Quasiparticles, Casimir effect and all that: Conceptual insights

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A few words about me:

I feel like an outsider on this conference.
- I don't work on electron and phonon transport.
- I am not even a condensed-matter physicist.

I work on *foundations* of physics.

For instance, to study condensed matter
one must first learn foundations such as
- principles of quantum mechanics
- principles of quantum field theory (= second quantization?)
- principles of statistical physics

I study such general principles of physics.

- Here I will talk about some conceptual foundations which may be relevant for general conceptual understanding of (some aspects of) condensed matter physics.

- What's the difference between particle and quasiparticle?
- Is quantum particle a pointlike object?
- Is quasiparticle (e.g. phonon) a pointlike object?
- Is vacuum a state without particles?
- Does Casimir effect originate from vacuum energy?

I will try to answer those and many related conceptual questions.

Part 1.

PARTICLES AND QUASIPARTICLES

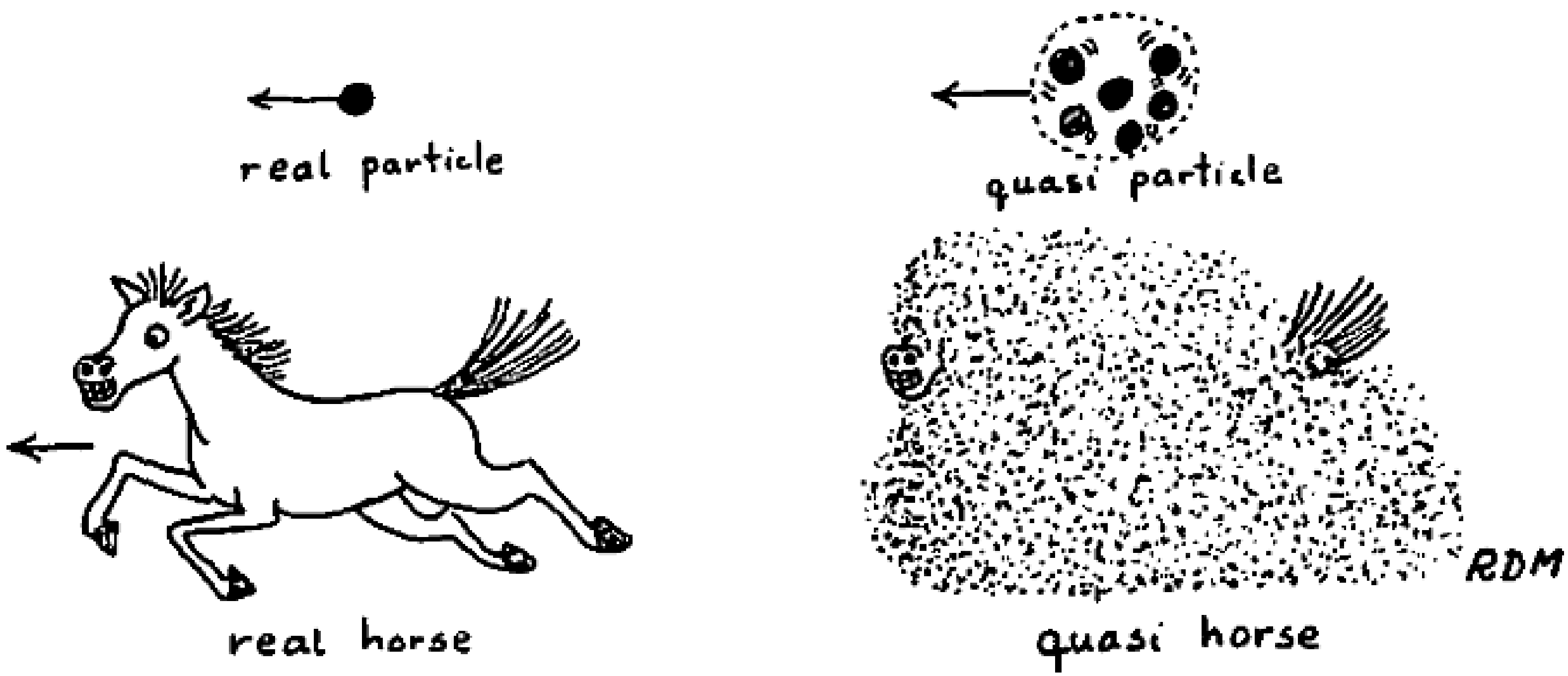
Standard and Bohmian Perspective

What is quasiparticle?

- Terminology is not unique.

- Mattuck (*A Guide to Feynman Diagrams in the Many-Body Problem*) distinguishes quasiparticles from collective excitations.

