Asymptotic safety

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The gospel according to de Witt

Expand

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

Gauge fix using a background gauge fixing condition e.g.

$$S_{GF}(\bar{g}, h) = \frac{1}{2}\int dx\sqrt{-\bar{g}}\,\bar{g}^{\mu\nu}\chi_\mu\chi_\nu\ ; \qquad \chi_\mu = \bar{\nabla}^\nu h_{\nu\mu} - \frac{1}{2}\bar{\nabla}_\mu h$$

Add ghost Lagrangian

$$S_{ghost}(\bar{g}, \bar{c}, c) = \int dx\sqrt{-\bar{g}}\,\bar{c}^\mu(-\delta^\nu_\mu\bar{\nabla}^2 - \bar{R}^\nu{}_\mu)c_\nu$$

Compute $\Gamma(\bar{g}, h)$.
Formalism preserves background gauge invariance:
$\delta_\epsilon\bar{g}_{\mu\nu} = \mathcal{L}_\epsilon\bar{g}_{\mu\nu}$, $\delta_\epsilon h_{\mu\nu} = \mathcal{L}_\epsilon h_{\mu\nu}$.

ISSUES

Non-renormalizable (Goroff and Sagnotti 1985)

- interaction strength grows like $\tilde{G} = Gk^2$
- violation of unitarity
- lack of predictivity

General EFT recipe

- fix the level of precision that is required in the calculation
- fix the ratio E/M. This determines the order of the expansion that will be required
- at the given order in the expansion, determine all the terms in the action that can contribute to the given process. By power counting there can only be finitely many.
- if the fundamental theory is known, their coefficients can in principle be calculated
- otherwise, measure them with as many experiments.
- use the EFT Lagrangian to compute the amplitudes to the desired precision

Chiral perturbation theory

Strong interactions at low energy described by chiral model

$$S = \int dx \left[\frac{f_\pi^2}{4} tr(U^{-1}\partial U)^2 + \ell_1 tr((U^{-1}\partial U)^2)^2 + \ell_2 tr((U^{-1}\partial U)^2)^2 + O(\partial^6) \right]$$

Expansion parameter $E/4\pi f_\pi$.
For low energy meson physics need f_π, ℓ_1, ℓ_2 and perhaps a few others. One loop in f_π, tree level in ℓ_1, ℓ_2.
Successfully describe a rich phenomenology.

Gravity

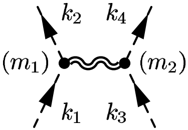
$$S = \int dx \sqrt{g} \left[2m_P^2 \Lambda - m_P^2 R + \ell_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \ell_2 R_{\mu\nu} R^{\mu\nu} + \ell_3 R^2 + O(\partial^6) \right]$$

$R \sim \Gamma\Gamma \sim (g^{-1}\partial g)^2$

m_P similar to f_π.

Concrete problem: quantum corrections to Newtonian potential

Newtonian potential



$$\sim \frac{Gm_1m_2}{\mathrm{q}^2}$$

$$V(r) = \int \frac{d^3q}{(2\pi)^3}\frac{Gm_1m_2}{\mathrm{q}^2}e^{i\mathrm{qr}} = -\frac{Gm_1m_2}{r}$$

Leading quantum correction

For dimensional reasons the leading quantum correction is

$$V(r) = -\frac{Gm_1m_2}{r}\left[1 + \beta\frac{G\hbar}{r^2c^3} + \dots\right]$$

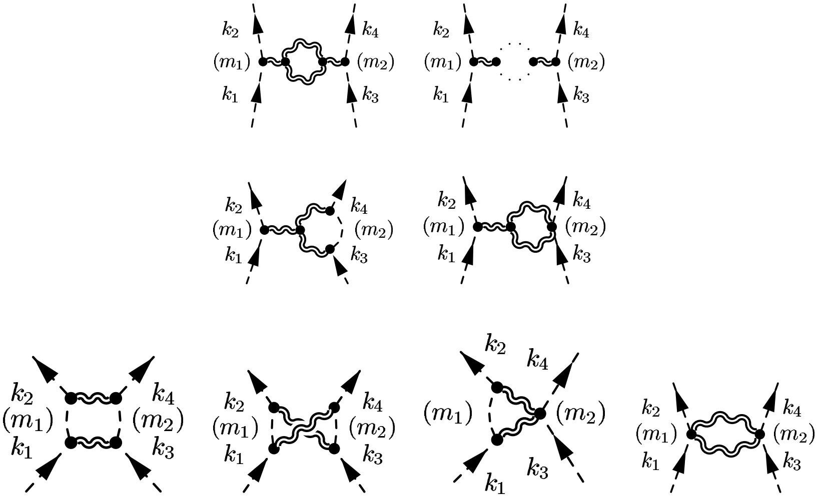
and

$$\int \frac{d^3q}{(2\pi)^3}\log\left(\frac{\mathrm{q}^2}{\mu^2}\right)e^{i\mathrm{qr}} = -\frac{1}{2\pi^2r^3}$$

Clearly distinct from contributions of local counterterms that give analytic corrections to the amplitude. E.g.

$$\int \frac{d^3q}{(2\pi)^3}\ell e^{i\mathrm{qr}} = \ell\delta(r)$$

One loop graphs



(from Bjerrum Bohr, Donoghue, Holstein, 2002)

Evaluation

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- H.W. Hamber, S. Liu, Phys. Lett. B357, 51 (1995)
- A. Akhundov, S. Bellucci, A. Shiekh, Phys. Lett. B395, 16 (1997)
- N.E.J. Bjerrum-Bohr (2002) Phys. Rev. D66, 084023
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- N.E.J. Bjerrum-Bohr, J.F. Donoghue, B.R. Holstein Phys. Rev. D68, 084005; Erratum-ibid.D71, 069904 (2005)
- N.E.J. Bjerrum-Bohr, J.F. Donoghue and B.R. Holstein Phys. Rev. D 67, 084033 (2003) [Erratum-ibid. D 71 (2005) 069903
- I.B. Khriplovich, G.G. Kirilin (2004) J. Exp. Theor. Phys. 98, 1063-1072

A prediction of quantum gravity

$$V(r) = -\frac{Gm_1m_2}{r}\left[1 + \frac{41}{10\pi}\frac{G\hbar}{r^2c^3} + \dots\right]$$

Comes from non-local terms in the effective action, e.g.

$$\int dx\sqrt{g}\,[RF_1(\Box)R + R_{\mu\nu}F_2(\Box)R^{\mu\nu}]$$

Local terms cannot be predicted

Lessons

- this is a quantum theory of gravity
- it agrees with all experimental data
- to some extent, background independent

- open issues in the UV, IR, strong field...
- not a quantum theory of spacetime

Before giving up QFT...

