Bohmian mechanics for instrumentalists

Hrvoje Nikolić

Rudjer Bošković Institute, Zagreb, Croatia

Long version, 2018

Outline:

Part 1. Philosophy

Part 2. Quantum theory of perceptibles

Part 3. Bohmian mechanics

Part 4. Beyond relativistic QFT

Part 1.
PHILOSOPHY

4 basic notions in philosophy of physics

Ontology:
- Things which are supposed to be there irrespective of (human) observations.

Determinism:
- Assumption that future is completely determined by the past (at least in principle, as e.g. in deterministic chaos).
- Says that fundamental laws of physics are not probabilistic.

Instrumentalism:
- The main goal of theoretical physics is to predict (and control) the macroscopic phenomena, especially the outcomes of scientific instruments.
- Most physicists are (at least partly) instrumentalists.

Instrumental interpretation of quantum mechanics (QM):
- Not deterministic (prescribes only probabilities).
- Says nothing about ontology.

(Does particle have a position before one measures it?)

The trouble with Bohmian mechanics

Bohmian mechanics (BM):

- Postulates that quantum particles are pointlike objects
with deterministic trajectories.
- Usually motivated by the goal of prescribing fundamental microscopic ontology. (Particle has a position even if one doesn't measure it.)
- Determinism is **not** the main goal of BM, it's only a byproduct!

- Typical instrumentalists don't care about ontology.
- "Ontology is not physics, it's metaphysics."

$\Rightarrow$ Instrumentalists don't find BM intuitive and well motivated.
$\Rightarrow$ BM is widely ignored or misunderstood in a wider physics community.

- The goal of this talk is to **reformulate** BM
such that it looks better motivated and more intuitive
to a wider physics community, especially instrumentalists.

3 funny "ble" nouns in QM

Observable:
- In QM it is a noun (in normal English it is an adjective).
- Hermitian operator in the Hilbert space.
- Related to (but not identical with) a measurable quantity.
- *"Quantum phenomena do not occur in a Hilbert space, they occur in a laboratory."* - Asher Peres (an instrumentalist)

Beable:
- Word coined by John Bell.
- Same as ontology: stuff which is there irrespective of measurement.
- This concept is central to Bohmians, but not to instrumentalists.

With a goal to make BM more meaningful to instrumentalists,
I introduce a new concept:

Perceptible:
- In physics it is a noun (in normal English it is an adjective).
- A thing or phenomenon amenable to **direct** human perception.
- Perceptibles: tables, chairs, Moon, macroscopic instrument, click in detector, picture of atom produced by electron microscope, ...
- Non-perceptibles: wave function, electron, photon, atom, ...

More on perceptibles

- The distinction between perceptibles and non-perceptibles is similar to the distinction between macroscopic and microscopic.
- All microscopic entities are non-perceptibles.
- However, a macroscopic entity does not necessarily need to be a perceptible (e.g. gravitational field, radio wave).
- Non-perceptibles are **theoretical constructs** that explain and predict properties of perceptibles, e.g.

perceptible	**explained by non-perceptible**
click in the detector	photon
picture by electron microscope	atom
falling apple	gravitational field
music from the radio	radio wave

- **To make a measurable prediction means to predict a property of a perceptible.**

- Just because a non-perceptible is a theoretical construct doesn't necessarily mean that it is not a beable.
- Beable is a theoretical construct itself:
The claim that something is a beable really means that it is beable **in a given theory**.
- For instance, point-like particles are beables in classical mechanics, but not in classical field theory (classical electrodynamics).
- It is impossible to know whether a non-perceptible beable "really" exists.
- However, it may be useful to **imagine** that it "really" exists because that sometimes helps in cognitive processes (intuition).

- Like with macroscopic/microscopic, there is no strict border between perceptible/non-perceptible.
- Is a one cell microorganism macroscopic or microscopic?
- Is perception of one cell microorganism by optical microscope direct or indirect? (It must be direct to be called perceptible.)
- Even though there is no strict border, the concepts are useful.