quasiparticle = original individual particle
+ cloud of disturbed neighbors



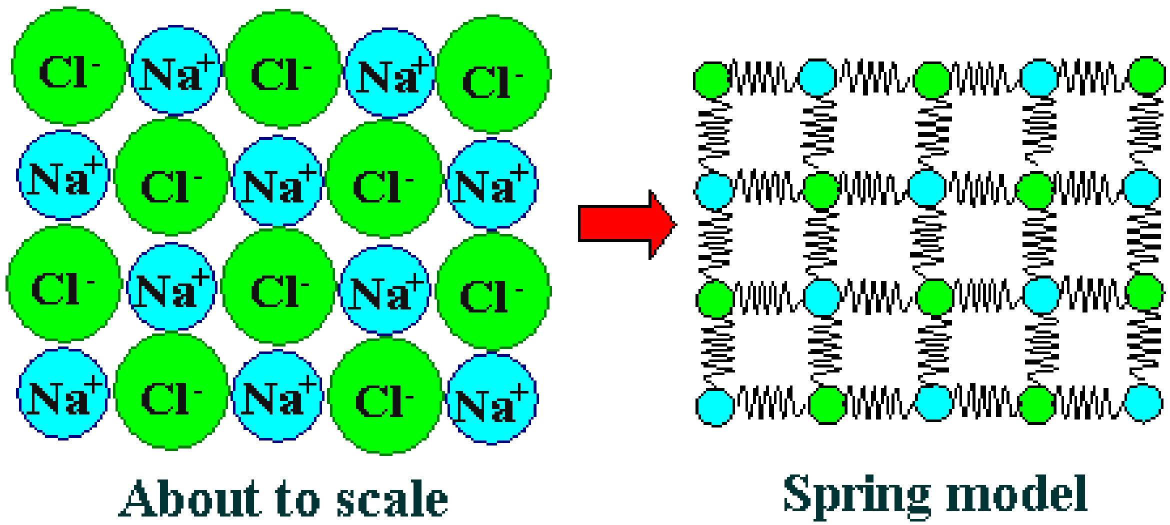
- Collective excitation (e.g. phonon) is not centered around individual particle

Alternative terminology (this talk):
- **collective excitations (e.g. phonons) also called quasiparticles.**

What makes phonons similar to particles?

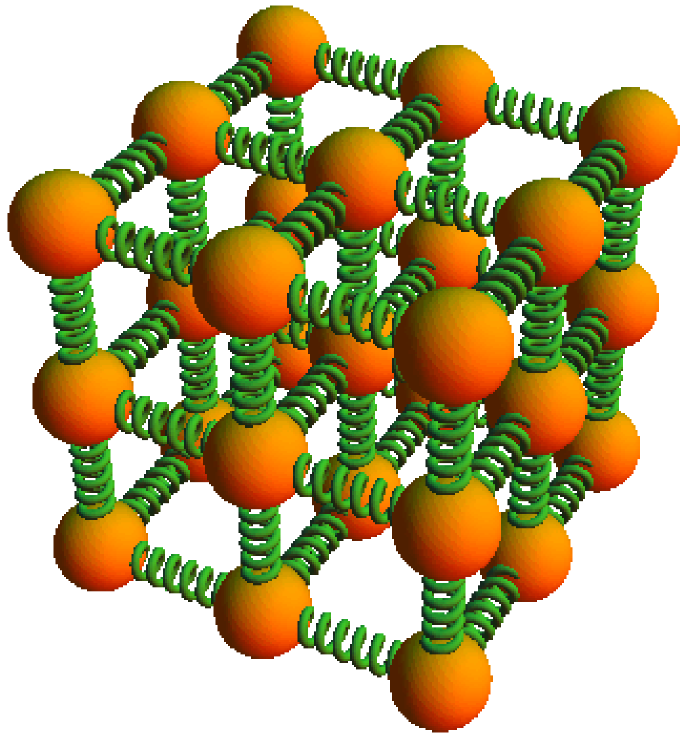
- The key is to approximate the system by a collection of harmonic oscillators.

2-dimensional lattice:



- For each h.o. the potential energy proportional to $(x_i - x_{i+1})^2$.

3-dimensional lattice:



Elementary QM: Each h.o. has energy spectrum of the form

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right), \quad n = 0, 1, 2, 3, \ldots$$

$\Rightarrow$ n can be thought of as a number of “quanta”.

$\Rightarrow$ n behaves like a number of “particles”.

More formally:

- the h.o.'s decouple in new collective coordinates

$k = 1, \dots, N$ - labels N decoupled harmonic oscillators $\Rightarrow$

$$\hat{H} = \sum_k \hbar\omega_k \left(\hat{n}_k + \frac{1}{2}\right), \quad \hat{n}_k = \hat{a}_k^\dagger \hat{a}_k, \quad [\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta_{kk'}$$

Complete set of eigenstates:

- groundstate: $|0\rangle$, satisfies $\hat{a}_k|0\rangle = 0$
- 1-"particle" states: $|k\rangle = \hat{a}_k^\dagger|0\rangle$
- 2-"particle" states: $|k_1, k_2\rangle = \hat{a}_{k_1}^\dagger \hat{a}_{k_2}^\dagger|0\rangle$
- 3-"particle" states: ...

- The formalism looks identical to QFT (quantum field theory) of elementary particles (e.g. **photons**).

- Due to this analogy, the above quanta of lattice vibrations are called **phonons**.

$\Rightarrow$ Formally, a phonon is not less a particle than a photon.

- Indeed, photon is also a collective excitation.
- It is a collective excitation of electromagnetic field.
- Electromagnetic field $\mathbf{E}(\mathbf{x})$, $\mathbf{B}(\mathbf{x})$ lives on a continuum 3d space, which can be thought of as a 3d lattice with spacing $l \to 0$.

- Why then photon is a "true" particle and phonon a "quasiparticle"?

- The difference is in the nature of lattice **vertices**!

- For phonons, the vertices are particles themselves - atoms.
- Phonons emerge from atoms (not the other way around), so atoms are **more fundamental** particles than phonons.
- In this sense, a phonon is "less" particle than an atom, so it makes sense to call it "quasiparticle".