Try to extend beyond Planck scale

- higher derivative gravity (renormalizable and AF)
- special combinations of gravity and matter (SUGRA)
- give up Lorentz invariance (Hořava)
- non-perturbative framework

The issues of QG, and the RG solution

Perhaps interaction strength $\tilde{G} = Gk^2$ stops growing:
$\tilde{G} = G(k)k^2 \to \tilde{G}_*$

More generally, define a theory space parameterized by $\tilde{\lambda}_i$, where

$$\Gamma_k(\bar{g}, h) = \sum_i \lambda_i(k)\mathcal{O}_i(\bar{g}, h)$$

and $\tilde{\lambda}_i = k^{-d_i}\lambda_i$

- RG trajectory is “renormalizable” or “asymptotically safe” if it flows to a FP in the UV

THE RG SOLUTION CONT'D

Issue of predictivity solved if the set of renormalizable trajectories is finite dimensional.

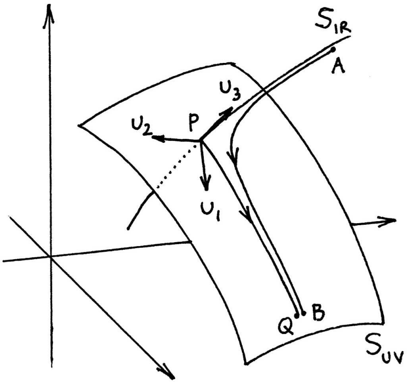
Define S_{UV} the basin of attraction of the fixed point. Require $\dim(S_{UV})$ to be finite.

Can be determined by studying the linearized flow

$$\partial_t(\tilde{\lambda}_i - \tilde{\lambda}_{i*}) = M_{ij}(\tilde{\lambda}_j - \tilde{\lambda}_{j*}) \; ; \qquad M_{ij} = \frac{\partial \tilde{\beta}_i}{\partial \tilde{\lambda}_j}$$

$\dim(S_{UV})$=# of negative eigenvalues of M.

General picture



Examples

QCD:

- Gaußian Fixed Point at $\tilde{\lambda}_{i*} = 0$.
- $\tilde{\lambda}_i = k^{-d_i} \lambda_i$
- $\tilde{\beta}_i = \partial_t \tilde{\lambda}_i = -d_i \tilde{\lambda}_i + k^{-d_i} \beta_i$
- $M_{ij}\Big|_* = \frac{\partial \tilde{\beta}_i}{\partial \tilde{\lambda}_j}\Big|_* = -d_i \delta_{ij}$
- relevant couplings=renormalizable couplings

Non-AF examples?

General NLSM

$$\frac{1}{2}Z\int d^dx\,\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta h_{\alpha\beta}(\varphi)$$

$Z = \frac{1}{g^2}$
$Z \approx \mathrm{mass}^{d-2}$, $g \approx \mathrm{mass}^{\frac{2-d}{2}}$
nonrenormalizable in $d > 2$

Ricci flow:

$$\frac{d}{dt}(Zh_{\alpha\beta}) = 2c_d k^{d-2}R_{\alpha\beta}$$

$c_d = \frac{1}{(4\pi)^{d/2}\Gamma(d/2+1)}$

$O(N)$ (SPHERICAL) NLSM

$d = 2$:

$$\beta_{g^2} = -2c_2(N-2)\tilde{g}^4$$

$d = 2 + \epsilon$:
$\tilde{g}^2 = k^{d-2} g^2 = k^{d-2}/Z$

$$\beta_{g^2} = \epsilon \tilde{g}^2 - 2c_d(N-2)\tilde{g}^4$$

non gaussian FP at

$$\tilde{g}_*^2 = \frac{d-2}{2} \frac{1}{c_d(N-2)} \, .$$

$\frac{d\beta}{d\tilde{g}}\Big|_* = 2 - d.$
Confirmed by Monte-Carlo calculations in $d = 3$.

Other old examples

Gross-Neveu model in $d = 3$

Gauge theories in $d = 4$: one loop

$$
\begin{aligned}
L_{YM} &= -\frac{1}{4}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} \\
L_F &= \bar{\psi} i \not{D} \psi
\end{aligned}
$$

$$\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$$\beta_g = -B\alpha_g^2$$

$$B = -\frac{4}{3}\epsilon \; ; \qquad \epsilon = \frac{N_F}{N_c} - \frac{11}{2}$$

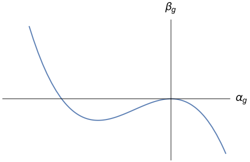
$$N_F < \tfrac{11}{2} N_c \Longrightarrow \epsilon < 0 \Longrightarrow B > 0 \Longrightarrow \mathrm{AF}$$

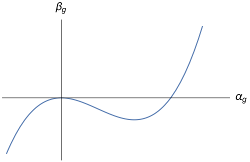
Gauge theories in $d = 4$: two loops

$$\beta_g = -B\alpha_g^2 + C\alpha_g^3$$

$$\alpha_{g*} = B/C \; ; \qquad \beta'(\alpha_{g*}) = B^2/C$$

UV FP if $C < 0$, IR FP if $C > 0$.





W.E. Caswell, *Asymptotic behavior of non-Abelian gauge theories to two loop order*, Phys. Rev. Lett. 33 (1974) 244 found C=25

T. Banks and A. Zaks, *On the phase structure of vector-like gauge theories with massless fermions*, Nucl. Phys. B 196 (1982)

From AF to AS: Yukawa couplings

$$L_H = \mathrm{tr}(\partial^\mu H)^\dagger(\partial_\mu H)$$
$$L_Y = y\,\mathrm{tr}(\bar{\psi}_L H \psi_R + \bar{\psi}_R H \psi_L)$$

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2}$$

$$\beta_g = \alpha_g^2 \left[\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{3} + \epsilon\right)\alpha_y\right]$$
$$\beta_y = \alpha_y \left[(13 + 2\epsilon)\alpha_y - 6\alpha_g\right]$$

Fixed points

$$(\alpha_{g*}, \alpha_{y*}) = \left(-\frac{4\epsilon}{75+26\epsilon}, 0\right)$$