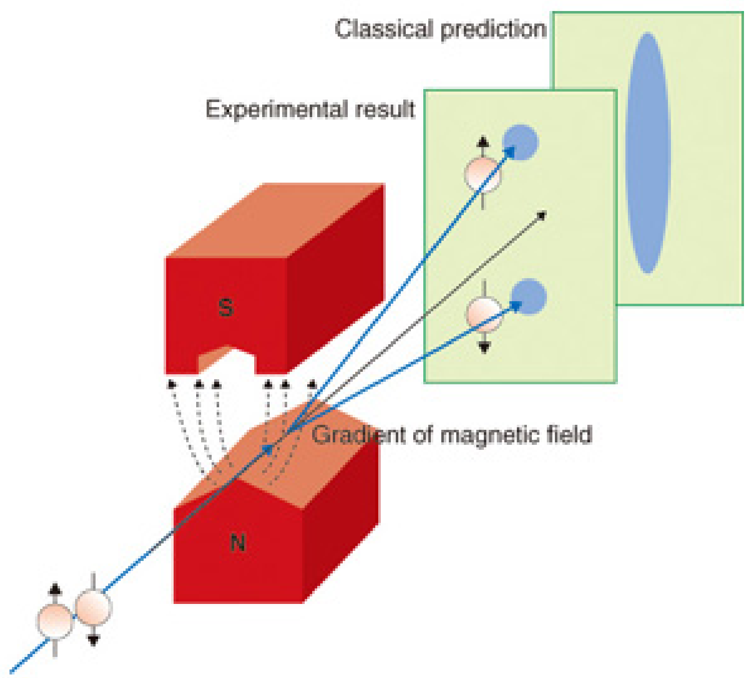
Part 2.

QUANTUM THEORY OF PERCEPTIBLES

All perceptibles can be reduced to macroscopic positions

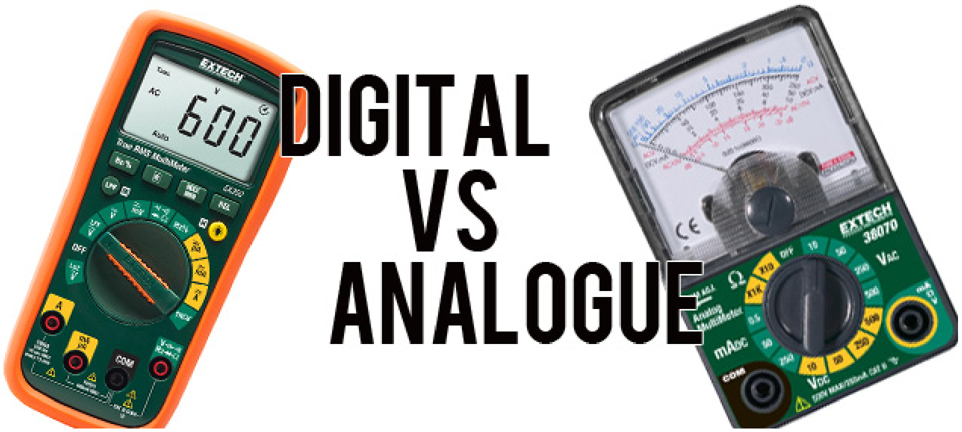
- All perceptibles are macroscopic, which means big in position space.
$\Rightarrow$ When 2 perceptibles can be distinguished, it means that they can be distinguished by macroscopic positions of something.

Measurement of spin:
- Spin is an observable, but not a perceptible.
- Spin is measured by Stern-Gerlach apparatus.
- Perceptible is a big dark spot on the screen.



More sophisticated instruments:

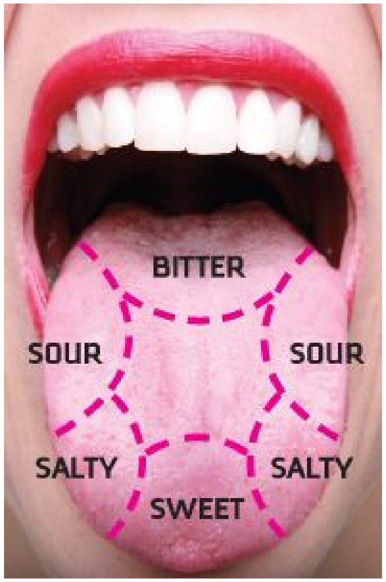
- Analog: position of macro pointer.
- Digital: positions of lines that make a digit.



