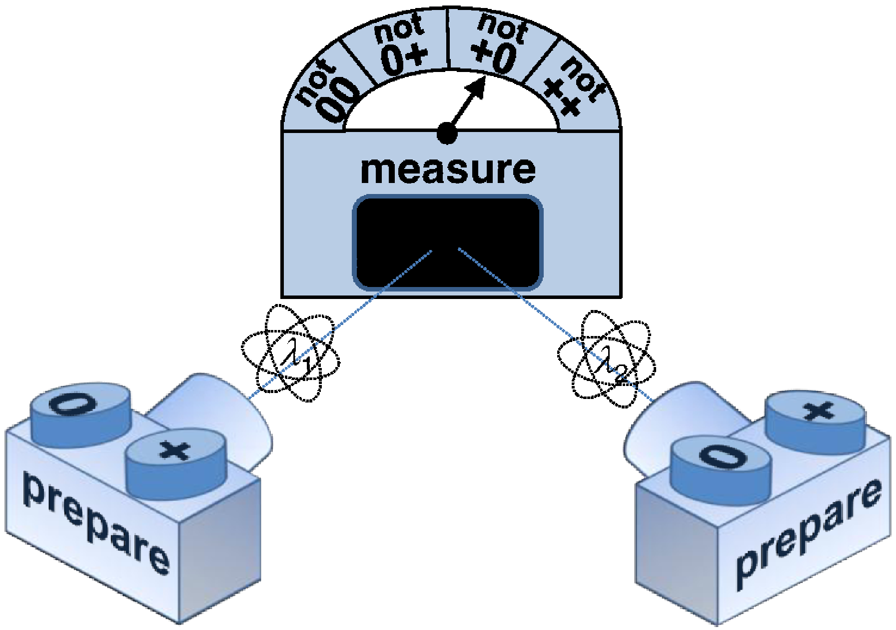
$$\langle 0|+\rangle = 1/\sqrt{2}$$

Goal: prove that $\mu_0(\lambda)\mu_+(\lambda) = 0 \ \forall\lambda$
Strategy: assume the opposite and derive a contradiction!

- assume overlap $\Rightarrow$ finite probability p that $\lambda \in$ overlap

Consider two similar systems:

- each prepared either in $|0\rangle$ or $|+\rangle$
  (but experimentalist does not know in which one it is prepared)



- probability of overlap in each is p
- assume they are statistically independent

$\Rightarrow$ probability of overlap in both is $P_{\text{joint}} = p \cdot p > 0$

$\Rightarrow$ Consequence of the assumed overlap:

There is a probability $P_{\text{joint}} > 0$ that
the outcome will be consistent with *all four*
possibilities for the initial preparation
($|0\rangle|0\rangle$, $|0\rangle|+\rangle$, $|+\rangle|0\rangle$, and $|+\rangle|+\rangle$)

Now compare it with predictions of QM:
- Measure the joint system in a specially chosen complete orthogonal basis:

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}[|0\rangle|1\rangle + |1\rangle|0\rangle], \qquad |\phi_2\rangle = \frac{1}{\sqrt{2}}[|0\rangle|-\rangle + |1\rangle|+\rangle],$$
$$|\phi_3\rangle = \frac{1}{\sqrt{2}}[|+\rangle|1\rangle + |-\rangle|0\rangle], \qquad |\phi_4\rangle = \frac{1}{\sqrt{2}}[|+\rangle|-\rangle + |-\rangle|+\rangle]$$

- This basis has the property (notation: $|ab\rangle \equiv |a\rangle|b\rangle$)

$$\langle\phi_1|00\rangle = 0, \quad \langle\phi_2|0+\rangle = 0, \quad \langle\phi_3|+0\rangle = 0, \quad \langle\phi_4|++\rangle = 0$$

$\Rightarrow$ Whatever the outcome of a single measurement will be ($|\phi_1\rangle$, $|\phi_2\rangle$, $|\phi_3\rangle$, or $|\phi_4\rangle$), it is *certain* that it will *eliminate* one of the possibilities ($|00\rangle$, $|0+\rangle$, $|+0\rangle$, or $|++\rangle$).
$\Rightarrow$ $P_{\text{joint}} = 0$
$\Rightarrow$ Contradiction with the Consequence above! *Q.E.D.*

5. DISCUSSION AND CONCLUSION

Explicit assumptions of the theorem:
- some fundamental reality λ exists
- separately prepared λ's are statistically independent
- statistical predictions of QM are correct
- definition of reality based on non-overlapping probability distributions

PBR theorem: under these assumptions, $|\psi\rangle$ is real!
(In other words, whatever the fundamental λ is,
if λ is given, then $|\psi\rangle$ can be determined *uniquely*.)

The theorem stimulated a lot of further research:

- further clarifications, refinements, and simplifications of the theorem
- e.g., a proof without using many copies of the system (Hardy, 2012)

Critiques of
- explicit assumptions
- implicit assumptions (not spelled out in the theorem)
- definition of reality
- physical relevance

Conclusion:
- The PBR theorem provided a strong argument (if not the ultimate proof) that $|\psi\rangle$ is real.
- It elevated the question of reality of $|\psi\rangle$ to a higher scientific level.

Thank You!