

Quantum Foundations III: Decoherence

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Decoherence in a nutshell:

- In QM relative phase is physical:

$$|\psi_1\rangle + e^{i\varphi}|\psi_2\rangle \text{ not equivalent to } |\psi_1\rangle + e^{i\varphi'}|\psi_2\rangle$$

- There are physical mechanisms by which information about the phase is effectively washed out:

$$|\psi_1\rangle + e^{i\varphi}|\psi_2\rangle \rightarrow |\psi_1\rangle + e^{i \cdot \text{unknown}}|\psi_2\rangle$$

- Coherent superposition: $e^{i\varphi}$ known.
- Incoherent superposition: $e^{i\varphi}$ unknown.
- Decoherence: transition from coherent \rightarrow incoherent.

- Today theoretically well understood and experimentally confirmed.
- Nobel prize 2012 (Haroche and Wineland).

The role of decoherence in quantum foundations:

- The mechanism of decoherence does **not depend** on interpretation (Copenhagen, many worlds, Bohmian, ...).
- Yet it significantly **helps** to understand why some quantum systems behave classically.
- For instance, why a quantum cat $|live\rangle + |dead\rangle$ appears either $|live\rangle$ or $|dead\rangle$.

However:

- Decoherence helps, but it's not enough!
- To explain the origin of classicality **completely**, it is necessary to use some specific interpretation.
- Here we shall deal mostly with interpretation-independent aspects.

Question: How to write down a state with unknown $e^{i\varphi}$?

Answer: With **density matrix**!

State with known $e^{i\varphi}$:

$$|\psi\rangle = \frac{e^{i\varphi_0}}{\sqrt{2}} (|\psi_1\rangle + e^{i\varphi}|\psi_2\rangle)$$

Density matrix

$$\rho = |\psi\rangle\langle\psi| = \frac{1}{2} (|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2| + e^{-i\varphi}|\psi_1\rangle\langle\psi_2| + e^{i\varphi}|\psi_2\rangle\langle\psi_1|)$$

- does not depend on the unphysical total phase $e^{i\varphi_0}$
- contains all the physical information as $|\psi\rangle$

In the representation $|\psi_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|\psi_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\rho \equiv \rho(\varphi) = \frac{1}{2} \begin{pmatrix} 1 & e^{-i\varphi} \\ e^{i\varphi} & 1 \end{pmatrix}$$

Now if $e^{i\varphi}$ is completely unknown, we **average** over all possible values of φ :

$$\tilde{\rho} = \frac{1}{2\pi} \int_0^{2\pi} d\varphi \rho(\varphi) = \frac{1}{2} (|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|) = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- There is **no** $|\tilde{\psi}\rangle$ such that $\tilde{\rho} = |\tilde{\psi}\rangle\langle\tilde{\psi}|$.
- State $|\psi\rangle$ or $\rho = |\psi\rangle\langle\psi|$ is called **pure** state.
- State $\tilde{\rho}$ is called **mixed** state.

More generally

$$|\psi\rangle = \sum_k c_k |k\rangle$$

$c_k = |c_k| e^{i\varphi_k}$, average over all phases $\varphi_k \Rightarrow$

$$\tilde{\rho} = \sum_k p_k |k\rangle\langle k| = \begin{pmatrix} p_1 & 0 & \cdots \\ 0 & p_2 & \cdots \\ \vdots & & \ddots \end{pmatrix}$$

where $p_k = |c_k|^2$ are probabilities.

\Rightarrow Average over φ_k destroys all non-diagonal elements of ρ in basis $\{|k\rangle\}$.

Destruction of phases equivalent to destruction of interference.

For instance, 2-slit experiment (slits called A and B):

$$\psi(x) = \psi_A(x) + \psi_B(x)$$

$$\psi_A(x) = e^{i\varphi_A(x)} |\psi_A(x)|, \quad \psi_B(x) = e^{i\varphi_B(x)} |\psi_B(x)|$$

Probability density in the position space $p(x) = |\psi(x)|^2 \Rightarrow$

$$p(x) = \underbrace{|\psi_A(x)|^2 + |\psi_B(x)|^2}_{\text{classical sum of probabilities}} + \underbrace{2 \cos [\varphi_A(x) - \varphi_B(x)] |\psi_A(x)| |\psi_B(x)|}_{\text{quantum interference}}$$

Average over phases \Rightarrow

$$\tilde{p}(x) = |\psi_A(x)|^2 + |\psi_B(x)|^2$$

\Rightarrow Decoherence implies that quantum addition of probability amplitudes gets replaced by classical addition of probabilities.