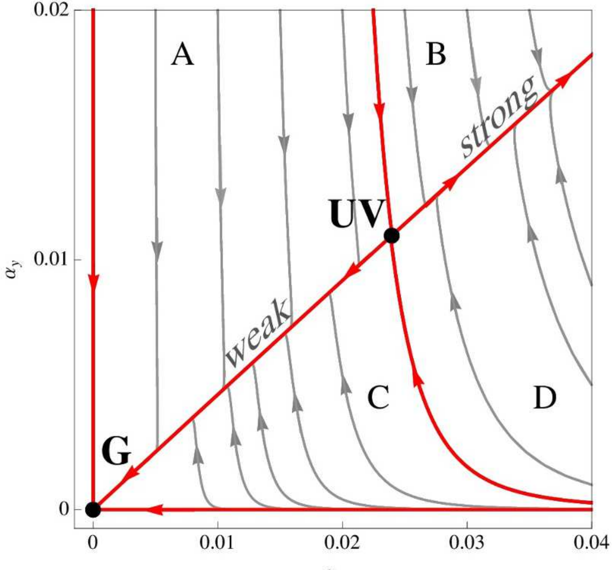
for $\epsilon < 0$, Banks-Zaks

$$\begin{aligned}(\alpha_{g*}, \alpha_{y*}) &= \left(\frac{2\left(13\epsilon + 2\epsilon^2\right)}{57-46\epsilon-8\epsilon^2}, \frac{12\epsilon}{57-46\epsilon-8\epsilon^2}\right) \\ &\approx (0.456\epsilon + O(\epsilon^2), 0.211\epsilon + O(\epsilon^2))\end{aligned}$$

for $\epsilon > 0$

D.F. Litim and F. Sannino, “Asymptotic safety guaranteed” JHEP 1412 (2014) 178

Phase diagram



With scalar interactions

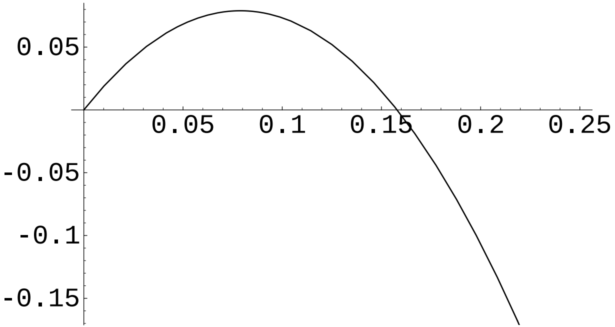
$$V = -u\,\mathrm{tr}((H^\dagger H)^2) - u\,(\mathrm{tr}(H^\dagger H))^2$$

Fixed point persists

Gravity in $d = 2 + \epsilon$

$d = 2 + \epsilon$

$$\tilde{G} = Gk^{\epsilon}$$
$$\beta_{\tilde{G}} = \epsilon\tilde{G} - \frac{38}{3}\tilde{G}^2$$



One Loop Corrections in Einstein's Theory

$$k\frac{d}{dk}\frac{1}{16\pi G(k)} = ck^{d-2}$$

$$k\frac{dG}{dk} = -16\pi c G^2 k^{d-2}$$

$$\tilde{G} = Gk^{d-2}$$

$$k\frac{d\tilde{G}}{dk} = (d-2)\tilde{G} - 16\pi c\tilde{G}^2$$

fixed point at $\tilde{G} = (d-2)/16\pi c$

$c = \frac{11}{3\pi}, \frac{35}{8\pi}, \frac{23}{3\pi}, \ldots$

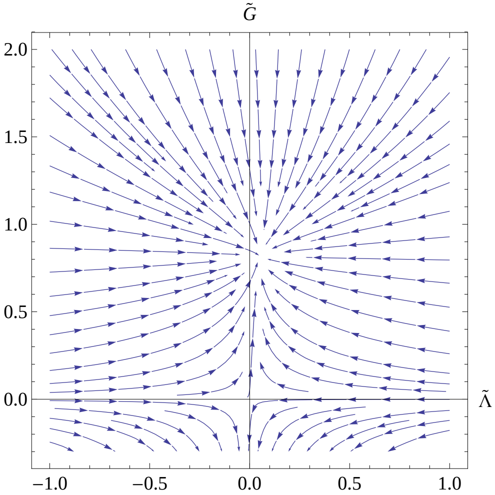
PERTURBATIVE BETA FUNCTIONS

$$\beta_{\tilde{G}} = 2\tilde{G} - \frac{46\tilde{G}^2}{6\pi},$$

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{2\tilde{G}}{4\pi} - \frac{16\tilde{G}\tilde{\Lambda}}{6\pi}$$

$$\tilde{\Lambda}_* = \frac{3}{62} \qquad \tilde{G}_* = \frac{12\pi}{46}$$

PERTURBATIVE FLOW



Topologically massive gravity

Action

$$S(g) = \frac{1}{16\pi G}\int d^3x\sqrt{g}\left(2\Lambda - R + \frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\Gamma^{\rho}_{\lambda\sigma}\left(\partial_\mu\Gamma^{\sigma}_{\nu\rho} + \frac{2}{3}\Gamma^{\sigma}_{\mu\tau}\Gamma^{\tau}_{\nu\rho}\right)\right)$$

Dimensionless combinations of couplings

$$\nu = \mu G\ ; \qquad \tau = \Lambda G^2\ ; \qquad \phi = \mu/\sqrt{|\Lambda|}$$

R.P., E. Sezgin, Class.Quant.Grav. 27 (2010) 155009, arXiv:1002.2640 [hep-th]

Recently extended to TM SUGRA: R.P., M. Perry, C. Pope, E. Sezgin, arXiv 1302.0868

Beta functions of

$$
\begin{aligned}
\beta_\nu &= 0\,, \\
\beta_{\tilde{G}} &= \tilde{G} + B(\tilde{\mu})\tilde{G}^2\,, \\
\beta_{\tilde{\Lambda}} &= -2\tilde{\Lambda} + \frac{1}{2}\tilde{G}\left(A(\tilde{\mu},\tilde{\Lambda}) + 2B(\tilde{\mu})\tilde{\Lambda}\right)
\end{aligned} \tag{1}
$$