Click in the detector:

- Sound determined by macroscopic oscillations
(e.g. membrane of the speaker).
- This oscillation is a macro position as a function of time.

What about senses such as color, taste and smell?
- Created in the eye, tongue or nose (and interpreted by brain).
- Determined by **which** nerve is stimulated.
- One can fool the brain: e.g. electro-stimulation
of the sweet nerve creates the illusion of sweetness.
- Different nerves have different macro positions
(most pronounced in the tongue).



⇒ Senses are perceptibles too, determined by macro positions of nerves.

The origin of Born rule in QM

- In any basis $\{|k\rangle\}$, the Born rule **postulates** probability

$$p_k = |\langle k|\psi\rangle|^2$$

- However it is not necessary to postulate it.
- We **derive** it from the Born rule in the position-space only.

- We need probabilities of perceptibles
(e.g. probability that detector will click).
$\Rightarrow$ Probabilities of perceptibles must be computed in the position space.
- However, there is no strict border between
perceptible and non-perceptible.
$\Rightarrow$ Compute **all** probabilities in the position space.

- Measure observable $\widehat{K}$ with eigenstates $|k\rangle$.
- Macroscopic apparatus (the perceptible) can be described by its quantum microscopic state $|\Phi\rangle$.
- Initial microscopic state of the apparatus $|\Phi_0\rangle$.
- Interaction $\Rightarrow$ unitary transition

$$|k\rangle|\Phi_0\rangle \to |k'\rangle|\Phi_k\rangle$$

$\Rightarrow$ Wave functions have a negligible overlap in multi-position space

$$\Phi_{k_1}(\vec{x})\Phi_{k_2}(\vec{x}) \simeq 0 \quad \text{for} \quad k_1 \neq k_2$$

where $\Phi_k(\vec{x}) \equiv \langle\vec{x}|\Phi_k\rangle$,

$$\vec{x} \equiv (\mathbf{x}_1, \ldots, \mathbf{x}_n)$$

n = number of particles constituting the apparatus

$$\int d\vec{x}\,|\Phi_k(\vec{x})|^2 = 1, \quad d\vec{x} \equiv d^{3n}x$$

- For a superposition $|\psi\rangle = \sum_k c_k|k\rangle$:

$$|\psi\rangle|\Phi_0\rangle \to \sum_k c_k|k'\rangle|\Phi_k\rangle$$

- A more realistic analysis includes also environment

$$|\psi\rangle|\Phi_0\rangle|E_0\rangle \to \sum_k c_k|k'\rangle|\Phi_k\rangle|E_k\rangle \equiv |\Psi\rangle$$

$$\Rightarrow \boxed{|\Psi\rangle = \sum_k c_k|\Phi_k\rangle|R_k\rangle,} \quad |R_k\rangle \equiv |k'\rangle|E_k\rangle$$

- $|\Phi_k\rangle$ describes the perceptible, $|R_k\rangle$ all the rest.

Multi-position representation

$$\Psi(\vec{x},\vec{y}) = \sum_k c_k \Phi_k(\vec{x}) R_k(\vec{y})$$

- Born rule in the multi-position space

$$\rho(\vec{x},\vec{y}) = |\Psi(\vec{x},\vec{y})|^2 \simeq \sum_k |c_k|^2 |\Phi_k(\vec{x})|^2 |R_k(\vec{y})|^2$$

$$\Rightarrow \rho^{(\text{appar})}(\vec{x}) = \int d\vec{y}\, \rho(\vec{x},\vec{y}) \simeq \sum_k |c_k|^2 |\Phi_k(\vec{x})|^2$$

$\Rightarrow$ Probability to find the apparatus particles in the support of $\Phi_k(\vec{x})$:

$$\boxed{p_k^{(\text{appar})} = \int_{\text{supp }\Phi_k} d\vec{x}\, \rho^{(\text{appar})}(\vec{x}) \simeq |c_k|^2}$$

- This coincides with the Born rule in arbitrary k-space. *Q.E.D.*

Generalized measurements

The **master formula** of quantum measurement:

$$\boxed{|\Psi\rangle = \sum_k c_k |\Phi_k\rangle |R_k\rangle}$$

- Not to be confused with master *equation* in quantum decoherence.
- $|\Phi_k\rangle$ micro state of the perceptible, $|R_k\rangle$ the rest.
- In derivation on the previous page, the label k had double meaning:

1) Eigenstates $|k\rangle$ of observable $\hat{K}$ with non-degenerate spectrum.
2) Label of distinct perceptibles.