- Important step towards understanding how classical physics emerges from fundamental quantum laws.

However, we still haven't answered the most important question:
Why is the phase unknown?

There can be various reasons, but the most common reason is:

Due to interaction with environment!

- This is called **environment-induced decoherence**.

- When physicists talk about “decoherence”, they usually mean **this** kind of decoherence.

⇒ In the rest we study only environment-induced decoherence.

Heuristic explanation:

$|\psi\rangle$ a state in the system we want to study

$|\phi\rangle$ a state in the environment

\Rightarrow A state in the full system is a superposition of terms like

$$(e^{i\varphi}|\psi\rangle)|\phi\rangle = |\psi\rangle(e^{i\varphi}|\phi\rangle) \equiv |\psi\rangle|\phi'\rangle$$

\Rightarrow The phase is “eaten up” by environment.

\Rightarrow If we don't know the state of environment,
the effect is the same as if we don't know the phase.

Why do we not know the state of environment?

- Sometimes simply because we choose not to measure it.
- More often, because it has large (say 10^{23}) number of degrees of freedom, so it's **impossible** to measure it in practice.

In the rest we make these heuristic ideas more precise.

Average = trace:

For pure state $|\psi\rangle$

$$\begin{aligned}\langle A \rangle &= \langle \psi | A | \psi \rangle = \sum_k \sum_{k'} \langle \psi | k' \rangle \langle k' | A | k \rangle \langle k | \psi \rangle = \sum_k \sum_{k'} \underbrace{\langle k | \psi \rangle \langle \psi | k' \rangle}_{\rho_{kk'}} \underbrace{\langle k' | A | k \rangle}_{A_{k'k}} \\ &= \sum_k (\rho A)_{kk} = \text{Tr} \rho A = \text{Tr} A \rho\end{aligned}$$

Probability: $p_k = |\langle k | \psi \rangle|^2 = \langle \psi | \overbrace{|k\rangle\langle k|}^{\pi_k} | \psi \rangle = \langle \pi_k \rangle$, π_k is projector.

For two subsystems with bases $\{|k\rangle\}$ and $\{|l\rangle\}$ the full basis is $\{|k\rangle|l\rangle\}$.

$$\text{Tr} O = \sum_k \sum_l \langle l | \langle k | O | k \rangle | l \rangle = \text{Tr}_2 \text{Tr}_1 O = \text{Tr}_1 \text{Tr}_2 O$$

where $\text{Tr}_1 O = \sum_k \langle k | O | k \rangle$ and $\text{Tr}_2 O = \sum_l \langle l | O | l \rangle$ are called **partial traces**.

For operator in the first subsystem $A = A_1 \otimes 1$

$$\text{Tr}A\rho = \text{Tr}_1\text{Tr}_2A\rho = \text{Tr}_1(A_1\text{Tr}_2\rho) = \text{Tr}_1A_1\rho_1$$

where

$$\rho_1 = \text{Tr}_2\rho$$

is called **reduced density matrix**.

\Rightarrow **All quantum information about the first subsystem is encoded in $\rho_1 = \text{Tr}_2\rho$.**

Example:

$$|\Psi\rangle = \sum_k c_k |k\rangle |\phi_k\rangle$$

The first subsystem is **not** $|\psi\rangle = \sum_k c_k |k\rangle$. Instead, if $\langle \phi_k | \phi_{k'} \rangle = \delta_{kk'}$

$$\rho_1 = \text{Tr}_2 |\Psi\rangle \langle \Psi| = \sum_k p_k |k\rangle \langle k| \neq |\psi\rangle \langle \psi|, \quad p_k = |c_k|^2$$

- $\Rightarrow \rho_1$ is a mixed state, the information about phases of c_k is “lost”!
- More precisely, in principle information is still there in the full $|\Psi\rangle$, but it's not visible in the first subsystem alone.
 - Similarly, it's not visible in the second subsystem alone:

$$\rho_2 = \text{Tr}_1 |\Psi\rangle \langle \Psi| = \sum_k p_k |\phi_k\rangle \langle \phi_k|$$

Partial decoherence:

What if $\langle \phi_k | \phi_{k'} \rangle \neq \delta_{kk'}$?