- For photons, the "vertices" are simply fields $\mathbf{E}$, $\mathbf{B}$ at point $\mathbf{x}$.
- There are no more fundamental particles at field vertices.
- Hence photon can be considered a fundamental particle, not "quasiparticle".
- At least this is our current understanding of photons.
- The future physicists might discover that photons are quasiparticles too, just like phonons.

Practical consequences:

- For experiments at large distances ($d \gg l$), phonons behave as particles just as photons do.
- In principle you can measure phonon position, but only with precision $\Delta x \gg l$.
- Position measurement may induce a phonon wave-function collapse, to a narrow (but not too narrow!) Gaussian with a width $\Delta x \gg l$.

Why does measurement induce a collapse?

- Is it just an axiom of QM?
- Or can it be explained by Schrodinger equation?

- To answer those questions we need to understand what is going on during a measurement.

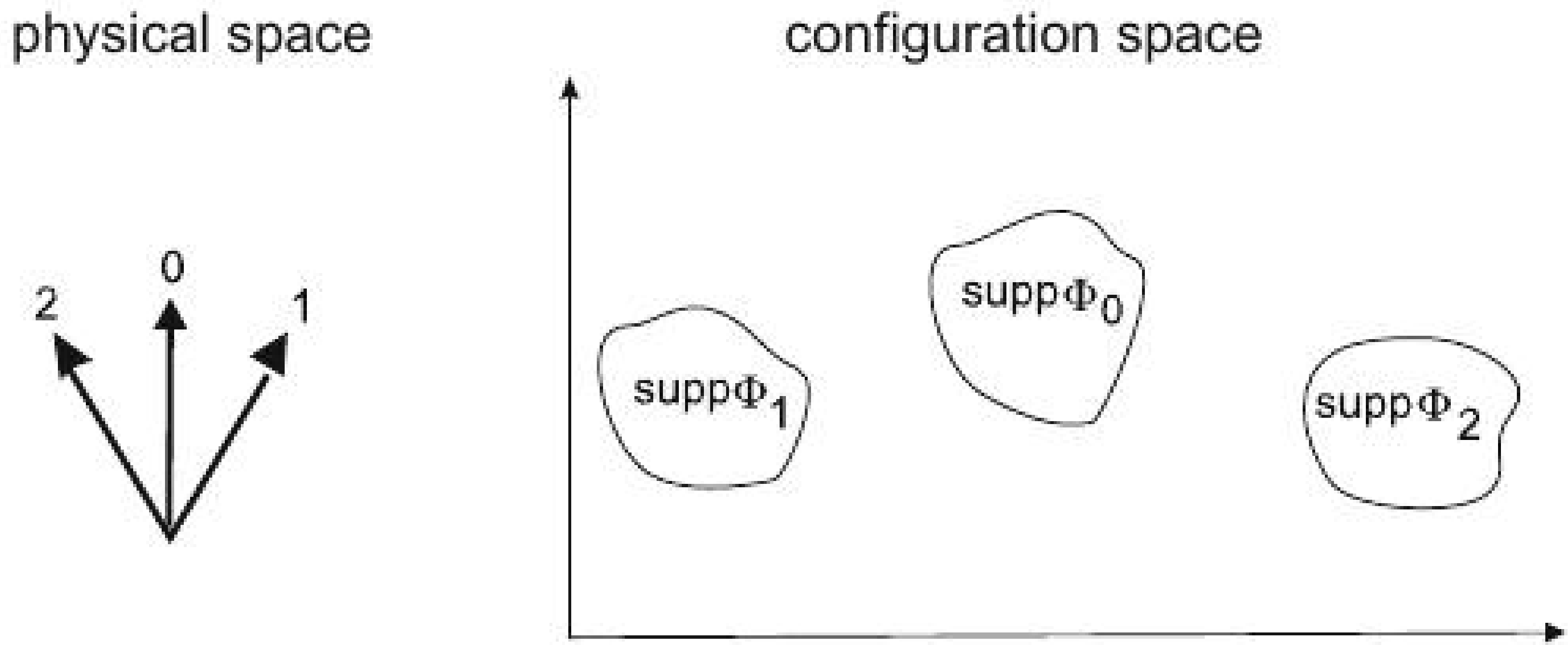
Quantum theory of measurement:

Measure observable $\hat{K}$ with eigenstates $|k\rangle$.
$|k\rangle$ - states of any quantum object
(electron, photon, **phonon**, ..., quantum "cat", ... whatever)

Measuring apparatus in initial state $|\Phi_0\rangle$.
Interaction $\Rightarrow$ unitary continuous transition

$$|k\rangle|\Phi_0\rangle \rightarrow |k'\rangle|\Phi_k\rangle$$

$|\Phi_0\rangle$ and $|\Phi_k\rangle$ are **macroscopically distinguishable** pointer states
$\Rightarrow$ wave functions have a negligible overlap in **configuration space**



$$\Phi_{k_1}(\vec{x})\Phi_{k_2}(\vec{x}) \simeq 0 \quad \text{for} \quad k_1 \neq k_2$$

where $\Phi_k(\vec{x}) \equiv \langle \vec{x}|\Phi_k\rangle$, $\vec{x} \equiv (\mathbf{x}_1, \ldots, \mathbf{x}_n)$,
n = number of particles constituting the apparatus

For a superposition $|\psi\rangle = \sum_k c_k |k\rangle$:

$$\boxed{|\psi\rangle|\Phi_0\rangle \to \sum_k c_k |k'\rangle|\Phi_k\rangle}$$

Why this "collapses" to $|k'\rangle|\Phi_k\rangle$?