In general, 1) is not true:

- Degenerate spectrum, photon position, measurement of time, ...
- Generalized measurements described by POVM formalism.
- Neumark theorem: any POVM can be reduced

to projective measurement in a **larger** Hilbert space.

$\Rightarrow$ The master formula with 2) is true for **any** measurement.

$$\Rightarrow \boxed{p_k^{\text{(appar)}} \simeq |c_k|^2}$$

always true if $\rho(\vec{x}, \vec{y}; t) = |\Psi(\vec{x}, \vec{y}, t)|^2$.

Part 3.
Bohmian mechanics

Motivation for BM

The main axiom for BM:

All perceptibles are beables.

- E.g. the Moon is there even if nobody observes it.
- Motivated by common sense.
- The opposite would be that the Moon is only in our mind.
- Impossible to prove or disprove by scientific method.
- It's only a thinking tool (hard to think the opposite).

Most of the motivation for BM arises from this common sense axiom!

Bell theorem expressed in the language of perceptibles:

If perceptibles are beables, then perceptibles are non-local.

- If the correlated, yet spatially separated, measurement outcomes are there even before a single local observer detects the correlation, then measurement outcomes are governed by non-local laws.
- Avoids talk about "hidden variables".
- Not depend on determinism.

- Perceptible is determined by microscopic positions $\vec{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_n)$ of apparatus particles.
$\Rightarrow$ The **simplest** possibility is that all $\vec{x}$ are beables.
- But there is no strict border between perceptible/non-perceptible.
$\Rightarrow$ The **simplest** possibility is that positions $\vec{y}$ of all the rest are also beables.

We have derived the QM Born rule in arbitrary k-space from the Born rule in position space.
$\Rightarrow$ **Any** theory for which

$$\rho(\vec{x}, \vec{y}; t) = |\Psi(\vec{x}, \vec{y}, t)|^2$$

has the same measurable predictions as QM.

Valid even for generalized measurements, e.g. measurement of time:
- There is no time operator $\hat{K} = \hat{T}$ with eigenstates $|k\rangle = |t\rangle$.
- Not problem because in the master formula

$$|\Psi\rangle = \sum_k c_k |\Phi_k\rangle |R_k\rangle$$

k labels distinct positions of the clock pointer.

So far we found motivation for two requirements:

1) perceptibles are beables (common sense)

2) $\rho(\vec{x},\vec{y};t) = |\Psi(\vec{x},\vec{y},t)|^2$ (QM)

- A simple theory that satisfies both requirements is:

All positions $\vec{q} = (\vec{x},\vec{y})$ are **beables** and **random**.

- Almost like standard QM, except that $\vec{q}$ are beables.
- However, such theory does **not explain** Born rule for $\vec{q}$.
- The Born rule for $\vec{q}$ is **postulated**.

Can we **explain** the Born rule for $\vec{q}$?

- $\vec{q}$ is beable $\Rightarrow$ it has a value $\vec{Q}(t)$ at each time t.
- In principle $\vec{Q}(t)$ could be stochastic (not deterministic).
- However, $\vec{Q}(t)$ must be compatible with $\rho(\vec{q};t) = |\Psi(\vec{q},t)|^2$,

which is a deterministic function of t.

$\Rightarrow$ Suggests (not proves) that $\vec{Q}(t)$ could be deterministic too.