Since $\nu = \mu G = \tilde{\mu}\tilde{G}$ is constant

can replace $\tilde{\mu}$ by $\nu/\tilde{G}$

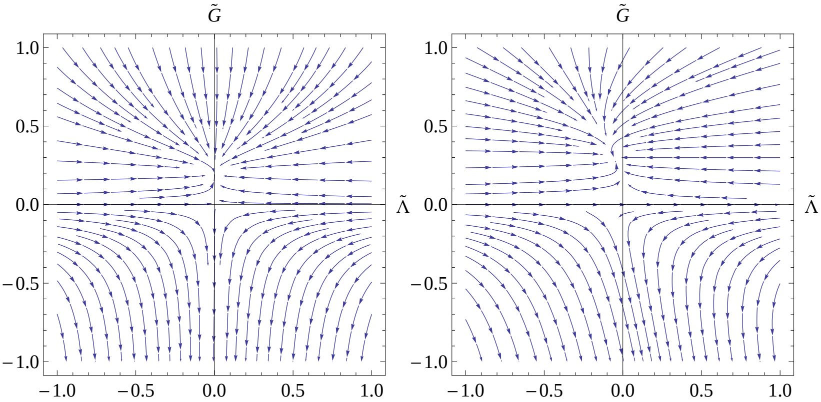


Figure: The flow in the $\tilde{\Lambda}$-$\tilde{G}$ plane for $\nu = 5$ (left) and $\nu = 0.1$ (right).

Higher derivative gravity

$$\Gamma_k = \int d^4x \sqrt{g} \left[2Z\Lambda - ZR + \frac{1}{2\lambda} \left(C^2 - \frac{2\omega}{3} R^2 + 2\theta E \right) \right]$$

$$Z = \frac{1}{16\pi G}$$

K.S. Stelle, Phys. Rev. **D16**, 953 (1977).
J. Julve, M. Tonin, Nuovo Cim. **46B**, 137 (1978).
E.S. Fradkin, A.A. Tseytlin,Phys. Lett. **104 B**, 377 (1981).
I.G. Avramidi, A.O. Barvinski,Phys. Lett. **159 B**, 269 (1985).
G. de Berredo–Peixoto and I. Shapiro, Phys.Rev. **D71** 064005 (2005).
A. Codello and R. P., Phys.Rev.Lett. **97** 22 (2006)
M. Niedermaier, Nucl. Phys. B833, 226-270 (2010)
N. Ohta and R.P. Class. Quant. Grav. **31** 015024 (2014); arXiv:1308.3398

BETA FUNCTIONS I

$$\beta_\lambda = -\frac{1}{(4\pi)^2}\frac{133}{10}\lambda^2$$

$$\beta_\omega = -\frac{1}{(4\pi)^2}\frac{25+1098\omega+200\omega^2}{60}\lambda$$

$$\beta_\theta = \frac{1}{(4\pi)^2}\frac{7(56-171\theta)}{90}\lambda$$

$$\lambda(k) = \frac{\lambda_0}{1+\lambda_0\frac{1}{(4\pi)^2}\frac{133}{10}\log\left(\frac{k}{k_0}\right)}$$

$$\omega(k) \to \omega_* \approx -0.0228$$

$$\theta(k) \to \theta_* \approx 0.327$$

Beta functions II

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{1}{(4\pi)^2}\left[\frac{1+20\omega^2}{256\pi\tilde{G}\omega^2}\lambda^2 + \frac{1+86\omega+40\omega^2}{12\omega}\lambda\tilde{\Lambda}\right]$$

$$-\frac{1+10\omega^2}{64\pi^2\omega}\lambda + \frac{2\tilde{G}}{\pi} - q(\omega)\tilde{G}\tilde{\Lambda}$$

$$\beta_{\tilde{G}} = 2\tilde{G} - \frac{1}{(4\pi)^2}\frac{3+26\omega-40\omega^2}{12\omega}\lambda\tilde{G} - q(\omega)\tilde{G}^2$$

where $q(\omega) = (83 + 70\omega + 8\omega^2)/18\pi$

Flow in $\tilde{\Lambda}$–$\tilde{G}$ plane I

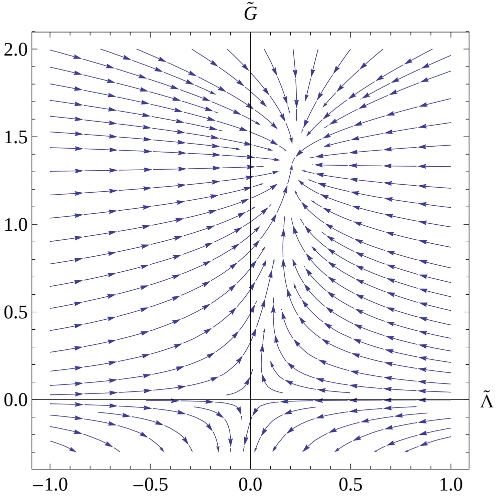
$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{2\tilde{G}}{\pi} - q_* \tilde{G}\tilde{\Lambda}$$

$$\beta_{\tilde{G}} = 2\tilde{G} - q_* \tilde{G}^2$$

where $q_* = q(\omega_*) \approx 1.440$

$$\tilde{\Lambda}_* = \frac{1}{\pi q_*} \approx 0.221 \,, \qquad \tilde{G}_* = \frac{2}{q_*} \approx 1.389 \,.$$

Flow in $\tilde{\Lambda}$–$\tilde{G}$ plane II



ERGE I: Definitions

$$e^{-W_k[J]} = \int (D\phi) \exp(-(S + \Delta S_k + \int J\phi))$$
$$\Delta S_k(\phi) = \frac{1}{2} \int d^4 q \phi(-q) R_k(q^2) \phi(q)$$

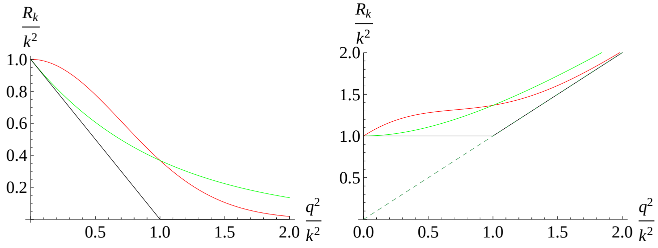


Figure: Left: cutoff. Right: $P_k(q^2) = q^2 + R_k(q^2)$

$$\Gamma_k[\phi] = W_k[J] - \int J\phi - \Delta S_k$$

ERGE III: One loop

$$\Gamma^{(1)} = S + \frac{1}{2}\ln\det\mathcal{O} \quad ; \qquad \mathcal{O} = \frac{\delta^2 S}{\delta\phi\delta\phi}$$

$$\Gamma_k^{(1)} = S + \Delta S_k + \frac{1}{2}\ln\det\mathcal{O}_k - \Delta S_k \quad ; \qquad \mathcal{O}_k = \frac{\delta^2(S+\Delta S_k)}{\delta\phi\delta\phi} = \mathcal{O} + R_k$$

$$\partial_t\Gamma_k^{(1)} = \frac{1}{2}(\det\mathcal{O}_k)^{-1}\partial_t\det\mathcal{O}_k = \frac{1}{2}\mathrm{Tr}\,\mathcal{O}_k^{-1}\partial_t\mathcal{O}_k$$