$|\Phi_k\rangle$ are macroscopically distinguishable.
$\Rightarrow$ Superposition consists of many distinguishable branches.
Each branch evolves as if other branches did not exist.
$\Rightarrow$ From perspective of any branch, other branches do not exist.
Explains the collapse if one remaining question can be answered:

Why should we take a view from the perspective of a branch as the physical one?

Further conceptual issues:
- In QM, particle becomes localized due to measurement.
- In classical physics, particle is localized even without measurement.

Can we think of quantum particle as localized without measurement?
- In standard formulation of QM - no!
- In Bohmian formulation of QM - yes!

How does Bohmian formulation work?

Wave function: $\psi(\mathbf{x},t) = \sqrt{\rho(\mathbf{x},t)}e^{iS(\mathbf{x},t)/\hbar}$
Complex Schrodinger equation

$$-\frac{\hbar^2\nabla^2\psi}{2m} + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

can be rewritten as two real equations;
1. "Continuity" equation:

$$\frac{\partial\rho}{\partial t} + \nabla(\rho\mathbf{v}) = 0, \quad \mathbf{v} \equiv \frac{\nabla S}{m}$$

2. "Hamilton-Jacobi" equation:

$$\frac{(\nabla S)^2}{2m} + V + Q = -\frac{\partial S}{\partial t}, \quad Q \equiv -\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}$$

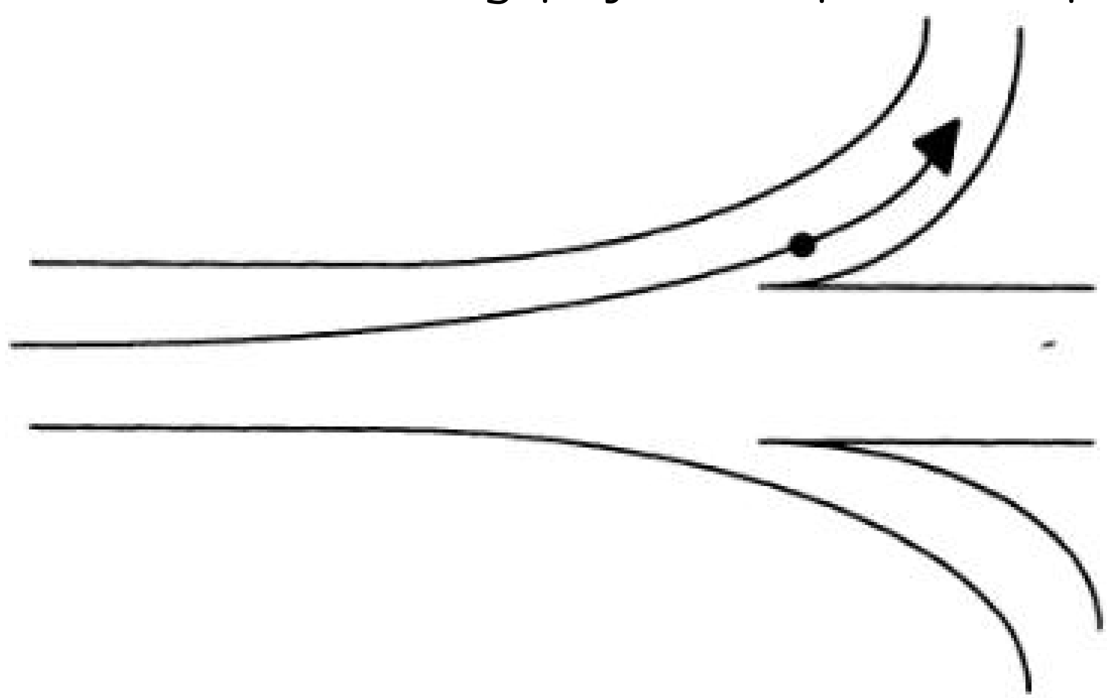
Bohmian formulation interprets these equations as "classical", i.e. it postulates that $\mathbf{v}$ is the velocity of point-like particle with trajectory $\mathbf{X}(t)$

$$\boxed{\frac{d\mathbf{X}(t)}{dt} = \mathbf{v}(\mathbf{X}(t),t)}$$

Quantum uncertainty interpreted as in classical statistical physics:
- Fundamental dynamics is deterministic.
- Uncertainty is emergent due to our ignorance of initial positions.

Let me show that measurable statistical predictions
are identical to standard QM:

- Particular branch becomes physical because
it becomes **filled with** something physical - pointlike particles:



$\Rightarrow$ Filling entity is described by position

$$\vec{X} \equiv (\mathbf{X}_1, \ldots, \mathbf{X}_n)$$

n = number of particles constituting the **apparatus**.

$\Rightarrow$ Apparatus made of pointlike particles.

- If apparatus particles are real and pointlike,
does it make sense to assume that so are other particles?

$$\mathbf{X}_1, \ldots, \mathbf{X}_n, \mathbf{X}_{n+1}, \ldots, \mathbf{X}_N$$

N = total number of particles in the laboratory

Probability density of particle positions:

$$\rho(\mathbf{x}_1, \ldots, \mathbf{x}_N, t) = |\Psi(\mathbf{x}_1, \ldots, \mathbf{x}_N, t)|^2$$

- cannot be tested in practice
 (one cannot observe all particles in the whole laboratory).

- One really observes macroscopic observable describing
the measuring apparatus.