Example:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle|\phi_1\rangle + |2\rangle|\phi_2\rangle)$$

$$\rho_1 = \text{Tr}_2 |\Psi\rangle\langle\Psi| \Rightarrow$$

$$\begin{aligned} \rho_1 &= \frac{1}{2}(|1\rangle\langle 1| + |2\rangle\langle 2| + |1\rangle\langle 2|\langle\phi_2|\phi_1\rangle + |2\rangle\langle 1|\langle\phi_1|\phi_2\rangle) \\ &= \frac{1}{2} \begin{pmatrix} 1 & \langle\phi_2|\phi_1\rangle \\ \langle\phi_1|\phi_2\rangle & 1 \end{pmatrix} \end{aligned}$$

Full decoherence (full diagonalisation): $\langle\phi_1|\phi_2\rangle = 0$

Full coherence: $|\langle\phi_1|\phi_2\rangle| = 1$

Partial decoherence: $0 < |\langle\phi_1|\phi_2\rangle| < 1$

Full coherence \Rightarrow no entanglement:

$$|\langle\phi_1|\phi_2\rangle| = 1 \Rightarrow |\phi_2\rangle = e^{i\varphi}|\phi_1\rangle$$

$$\Rightarrow |\Psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle + e^{i\varphi}|2\rangle)|\phi_1\rangle \equiv |\psi\rangle|\phi_1\rangle$$

- When the second subsystem has **many** degrees of freedom then decoherence is usually almost full: $|\langle\phi_1|\phi_2\rangle| \ll 1$.

Simple explanation:

- For $N \gg 1$ degrees, usually they are statistically quite independent.
- Independent means not correlated,

which in QM means not entangled: $|\phi_k\rangle \simeq |\chi_{k1}\rangle \cdots |\chi_{kN}\rangle$

$$\Rightarrow \langle\phi_1|\phi_2\rangle \simeq \langle\chi_{11}|\chi_{21}\rangle \cdots \langle\chi_{1N}|\chi_{2N}\rangle, \quad |\langle\chi_{1j}|\chi_{2j}\rangle| \equiv e^{-\alpha_j} < 1$$

For simplicity we can take all α_j to be roughly the same $\alpha_j \sim \alpha$

$$\Rightarrow |\langle\phi_1|\phi_2\rangle| \sim e^{-\alpha N} \ll 1$$

- The second system with many degrees usually called **environment**.
- A particularly useful kind of environment: **macroscopic measuring apparatus**.

Quantum measurement:

- Suppose that $|\phi_1\rangle$ and $|\phi_2\rangle$ are two macroscopic states of the measuring apparatus.
- In a practical sense, the apparatus “measures” only if the two states can be **distinguished** $\Rightarrow |\langle\phi_1|\phi_2\rangle| \ll 1$.

\Rightarrow **Measurement is associated with (almost) full decoherence.**

Generalized measurement with n possible outcomes:

$$|\Psi\rangle = \sum_{l=1}^n c_l |\psi_l\rangle |\phi_l\rangle, \quad \langle\phi_l|\phi_{l'}\rangle \simeq \delta_{ll'}$$

Counter-example:

- In quantum optics, a mirror does not destroy coherence of a photon.

⇒ Mirror does not cause decoherence.

⇒ Mirror does not measure the photon. Why?