One–loop RG equation

$$\partial_t\Gamma_k^{(1)} = \frac{1}{2}\mathrm{Tr}\left(\frac{\delta^2 S}{\delta\phi\delta\phi} + R_k\right)^{-1}\partial_t R_k$$

ERGE II: Derivation

$$\partial_t W_k = -\partial_t \langle \Delta S_k \rangle = -\mathrm{Tr}\langle \phi\phi \rangle \partial_t R_k$$

$$\begin{aligned}\partial_t \Gamma_k[\phi] =& \partial_t W_k[J] - \partial_t \Delta S_k[\phi] = \\ &= \mathrm{Tr}(\langle \phi\phi \rangle - \langle \phi \rangle \langle \phi \rangle) \partial_t R_k \\ &= -\mathrm{Tr} \frac{\delta^2 W_k}{\delta J \delta J} \partial_t R_k\end{aligned}$$

$$\frac{\delta^2 W_k}{\delta J \delta J} = -\left(\frac{\delta^2 \tilde{\Gamma}_k}{\delta \phi \delta \phi} \right)^{-1}$$

$$\boxed{\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \left(\frac{\delta^2 \Gamma_k}{\delta \phi \delta \phi} + R_k \right)^{-1} \partial_t R_k}$$

[C. Wetterich, Phys. Lett. B **301** (1993) 90.]

ERGE IV: Beta functions

$$\Gamma_k(\phi) = \sum_i g_i(k)\mathcal{O}_i(\phi)$$

$$\partial_t \Gamma_k = \sum_i \partial_t g_i \mathcal{O}_i = \sum_i \beta_{g_i} \mathcal{O}_i$$

compare with

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}\left(\frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi} + R_k\right)^{-1} \partial_t R_k$$

read off beta functions.

FUNCTIONAL RENORMALIZATION

Define $\Gamma_k(\bar{g}_{\mu\nu}, h_{\mu\nu})$. It satisfies a simple functional differential equation

$$k\frac{d\Gamma_k}{dk} = \beta(\bar{g}_{\mu\nu}, h_{\mu\nu})$$

The quantity β is UV and IR finite.
Since $\lim_{k\to 0}\Gamma_k = \Gamma$, can use FRGE to calculate the effective action.

Single-field truncations:

$$\Gamma_k(g_{\mu\nu}) = \Gamma_k(g_{\mu\nu}, 0)$$

Einstein–Hilbert truncation I

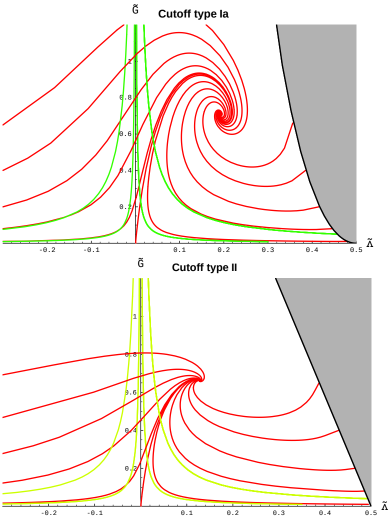
$$\Gamma_k(\bar{g}_{\mu\nu}, h_{\mu\nu}) = S_{EH}(\bar{g}_{\mu\nu}+h_{\mu\nu})+S_{GF}(\bar{g}_{\mu\nu}, h_{\mu\nu})+S_{ghost}(\bar{g}_{\mu\nu}, \bar{C}^{\mu}, C_{\nu})$$

$$S_{EH}(g_{\mu\nu}) = \int dx\sqrt{g}Z(2\Lambda - R)\ ; \quad Z = \frac{1}{16\pi G}$$

$$\beta_{\tilde{\Lambda}} = \frac{-2(1-2\tilde{\Lambda})^2\tilde{\Lambda} + \frac{36-41\tilde{\Lambda}+42\tilde{\Lambda}^2-600\tilde{\Lambda}^3}{72\pi}\tilde{G} + \frac{467-572\tilde{\Lambda}}{288\pi^2}\tilde{G}^2}{(1-2\tilde{\Lambda})^2 - \frac{29-9\tilde{\Lambda}}{72\pi}\tilde{G}}$$

$$\beta_{\tilde{G}} = \frac{2(1-2\tilde{\Lambda})^2\tilde{G} - \frac{373-654\tilde{\Lambda}+600\tilde{\Lambda}^2}{72\pi}\tilde{G}^2}{(1-2\tilde{\Lambda})^2 - \frac{29-9\tilde{\Lambda}}{72\pi}\tilde{G}}$$

Einstein–Hilbert truncation III



FOURTH ORDER GRAVITY

- R^2 O.Lauscher, M. Reuter, Phys. Rev. D 66, 025026 (2002) arXiv:hep-th/0205062
- $R^2 + C^2$ D. Benedetti, P.F. Machado, F. Saueressig, Mod. Phys. Lett. A24, 2233-2241 (2009) arXiv:0901.2984 [hep-th] Nucl. Phys. B824, 168-191 (2010), arXiv:0902.4630 [hep-th]
- M. Niedermaier, Nucl. Phys. B833, 226-270 (2010)

$f(R)$ GRAVITY

$$\Gamma_k(g_{\mu\nu}) = \int d^4x \sqrt{g} f(R)$$

$$f(R) = \sum_{i=0}^{n} g_i(k) R^i$$

n=6
A. Codello, R.P. and C. Rahmede Int.J.Mod.Phys.A23:143-150 arXiv:0705.1769 [hep-th];
n=8
A. Codello, R.P. and C. Rahmede Annals Phys. 324 414-469 (2009) arXiv: arXiv:0805.2909;
P.F. Machado, F. Saueressig, Phys. Rev. **D** arXiv: arXiv:0712.0445 [hep-th]
n=35
K. Falls, D.F. Litim, K. Nikolakopoulos, C. Rahmede, arXiv:1301.4191 [hep-th]
n=∞
Dario Benedetti, Francesco Caravelli, JHEP 1206 (2012) 017, Erratum-ibid. 1210 (2012) 157 arXiv:1204.3541 [hep-th]
Juergen A. Dietz, Tim R. Morris,JHEP 1301 (2013) 108 arXiv:1211.0955 [hep-th]
Dario Benedetti, arXiv:1301.4422 [hep-th]