$\Rightarrow$ Phenomenologically more interesting is apparatus probability density

$$\rho^{(\text{appar})}(\mathbf{x}_1, \ldots, \mathbf{x}_n, t) = \int d^3x_{n+1} \cdots d^3x_N \, \rho(\mathbf{x}_1, \ldots, \mathbf{x}_n, \mathbf{x}_{n+1}, \ldots, \mathbf{x}_N, t)$$

$\Rightarrow$

$$\rho^{(\text{appar})}(\vec{x}) \simeq \sum_k |c_k|^2 \, |\Phi_k(\vec{x})|^2$$

$\Rightarrow$ Probability to find the apparatus particles in the support of $\Phi_k(\vec{x})$:

$$\boxed{p_k = \int_{\text{supp } \Phi_k} d^{3n}x \, \rho^{(\text{appar})}(\vec{x}) \simeq |c_k|^2}$$

- this is the Born rule.

$\Rightarrow$ We **derived** Born rule in arbitrary k-basis
from assumption of Born rule in position basis.

$\Rightarrow$ It is crucial that **apparatus particles** exist
and have the quantum probability distribution.
- not so important whether positions
of the observed system (photon, phonon, ...) exist.

Bohmian formulation used in two ways:
- As a fundamental interpretation of QM (alternative to Copenhagen): assumes that particle trajectories really exist in Nature.
- As a practical tool for computations (e.g. Xavier Oriols *et al*).

- Bohmian formulation often used for electrons.
- Can Bohmian formulation be used for phonons?

As a fundamental interpretation of phonons:
- No, because we know that phonon is not a fundamental particle, but emerges from collective motion of atoms.

As a practical tool for phonon computations:
- Yes, because
(when phonon can be described by a Schrodinger-like equation)
Bohmian formulation will lead to same measurable predictions
as standard theory of phonons.

Final warnings:

- Be careful not to take seriously the phonon theory
(either standard or Bohmian) at small distances.

- Take phonons seriously only at distances
much larger than the interatomic distance.

- At smaller distances reformulate your problem
in terms of more fundamental particles (atoms, electrons, photons, ...).

Part 2.

THE ORIGIN OF CASIMIR EFFECT

Vacuum Energy or van der Waals Force?

- Spectrum of h.o.

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

$\Rightarrow$ Energy of the ground state $E_0 = \hbar\omega/2$.
- Is this energy physical?

- Standard answer - no, because we only measure energy **differences**.
$\Rightarrow$ We can subtract this constant without changing physics

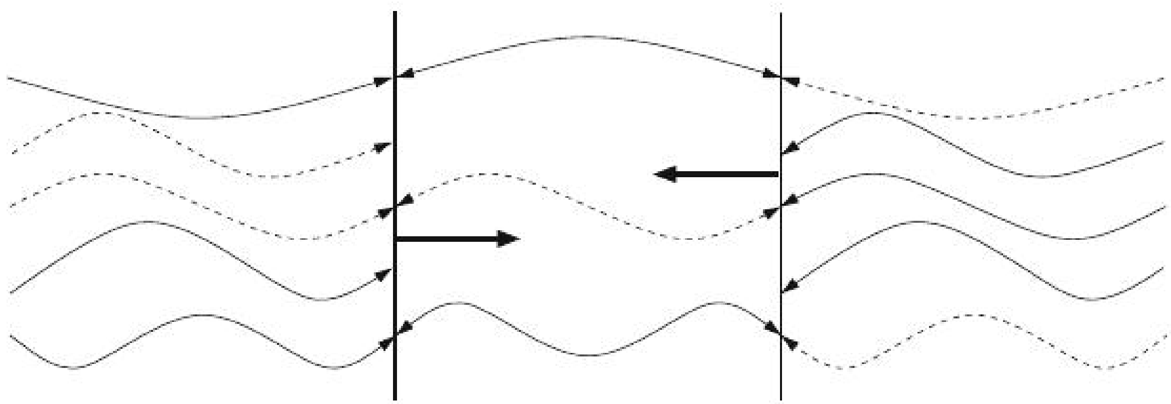
$$\Rightarrow \quad E_n = \hbar\omega n$$

- On the other hand, often claimed in literature that Casimir effect is a counter-example.
- Is Casimir effect evidence that vacuum energy is physical?

Casimir effect = attractive force between electrically neutral plates

Two explanations:
1) vacuum energy of electro-magnetic field
2) van der Waals force
- Which explanation is correct?

1) Vacuum-energy explanation:



- field vanishes at perfectly conducting plates

$\Rightarrow$ some wavelengths impossible between the plates

$\Rightarrow$ Hamiltonian does not contain those modes

$\Rightarrow$ those modes do not contribute to vacuum energy E_{vac}

$\Rightarrow$ E_{vac} depends on the distance y between the plates

$\Rightarrow$ Casimir force

$$F_{\text{vac}} = -\frac{\partial E_{\text{vac}}(y)}{\partial y} = -\frac{\pi^2}{240}\frac{\hbar c}{y^4}$$