Construction of BM

- How can a deterministic law for $\vec{Q}(t)$ be compatible with probability $\rho(\vec{q};t) = |\Psi(\vec{q},t)|^2$?
- The condition is that $\vec{Q}(t)$ is determined by a law of the form

$$\frac{d\vec{Q}(t)}{dt} = \vec{v}(\vec{Q}(t),t)$$

where $\vec{v}(\vec{q},t)$ is a function that satisfies the continuity equation

$$\boxed{\frac{\partial|\Psi|^2}{\partial t} + \vec{\nabla}(|\Psi|^2\vec{v}) = 0}$$

- If $\rho(\vec{q};t_0) = |\Psi(\vec{q},t_0)|^2$ for initial t_0,
then continuity equation $\Rightarrow$ $\rho(\vec{q};t) = |\Psi(\vec{q},t)|^2$ for $\forall t$.

- Continuity equation analogous to Liouville equation in classical statistical mechanics.
$\Rightarrow$ $\rho(\vec{q};t) = |\Psi(\vec{q},t)|^2$ is quantum equilibrium, can be explained even without assuming initial $\rho(\vec{q};t_0) = |\Psi(\vec{q},t_0)|^2$.
- Two approaches: typicality and H-theorem.
- Review: T. Norsen, Entropy **20**, 422 (2018).

- Is there such $\vec{v} = (\mathbf{v}_1, \ldots, \mathbf{v}_N)$? ($N$ = number of particles)

- In non-relativistic QM it is well-known that Schrödinger equation itself implies a continuity equation of that form, with

$$\mathbf{v}_a = \frac{-i\hbar}{2m_a} \frac{\Psi^* \overleftrightarrow{\boldsymbol{\nabla}}_a \Psi}{\Psi^* \Psi} = \frac{\mathrm{Re}(\Psi^* \hat{\mathbf{v}}_a \Psi)}{\Psi^* \Psi}$$

$\hat{\mathbf{v}}_a = \hat{\mathbf{p}}_a / m_a$ = velocity operator
$\hat{\mathbf{p}}_a = -i\hbar \boldsymbol{\nabla}_a$ = momentum operator

Spin: $\Psi^* \cdots \Psi \to \Psi^\dagger \cdots \Psi = \sum_\alpha \Psi_\alpha^* \cdots \Psi_\alpha$
(sum over all spin indices)

$\Rightarrow$ BM works for non-relativistic QM.

Robustness of long distance physics

A general rule in physics:

The laws of long distance physics do not depend on details of small distance physics.

Examples:
- Fluid mechanics and thermodynamics do not depend on details of atomic physics.
- Atomic physics does not depend on details of nuclear physics.
- Nuclear physics does not depend on details of quarks.
- QCD (quarks and gluons) ... of string theory.

Formalized more generally by Wilson renormalization theory:
- Long distance physics obtained from microscopic theory by **integrating out** small distance degrees of freedom.

Robustness of measurable predictions by BM

- Similarly, perceptibles in BM do not depend on details of particle trajectories.
- Recall: probability of perceptible obtained by **integrating out** over all microscopic positions:

$$p_k^{(\text{appar})} = \int_{\text{supp } \Phi_k} d\vec{x} \int d\vec{y} \, |\Psi(\vec{x}, \vec{y})|^2$$

- That's why BM (with trajectories) makes the same measurable predictions as standard QM (without trajectories).

How to make a **false** "prediction" of BM that differs from standard QM?
- By putting too much emphasis on trajectories and ignoring the perceptibles!
- A lot of wrong "disproofs of BM" of that kind are published.

Even distinguished Bohmians sometimes fall into this trap:
- By computing arrival time of microscopic BM trajectories (microscopic trajectories are not perceptibles).
- By computing gravitational field $g_{\mu\nu}(\mathbf{x}, t)$ in Bohmian quantum cosmology (gravitational field is macroscopic, but not a perceptible).

- BM is deterministic, so why can't it make deterministic predictions of measurement outcomes?
- Because of quantum equilibrium - analogous to thermal equilibrium.
- In full thermal equilibrium, macroscopic changes can only happen due to rare statistical fluctuations.
- Thermodynamics makes deterministic predictions of macroscopic changes only when the full system is **not** in thermal equilibrium.
- Equilibrium does not need an explanation.
- It's the **absence** of equilibrium that needs explanation (still not clear why is Universe not in thermal equilibrium).