$f(R)$ GRAVITY $n = 8$

Position of FixedPoint ($\times 10^{-3}$)

n	$\tilde{g}_{0*}$	$\tilde{g}_{1*}$	$\tilde{g}_{2*}$	$\tilde{g}_{3*}$	$\tilde{g}_{4*}$	$\tilde{g}_{5*}$	$\tilde{g}_{6*}$	$\tilde{g}_{7*}$	$\tilde{g}_{8*}$
1	5.23	-20.1							
2	3.29	-12.7	1.51						
3	5.18	-19.6	0.70	-9.7					
4	5.06	-20.6	0.27	-11.0	-8.65				
5	5.07	-20.5	0.27	-9.7	-8.03	-3.35			
6	5.05	-20.8	0.14	-10.2	-9.57	-3.59	2.46		
7	5.04	-20.8	0.03	-9.78	-10.5	-6.05	3.42	5.91	
8	5.07	-20.7	0.09	-8.58	-8.93	-6.81	1.17	6.20	4.70

Critical exponents

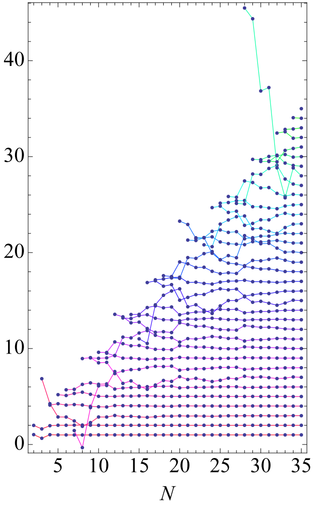
n	$Re\vartheta_1$	$Im\vartheta_1$	ϑ_2	ϑ_3	$Re\vartheta_4$	$Im\vartheta_4$	ϑ_6	ϑ_7	ϑ_8
1	2.38	2.17							
2	1.38	2.32	26.9						
3	2.71	2.27	2.07	-4.23					
4	2.86	2.45	1.55	-3.91	-5.22				
5	2.53	2.69	1.78	-4.36	-3.76	-4.88			
6	2.41	2.42	1.50	-4.11	-4.42	-5.98	-8.58		
7	2.51	2.44	1.24	-3.97	-4.57	-4.93	-7.57	-11.1	
8	2.41	2.54	1.40	-4.17	-3.52	-5.15	-7.46	-10.2	-12.3

$f(R)$ GRAVITY $n=8$ PREDICTIONS

Critical surface:

$$\tilde{g}_3 = 0.00061243 + 0.06817374\,\tilde{g}_0 + 0.46351960\,\tilde{g}_1 + 0.89500872\,\tilde{g}_2$$
$$\tilde{g}_4 = -0.00916502 - 0.83651466\,\tilde{g}_0 - 0.20894019\,\tilde{g}_1 + 1.62075130\,\tilde{g}_2$$
$$\tilde{g}_5 = -0.01569175 - 1.23487788\,\tilde{g}_0 - 0.72544946\,\tilde{g}_1 + 1.01749695\,\tilde{g}_2$$
$$\tilde{g}_6 = -0.01271954 - 0.62264827\,\tilde{g}_0 - 0.82401181\,\tilde{g}_1 - 0.64680416\,\tilde{g}_2$$
$$\tilde{g}_7 = -0.00083040 + 0.81387198\,\tilde{g}_0 - 0.14843134\,\tilde{g}_1 - 2.01811163\,\tilde{g}_2$$
$$\tilde{g}_8 = 0.00905830 + 1.25429854\,\tilde{g}_0 + 0.50854002\,\tilde{g}_1 - 1.90116584\,\tilde{g}_2$$

$f(R)$ GRAVITY $n = 35$



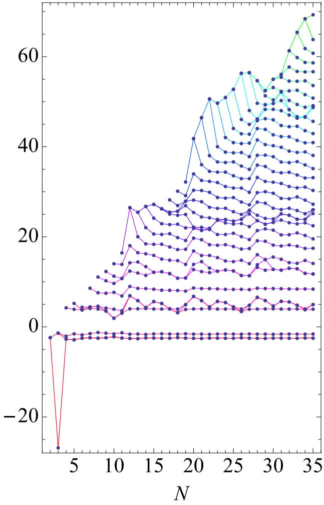
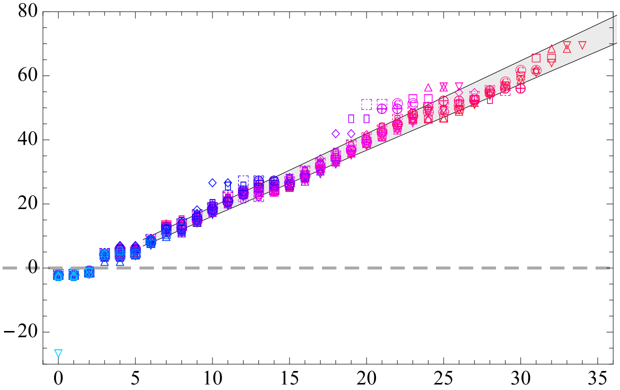


Figure: Left: couplings Right: scaling exponents

$f(R)$ GRAVITY $n = 35$



ENTER MATTER

- because it’s there
- because it may help (large *N* limit)
- because pure gravity has no local observables
- because experimental constraints more likely

[P. Donà, A. Eichhorn, R.P. arXiv:1311.2898 [hep-th](2013)]

Perturbative beta functions with matter

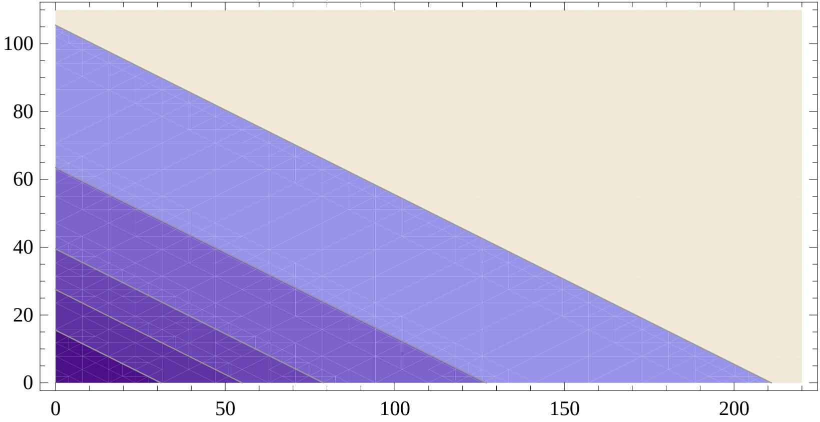
$$\beta_{\tilde{G}} = 2\tilde{G} + \frac{\tilde{G}^2}{6\pi}\left(N_S + 2N_D - 4N_V - 46\right),$$