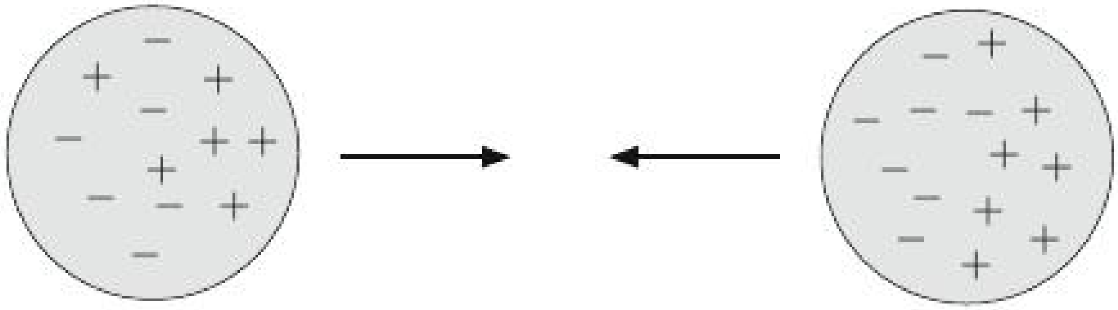
Advantages:
- calculation relatively simple
- presented in many textbooks

Disadvantages:
- Electromagnetic forces are forces between charges,
but where are the charges?
- Force originates from boundary conditions, but microscopic origin of boundary conditions not taken into account.
$\Rightarrow$ Vacuum-energy explanation is not a fully microscopic explanation.

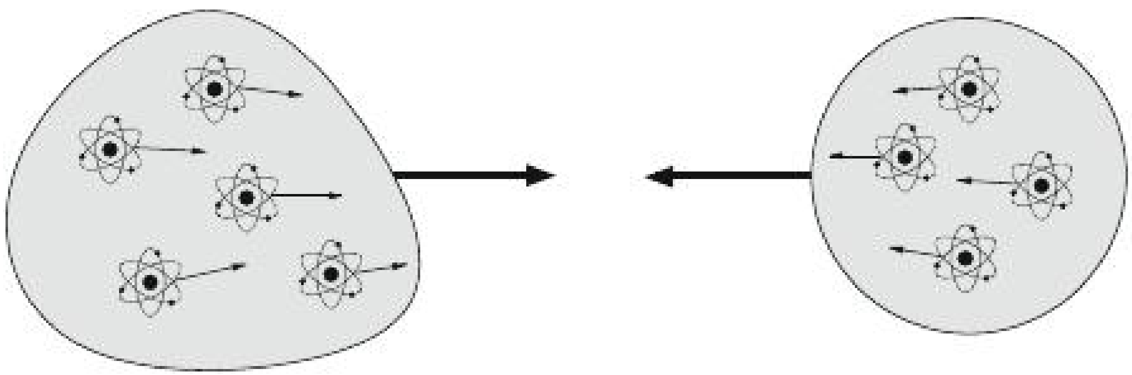
Those disadvantages avoided by van der Waals force approach.

2) Van der Waals force explanation:

- Force explained by polarization of the medium:



- Polarizability of the medium traced down to microscopic polarizability of atoms:



- calculation more complicated (Lifshitz theory)
- the final result is the same $F_{\mathrm{vdW}} = F_{\mathrm{vac}}$

Why do two different explanations give the same result?

Qualitative explanation:

- vacuum-energy explanation originates from boundary conditions
- boundary conditions originate from $\mathbf{E} = 0$ in a perfect conductor
- $\mathbf{E} = 0$ originates from rearrangement of charges

so that any external $\mathbf{E}_{\text{ext}}$ is canceled

- rearrangement of charges $=$ polarization $\mathbf{P}(\mathbf{x})$

(electric dipole moment per volume)

- such a system is simpler to describe by electric displacement

$$\mathbf{D} = \mathbf{E} + \mathbf{P}$$

- $\mathbf{P}$ is induced by $\mathbf{E}$, so approximately $\mathbf{P} \propto \mathbf{E}$

$\Rightarrow$ $\mathbf{D} = \epsilon\mathbf{E}$ (ϵ is dielectric constant) $\Rightarrow$

$$\mathbf{P} = (\epsilon - 1)\mathbf{E}$$

- energy density in dielectric medium (Jackson, *Classical Electrodyn.*)

$$\mathcal{H} = \frac{\mathbf{D} \cdot \mathbf{E}}{2}$$

- combining all the equations above $\Rightarrow$

$$\mathcal{H} = \frac{\mathbf{E}^2}{2} + \frac{\mathbf{P}\cdot\mathbf{E}}{2}$$

- assume there is no external electric field $\Rightarrow$ average field vanishes, i.e.

$$\langle \mathbf{E} \rangle = \langle \mathbf{P} \rangle = 0$$

- however there are quantum fluctuations $\langle \mathbf{E}^2 \rangle \neq 0 \Rightarrow$

$$\boxed{\langle \mathcal{H}_{\text{int}} \rangle = \frac{\langle \mathbf{P}\cdot\mathbf{E} \rangle}{2} = \frac{\langle \mathbf{P}^2 \rangle}{2(\epsilon - 1)} = \frac{\epsilon - 1}{2}\langle \mathbf{E}^2 \rangle}$$

$\Rightarrow$ interaction energy originates from correlation $\langle \mathbf{P}\cdot\mathbf{E} \rangle$
- this is van der Waals energy
- this is **fundamental** because it does not depend
on phenomenological macroscopic parameter ϵ.

At a **phenomenological** macroscopic ϵ-dependent level,
can also be interpreted as:
- energy of polarization fluctuations $\langle \mathbf{P}^2 \rangle$, or
- energy of electric field fluctuations $\langle \mathbf{E}^2 \rangle$
  (the "vacuum"-energy description of Casimir effect)

A toy model:

- The full quantum description is very complicated.
- To gain intuitive understanding of full quantum description, I present a simple toy model with many qualitative features analogue to Casimir effect.