- Why can't BM trajectories be directly observed?
- Because there are no local interactions between BM particles.
- That's like trying to observe Moon's trajectory by watching tides.
- Gravity is a long range interaction $\Rightarrow$ observation of effect on B caused by A does not directly reveal the position of A.
- That's why there is no direct evidence for astrophysical dark matter (hypothetic matter with negligible interactions, except gravitational).

$\Rightarrow$ The absence of direct evidence for BM trajectories analogous to the absence of direct evidence for dark matter.

Part 4.

Beyond relativistic QFT

What particles is BM about?

So far we didn't specify what kind of particles are we talking about.
- Atoms? Protons? Electrons? Quarks? Photons? Higgs?
- Perhaps quasiparticles (collective excitations), like phonons?

- Predictions on perceptibles do not depend much on those details.
- Yet details are important for their own sake.

- Phonon trajectory is certainly not beable because we **know** that 1 phonon is a collective motion of many atoms.
- But do we know that photon or electron is **not** a collective excitation?
- **We don't!**

- Theories which serve as good approximations at longer distances, but not at smaller distances, are called **effective theories**.
- Theory of phonons is certainly an effective theory.
- Widely believed that Standard Model of “elementary particles” is an effective theory too.

$\Rightarrow$ The “elementary particles” like electrons, quarks, photons, ... might be collective excitations too.

- Collective excitations of what?
- Of truly elementary particles.
- What these truly elementary particles are?
- We don’t know! (We still don’t have the theory of everything.)
- But whatever they are, BM trajectories can only be beables for those truly fundamental particles.

$\Rightarrow$ It is very likely that:

BM trajectories are **not** beables for Standard Model “elementary particles” like electrons, quarks, photons, or Higgs.

Bypassing relativistic QFT

We found an explicit construction of BM for non-relativistic QM.
- How about relativistic quantum field theory (QFT)?

- "Elementary particles" (electrons, photons ...)
described by relativistic QFT.
- We argued that we don't need BM trajectories for them.
- BM trajectories only for truly elementary particles.
- Possible that truly e.p. not described by relativistic QFT.
- If so, then BM bypasses relativistic QFT.

Can also be interpreted as a generic **measurable prediction** of BM:
- The simplest formulation of BM requires that the most fundamental degrees are described by non-relativistic QM.
$\Rightarrow$ The simplest BM predicts that at some very small distances
(not yet amenable to our current experimental technologies)
we should see violation of Lorentz invariance.
- Differs e.g. from generic predictions of string theory.

How could it be that non-relativistic QM is fundamental and that relativistic QFT is only an approximation?

- It is usually considered that relativistic QFT is fundamental, while non-relativistic QM is only an approximation.
- I propose that the opposite is the case. How could that be?
- The basic idea presented in most textbooks on condensed matter!

Sound satisfies the wave equation

$$\frac{1}{c_s^2}\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi = 0$$

- Lorentz invariant (with speed of sound c_s instead of c).
- Valid only at distances much larger than interatomic distances.
- Derived from non-relativistic motion of atoms.
- Atoms make the “ether” for sound waves.
- If one observed **only** the sound and nothing else, it would look as if there was no “ether” for sound.

Quantization of sound:

- First quantization of $\psi \Rightarrow$ QM of a single phonon.
- Second quantization of $\psi \Rightarrow$ QFT of phonons.
- Standard tools in condensed matter.
- Derived from non-relativistic QM of atoms (nuclei + electrons).
- Creation/destruction of phonons from fixed number of atoms.

- By analogy, all relativistic "elementary particles" of Standard Model (photons, electrons, ...) might be derivable from hypothetic more fundamental non-relativistic particles.
- The world looks "fundamentally" relativistic only because we don't yet see those more fundamental degrees.

- It's a neo-Lorentzian ether theory.
- Michelson-Morley experiment ruled out possibility that Earth moves **through** ether.
- No experiment ruled out possibility that Earth (and everything else) is **made of** ether.