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{\tilde{G}}{4\pi}\left(N_S - 4N_D + 2N_V + 2\right) + \frac{\tilde{G}\tilde{\Lambda}}{6\pi}\left(N_S + 2N_D - 4N_V - 16\right)$$

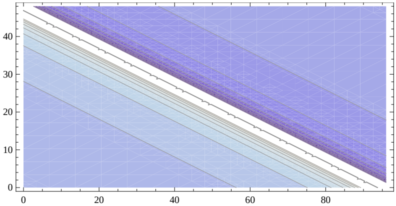
$$\tilde{\Lambda}_* = -\frac{3}{4}\frac{N_S - 4N_D + 2N_V + 2}{N_S + 2N_D - 4N_V - 31},$$

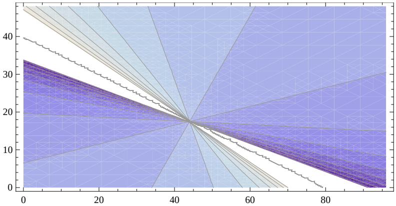
$$\tilde{G}_* = -\frac{12\pi}{N_S + 2N_D - 4N_V - 46}.$$

Exclusion plots $N_V = 0, 6, 12, 24, 45$



Position of FP for $N_V = 12$



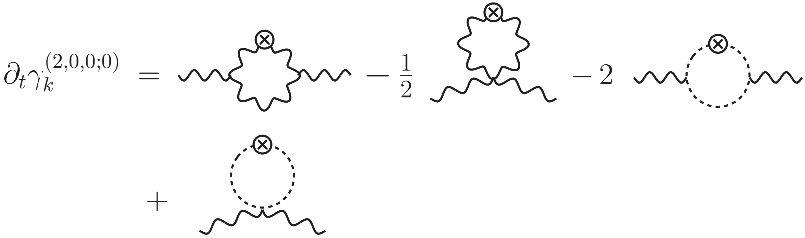


Truncated FRGE, bimetric formalism

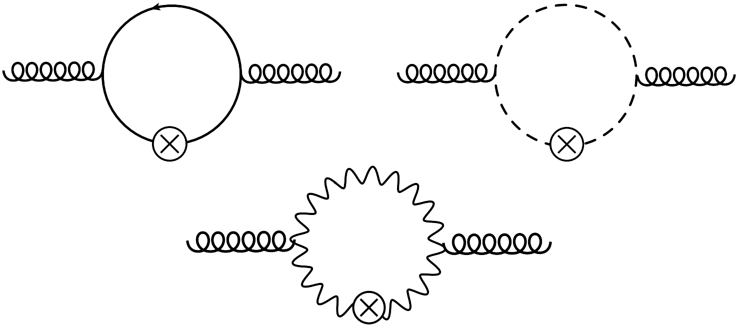
$$
\begin{aligned}
\Gamma_k(\bar{g}, h) &= \frac{1}{16\pi G}\int d^dx\sqrt{\bar{g}}\left(-\bar{R}+2\Lambda\right) \\
&+\frac{Z_h}{2}\int d^dx\sqrt{\bar{g}}\,h_{\mu\nu}K^{\mu\nu\alpha\beta}((-\bar{D}^2-2\Lambda)\mathbf{1}^{\rho\sigma}_{\alpha\beta}+W^{\rho\sigma}_{\alpha\beta})h_{\rho\sigma} \\
&-\sqrt{2}Z_c\int d^dx\sqrt{\bar{g}}\;\bar{c}_\mu\Big(\bar{D}^\rho\bar{g}^{\mu\kappa}g_{\kappa\nu}D_\rho+\bar{D}^\rho\bar{g}^{\mu\kappa}g_{\rho\nu}D_\kappa-\bar{D}^\mu\bar{g}^{\rho\sigma}g_{\rho\nu}D_\sigma\Big)c^\nu
\end{aligned}
$$

$$
\begin{aligned}
S_S &= \frac{Z_S}{2}\int d^dx\sqrt{g}\,g^{\mu\nu}\sum_{i=1}^{N_S}\partial_\mu\phi^i\partial_\nu\phi^i \\
S_D &= iZ_D\int d^dx\sqrt{g}\sum_{i=1}^{N_D}\bar{\psi}^i\nabla\!\!\!\!/\,\psi^i, \\
S_V &= \frac{Z_V}{4}\int d^dx\sqrt{g}\sum_{i=1}^{N_V}g^{\mu\nu}g^{\kappa\lambda}F^i_{\mu\kappa}F^i_{\nu\lambda}+\ldots
\end{aligned}
$$