(H.N., Annals of Physics 383 (2017) 181, arXiv:1702.03291)

- Electromagnetic field $\mathbf{E}(\mathbf{x})$, $\mathbf{B}(\mathbf{x})$ $\rightarrow$ mimic by single degree x_1
- Charged particles $\rightarrow$ mimic by single degree x_2
- Distance between the plates $\rightarrow$ mimic by the third degree y

Hamiltonian:

$$H = \left(\frac{p_1^2}{2m} + \frac{kx_1^2}{2} \right) + \left(\frac{p_2^2}{2m} + \frac{kx_2^2}{2} \right) + \frac{p_y^2}{2M} + g(y)x_1x_2$$

Force on y:

$$\boxed{F = -\frac{\partial H}{\partial y} = -g'(y)x_1x_2}$$

To decouple x_1 and x_2, introduce new canonical variables

$$x_\pm = \frac{x_1 \pm x_2}{\sqrt{2}}, \quad p_\pm = \frac{p_1 \pm p_2}{\sqrt{2}}$$

$\Rightarrow$

$$H = H_+ + H_- + \frac{p_y^2}{2M}$$

where

$$H_\pm = \frac{p_\pm^2}{2m} + \frac{k_\pm(y) x_\pm^2}{2}, \quad k_\pm(y) = k \pm g(y)$$

Force on y in new variables:

$$\boxed{F = -\frac{g'(y)\,(x_+^2 - x_-^2)}{2}}$$

To quantize the theory we make an approximation:

- treat y as a classical background

$\Rightarrow$ quantize only the effective Hamiltonian

$$H^{(\text{eff})} = H_+ + H_-$$

$\Rightarrow$ two (quantum) uncoupled harmonic oscillators

$$H_\pm = \hbar\Omega_\pm(y)\left(a_\pm^\dagger a_\pm + \frac{1}{2}\right), \quad \Omega_\pm^2(y) = \frac{k \pm g(y)}{m}$$

effective vacuum $a_\pm|\tilde{0}\rangle = 0 \Rightarrow$

$$E_{\text{vac}}^{(\text{eff})} = \langle\tilde{0}|H^{(\text{eff})}|\tilde{0}\rangle = \frac{\hbar\Omega_+(y)}{2} + \frac{\hbar\Omega_-(y)}{2}$$

$\Rightarrow$ Casimir-like force

$$\boxed{F = -\frac{\partial E_{\text{vac}}^{(\text{eff})}}{\partial y} = -\frac{\hbar\Omega'_+(y)}{2} - \frac{\hbar\Omega'_-(y)}{2} = \frac{-\hbar g'(y)}{4m\Omega_+(y)} + \frac{\hbar g'(y)}{4m\Omega_-(y)}}$$

- Not clear how is this quantum force related to the classical force?

A Lifshitz-like approach to calculate the force:

Quantum expectation of the "classical" force operator

$$\boxed{F = -\frac{g'(y)\,\langle\tilde{0}|(x_+^2 - x_-^2)|\tilde{0}\rangle}{2}}$$

Elementary property of harmonic oscillator:

$$\langle\tilde{0}|x_\pm^2|\tilde{0}\rangle = \frac{\hbar}{2m\Omega_\pm}$$

$\Rightarrow$

$$F = \frac{-\hbar g'(y)}{4m\Omega_+(y)} + \frac{\hbar g'(y)}{4m\Omega_-(y)}$$

- the same result as with the Casimir-like approach

In both approaches, the force originates from coupling function $g(y)$.

The structure of the interacting vacuum:

In the absence of coupling $g(y) \to 0$,

- different frequency

$$\omega = \frac{k}{m} \neq \Omega_\pm$$

- different creation/destruction operators $a_{1,2} \neq a_\pm$:

$$a_j = \sqrt{\frac{m\omega}{2\hbar}} x_j + \frac{i}{\sqrt{2m\hbar\omega}} p_j$$

$$a_\pm = \sqrt{\frac{m\Omega_\pm}{2\hbar}} x_\pm + \frac{i}{\sqrt{2m\hbar\Omega_\pm}} p_\pm$$

Related by a Bogoliubov transformation:

$$a_\pm = \sum_{j=1,2} \alpha_{j\pm} a_j + \beta_{j\pm} a_j^\dagger$$

Bogoliubov coefficients:

$$\alpha_{1\pm} = \frac{\Omega_\pm + \omega}{2\sqrt{2\Omega_\pm \omega}}, \quad \alpha_{2\pm} = \pm \alpha_{1\pm}$$

$$\beta_{1\pm} = \frac{\Omega_\pm - \omega}{2\sqrt{2\Omega_\pm \omega}}, \quad \beta_{2\pm} = \pm \beta_{1\pm}$$

Two different vacuums $|0\rangle \neq |\tilde{0}\rangle$:

$$a_j|0\rangle = 0, \quad a_\pm|\tilde{0}\rangle = 0$$

$\Rightarrow$ The average number of free quanta $N_j = a_j^\dagger a_j$
is not zero in interacting vacuum $|\tilde{0}\rangle$:

$$\boxed{\langle \tilde{0}|N_j|\tilde{0}\rangle = \beta_{j+}^2 + \beta_{j-}^2}$$

How is this toy model related to the real Casimir effect?