$|\phi_1\rangle$ - state of mirror when it is not hit by the photon

$|\phi_2\rangle$ - state of mirror when it is hit by the photon

- When photon hits the mirror, it transfers momentum \mathbf{p} to the mirror.

x = position of the mirror center of mass in the direction of \mathbf{p}

... = all the other mirror particle positions

- Mirror is a correlated system of particles, all momentum transferred to x

$$\Rightarrow \phi_2(x, \dots) = e^{ipx/\hbar} \phi_1(x, \dots) \Rightarrow \langle \phi_1 | \phi_2 \rangle \propto \int dx e^{ipx/\hbar} |\phi_1(x, \dots)|^2$$

- Center of mass is well defined, i.e. $\phi_1(x, \dots)$ is narrow Gaussian in x .

⇒ $e^{ipx/\hbar}$ nearly a constant inside the Gaussian

(unless the transferred momentum p is very big)

$$\Rightarrow |\langle \phi_1 | \phi_2 \rangle| \simeq 1$$

i.e. the photon is not much decohered by mirror.

Dynamics of decoherence:

- Evolution of the full closed system is unitary: $|\Psi(t)\rangle = e^{-iHt/\hbar}|\Psi(0)\rangle$
- Due to interactions, evolution of a subsystem is usually not unitary.
 $\Rightarrow |\langle\phi_1(t)|\phi_2(t)\rangle| \equiv r(t)$ (decoherence factor) changes with time.

Heuristic description:

- Initially (before interaction) $r(0) = 1$.
- During a short time δt it slightly lowers

$$r(\delta t) = r(0) - \Gamma\delta t = r(0)[1 - \Gamma\delta t] \simeq r(0)e^{-\Gamma\delta t}$$

- Due to many degrees of freedom, the system soon “forgets” its initial state, so it always repeats the same decay pattern:

$$r(2\delta t) \simeq r(\delta t)e^{-\Gamma\delta t} \simeq r(0)e^{-\Gamma 2\delta t}$$

\Rightarrow For long times exponential decay

$$r(t) \simeq r(0)e^{-\Gamma t}$$

- More careful computations and simulations qualitatively confirm.

- At extremely long times ($t \gg$ age of the universe) $r(t)$ can grow (quantum Poincare recurrence).
- Irrelevant in practice.
- For all practical purposes (FAPP), decoherence is irreversible.
- It's a statistical law, closely related to 2nd law of thermodynamics.

Γ depends on details of the interaction Hamiltonian.

- For N degrees of freedom, usually $\Gamma \propto N$.

\Rightarrow For macroscopic objects decoherence is usually **very fast**.

Typical decoherence times (in seconds) $\tau = 1/\Gamma$ for some cases:

Environment	Dust grain	Large molecule
Cosmic background radiation	1	10^{24}
Photons at room temperature	10^{-18}	10^6
Best laboratory vacuum	10^{-14}	10^{-2}
Air at normal pressure	10^{-31}	10^{-19}

Environment-induced superselection:

- Decoherence: mechanism by which ρ_1 becomes (nearly) diagonal.
- But if ρ_1 is diagonal in one basis, it's not diagonal in another.

Question: What determines the basis in which ρ_1 is diagonal?

Answer: The Hamiltonian (of the full closed system)!

- If $H|\Psi(t)\rangle \neq E|\Psi(t)\rangle \Rightarrow |\Psi(t)\rangle$ is non-stationary, changes with time.
- If $H|\Psi(t)\rangle = E|\Psi(t)\rangle \Rightarrow |\Psi(t)\rangle = e^{-iEt/\hbar}|\Psi(0)\rangle$ does not change (the overall phase $e^{-iEt/\hbar}$ is irrelevant).
- $\Rightarrow H$ -eigenstates are stable, other states are unstable.
- \Rightarrow Decoherence factors $r(t) \simeq e^{-\Gamma t}$ decay in the basis of H -eigenstates.
- \Rightarrow After time $t \sim \Gamma^{-1}$, ρ_1 settles down into a matrix (nearly) diagonal in the basis of H -eigenstates.
- $\Rightarrow H$ defines a preferred basis.
- In this basis, coherent superpositions are often (FAPP) impossible.
- In general, impossibility of a certain kind of coherent superposition is called **superselection**.
- Decoherence explains superselection via interaction with environment.

Examples:

Charge Q :

- Exactly conserved $\Rightarrow [Q, H] = 0$.