Explicit models in which various qualitative properties of the Standard Model of "elementary particles" derived from condensed-matter systems:

- G.E. Volovik, *The Universe in a Helium Droplet* (Oxford, 2009)

- X.-G. Wen, *Quantum Field Theory of Many-body Systems: From the Origin of Sound to an Origin of Light and Electrons* (Oxford, 2004)

Example: A Phonon and its Bohmian interpretation

- Crystal lattice made of N atoms with positions

$$\vec{q} = (\mathbf{q}_1, \ldots, \mathbf{q}_N)$$

- Wave function $\Psi(\vec{q}, t)$ satisfies non-relativistic Schrödinger equation

$$\left[\sum_{a=1}^{N} \frac{\hat{\mathbf{p}}_a^2}{2m_a} + V(\vec{q})\right] \Psi = i\hbar\partial_t \Psi$$

- Let $\Psi_{\mathbf{p}}(\vec{q}, t)$ = solution corresponding to 1 (acoustic) phonon with momentum $\mathbf{p}$.

$\Rightarrow$ Most general 1-phonon solution

$$\Psi(\vec{q}, t) = \sum_{\mathbf{p}} c_{\mathbf{p}} \Psi_{\mathbf{p}}(\vec{q}, t).$$

- In the abstract Hilbert space this state is

$$|\Psi(t)\rangle = \sum_{\mathbf{p}} c_{\mathbf{p}} |\Psi_{\mathbf{p}}(t)\rangle$$

which can also be represented by a 1-quasiparticle wave function

$$\psi(\mathbf{x}, t) = \sum_{\mathbf{p}} c_{\mathbf{p}} e^{-i[\omega(\mathbf{p})t - \mathbf{p}\cdot\mathbf{x}]}$$

- units $\hbar = 1$, $\omega(\mathbf{p}) = c_s|\mathbf{p}|$ - Lorentz invariant dispersion relation

- The 1-quasiparticle wave function $\psi(\mathbf{x},t)$ satisfies wave equation

$$\frac{1}{c_s^2}\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi = 0$$

Bohmian interpretation 1:

$\psi(\mathbf{x},t)$ suggests phonon position $\mathbf{X}(t)$.

- Makes sense if one imagines that phonon is fundamental.

Bohmian interpretation 2:

- Denies $\mathbf{X}(t)$, but $\Psi(\vec{q},t)$ suggests atom positions $\vec{Q}(t)$.
- Makes sense if one imagines that atoms are fundamental.

Contains 3 wave-like objects:

1) 1-phonon wave function $\psi(\mathbf{x},t)$, relativistic, not fundamental.
2) Multi-atom wave function $\Psi(\vec{q},t)$, non-relativistic, fundamental.
3) Collective motion of atoms $\vec{Q}(t)$, non-relativistic, fundamental.

Bohmian interpretation 3:
- Denies $\vec{Q}(t)$, but accepts $\vec{Q}_{\text{quarks \& electrons}}(t)$.
- Makes sense if one imagines that quarks & electrons are fundamental.
- Requires relativistic BM, hard problem.

Bohmian interpretation 4:
- Denies $\vec{Q}_{\text{quarks \& electrons}}(t)$, but accepts existence of as yet unknown **truly** fundamental particles with $\vec{Q}_{\text{truly fundamental}}(t)$.
- Truly fundamental particles are not created and destroyed.
$\Rightarrow$ Described by non-relativistic QM.
- Bypasses the hard problem of relativistic BM
in Bohmian interpretation 3.
- This is the version of BM that I actually propose.

Summary

- Perceptibles: macroscopic entities that we observe directly.
- Perceptibles distinguished in position space.
- Non-perceptibles: wave function, atom, photon ...
theoretical constructs that explain the perceptibles.
- Main axiom: perceptibles are beables.
(The Moon is there even when we don't observe it).
- No strict border between perceptibles and non-perceptibles.
$\Rightarrow$ Suggests that microscopic positions are also beables.
$\Rightarrow$ Suggests BM - deterministic particle positions.

- What particles? Only the fundamental ones.
- Indications that Standard Model particles are not fundamental.
- Measurable prediction by the simplest BM: fundamental particles obey non-relativistic QM.
- Analogy with phonons indicates how fundamental non-relativistic QM may lead to non-fundamental relativistic QFT.