- first free oscillator analogous to electromagnetic Hamiltonian

$$\frac{p_1^2/m + kx_1^2}{2} \leftrightarrow \int d^3x \, \frac{\mathbf{E}^2 + \mathbf{B}^2}{2}$$

- second free oscillator analogous to polarization field of charged matter (J.J. Hopfield, Phys. Rev. **112**, 1555 (1958))

- the interaction term analogous to interaction between charges and electromagnetic field

$$gx_1x_2 \leftrightarrow \int d^3x \, A_\mu j^\mu$$

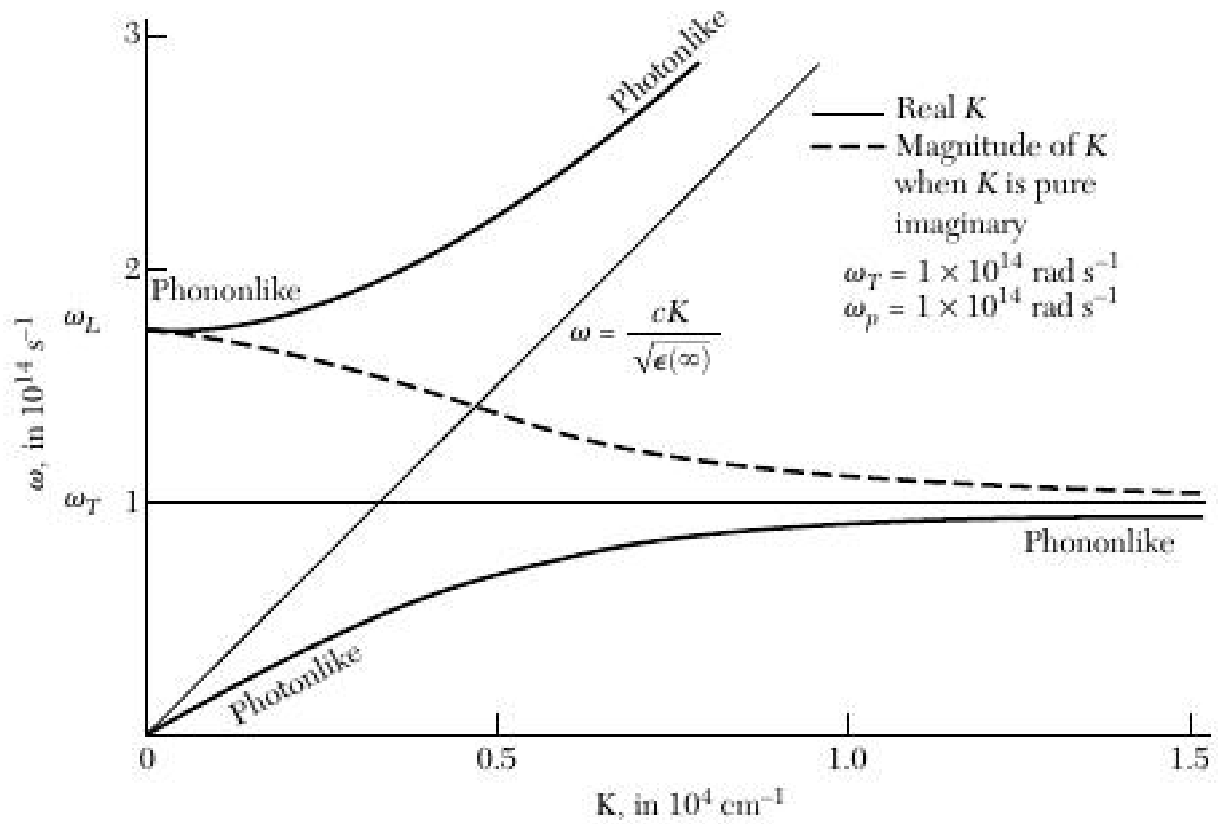
A_μ is electromagnetic 4-potential, j^μ is charged 4-current

- mixture of fundamental degrees:

$$x_+ = \frac{x_1 + x_2}{\sqrt{2}} \;\leftrightarrow\; \mathbf{D} = \mathbf{E} + \mathbf{P}$$

$\mathbf{P(x)}$ polarization (dipole moment per volume),
$\mathbf{D(x)}$ electric displacement (defined by Eq. above)

- More precisely, two frequencies $\Omega_\pm \leftrightarrow$ two branches $\omega_\pm(K)$ of the dispersion relation in a dielectric medium:



- free vacuum $|0\rangle \leftrightarrow$ state without photons and polarization quanta
- interacting vacuum $|\tilde{0}\rangle \leftrightarrow$ Casimir vacuum

$\Rightarrow$ Casimir vacuum is not a state without photons
(H.N., Phys. Lett. B **761**, 197 (2016); arXiv:1605.04143)

- Casimir vacuum is a state without **polaritons**.
(W.M.R. Simpson (2015), *Surprises in Theoretical Casimir Physics*)
- Polariton is a **quasiparticle**, a complicated mixture
of photons and polarization quanta.

The final question: What is vacuum?

In physics, there are different definitions of the word "vacuum":

1) - state without any particles

2) - state without photons

3) - state annihilated by **some** lowering operators $a_k|0\rangle = 0$

4) - local minimum of a classical potential

5) - state with lowest possible energy (ground state)

- Casimir vacuum is only 3),
it has zero number of quasiparticles (polaritons).

- Casimir vacuum is **not** 5),
for otherwise Casimir force could not attract the plates
to a state of even lower energy.