\Rightarrow Impossible to find a coherent superposition of states with different charges, e.g. $|1 \text{ electron}\rangle + |2 \text{ electrons}\rangle$

Energy of the 1st subsystem:

$$H = H_1 + H_2 + H_{\text{int}}$$

- If $H_{\text{int}} \ll H_1 \Rightarrow H$ -eigenstates are close to H_1 -eigenstates.

$\Rightarrow \rho_1$ is nearly diagonal in the basis of H_1 -eigenstates (e.g. a stable or quasi-stable atom).

Interaction energy:

- If $H_{\text{int}} \gg H_1 \Rightarrow H$ -eigenstates are close to H_1 -eigenstates.
 $\Rightarrow \rho_1$ is nearly diagonal in the basis of H_{int} -eigenstates.
- Usually H_{int} depends only on positions, e.g. $H_{\text{int}} = V(r)$.
- Explains why do we not observe macroscopic superpositions of states at different places, e.g. $|\text{cat here}\rangle + |\text{cat there}\rangle$.

Application to Schrödinger cat:

- Difference between life and death can be reduced to a difference between positions of some macroscopic objects (e.g. eyelid open vs eyelid closed).
 \Rightarrow Decoherence explains why there are **no** coherent superpositions $|\text{cat live}\rangle + |\text{cat dead}\rangle$.

Does decoherence explain the wave-function collapse?

- Not quite!
- Decoherence eliminates certain **coherent** superpositions, but it still contains **incoherent** superpositions.

For coherent superposition $|\psi\rangle = \frac{1}{\sqrt{2}}(|\text{live}\rangle + |\text{dead}\rangle)$

in the representation $|\text{live}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|\text{dead}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ we have

$$|\psi\rangle\langle\psi| = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \xrightarrow{\text{decoherence}} \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

On the other hand, collapse corresponds to an additional transition:

$$\frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \xrightarrow{\text{collapse}} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{or} \quad \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \xrightarrow{\text{collapse}} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

- Mixed state ρ always related to a lack of knowledge.
- Physical reasons for the lack of knowledge may be different, but ρ may look the same.

Improper mixture:

- Full system $|\Psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle|\phi_1\rangle + |2\rangle|\phi_2\rangle)$
- Someone who doesn't measure the 2nd subsystem, describes the 1st subsystem as $\rho_1 = \frac{1}{2}|1\rangle\langle 1| + \frac{1}{2}|2\rangle\langle 2|$

Proper mixture:

- An apparatus prepares a pure state, either $|1\rangle$ or $|2\rangle$.
- The decision is made by a classical pseudo-random mechanism.
- Someone who doesn't know what the decision was, describes her knowledge as $\rho = \frac{1}{2}|1\rangle\langle 1| + \frac{1}{2}|2\rangle\langle 2|$
- For given ρ , the interpretation as a proper mixture is not unique.
E.g. $|1\rangle\langle 1| + |2\rangle\langle 2| = |+\rangle\langle +| + |-\rangle\langle -|$, where $|\pm\rangle = \frac{1}{\sqrt{2}}(|1\rangle \pm |2\rangle)$

The problem of measurement in QM can be expressed as:

- How does an improper mixture turn into a proper mixture?
 - The answer depends on the interpretation of QM (Copenhagen, many worlds, Bohmian, ...)
- ⇒ Decoherence **alone** doesn't solve the measurement problem!

Nevertheless, decoherence has far reaching implications:

- Coherent superposition of a macroscopic object (e.g. Schrödinger cat) can live only for a very short time.
- Once coherence gets destroyed, it remains destroyed “forever” (FAPP).
- Quantum computer with many qbits is very hard to make.

Relevance for solution of the measurement problem:

- Fast decoherence **helps** to explain why macro objects obey classical laws.
- Irreversibility of decoherence **helps** to explain the (illusion of) wave-function collapse.
- A **full** solution requires a use of some specific interpretation (Copenhagen, many worlds, Bohmian, ...)
not to be discussed today.

Forthcoming talks:

Quantum Foundations IV:

Quantum theory of measurement and Bohmian mechanics

Quantum Foundations V:

Relativistic QFT from a Bohmian perspective: A proof of concept

Quantum Foundations VI:

Suggestions welcome