Graviton+ghost contributions to η_h

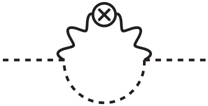


MATTER CONTRIBUTION TO η_h

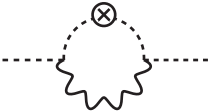


GRAVITON+GHOST CONTRIBUTIONS TO η_C

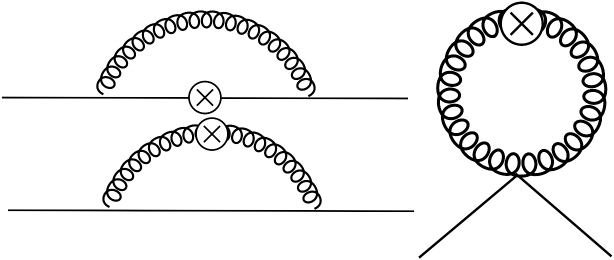
$$\partial_t \gamma_k^{(0,1,1;0)} =$$



$+$



GRAVITON CONTRIBUTION TO MATTER η



General structure of anomalous dimensions

For $\Psi = h, c, S, D, V$,

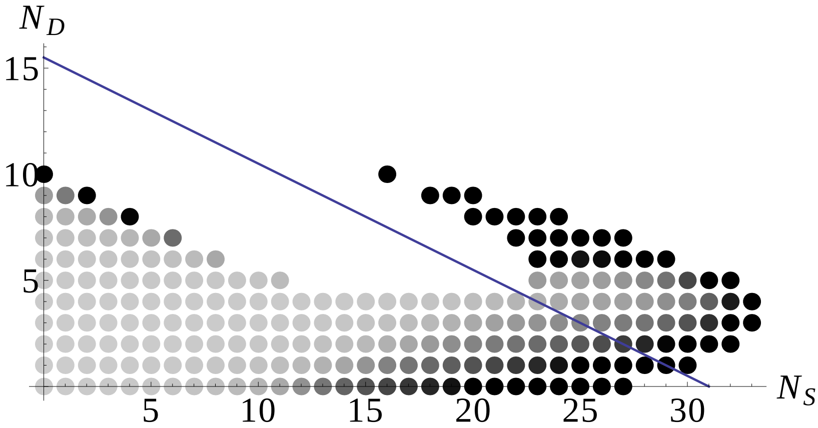
$$\eta_\Psi = -\frac{1}{Z_\Psi} k \frac{dZ_\Psi}{dk}$$

$$\vec{\eta} = (\eta_h, \eta_c, \eta_S, \eta_D, \eta_V)$$

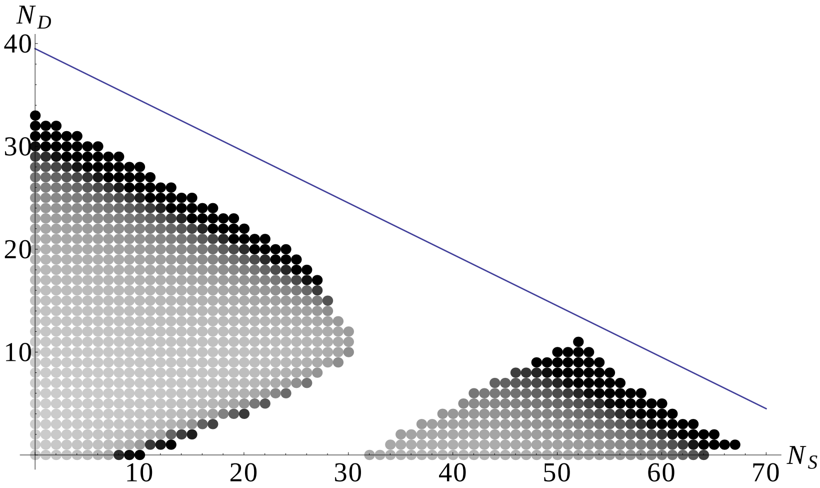
$$\vec{\eta} = \vec{\eta}_1(\tilde{G}, \tilde{\Lambda}) + \mathbf{A}(\tilde{G}, \tilde{\Lambda})\vec{\eta}$$

- one loop anomalous dimensions $\vec{\eta} = \vec{\eta}_1$
- RG improved anomalous dimensions $\vec{\eta} = (\mathbf{1} - \mathbf{A})^{-1}\vec{\eta}_1$

Exclusion plot $N_V = 0$



EXCLUSION PLOT $N_V = 12$



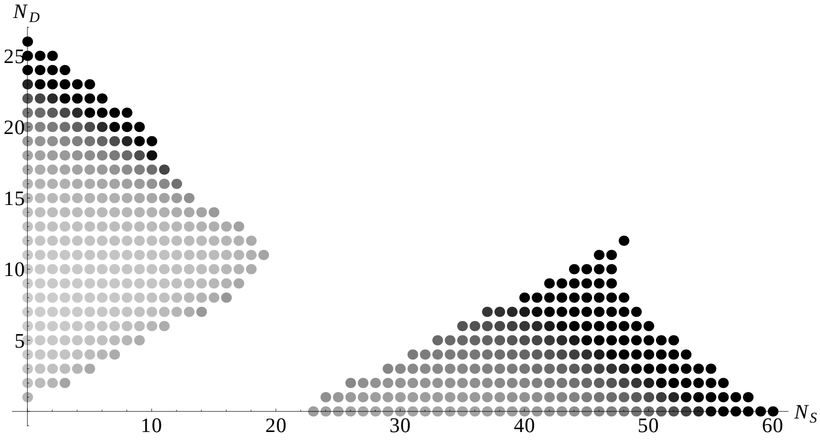
STANDARD MODEL MATTER

	1L-II	full-II	1L-Ia	full-Ia
$\tilde{\Lambda}_*$	-2.399	-2.348	-3.591	-3.504
$\tilde{G}_*$	1.762	1.735	2.627	2.580
θ_1	3.961	3.922	3.964	3.919
θ_2	1.644	1.651	2.178	2.187
η_h	2.983	2.914	4.434	4.319
η_c	-0.139	-0.129	-0.137	-0.125
η_S	-0.076	-0.072	-0.076	-0.073
η_D	-0.015	0.004	-0.004	0.016
η_V	-0.133	-0.145	-0.144	-0.158

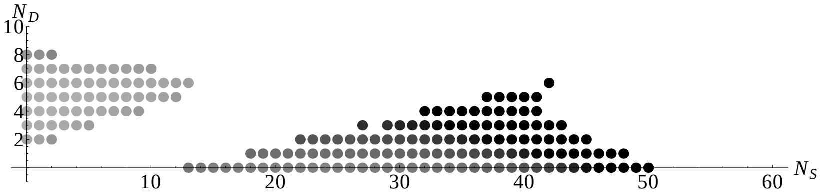
SPECIFIC MODELS

model	N_S	N_D	N_V	$\tilde{G}_*$	$\tilde{\Lambda}_*$	θ_1	θ_2	η_h
no matter	0	0	0	0.77	0.01	3.30	1.95	0.27
SM	4	45/2	12	1.76	-2.40	3.96	1.64	2.98
SM +dm scalar	5	45/2	12	1.87	-2.50	3.96	1.63	3.15
SM+ 3 ν's	4	24	12	2.15	-3.20	3.97	1.65	3.71
SM+3ν's + axion+dm	6	24	12	2.50	-3.62	3.96	1.63	4.28
MSSM	49	61/2	12	-	-	-	-	-
SU(5) GUT	124	24	24	-	-	-	-	-
SO(10) GUT	97	24	45	-	-	-	-	-

EXCLUSION PLOT $N_V = 12$, $d = 5$



Exclusion plot $N_V = 12$, $d = 6$



Functional truncations

ERGE well suited to study flow of potential

$$\Gamma_k[\phi] = \int d^d x \left(V(\phi^2) + \frac{1}{2}(\partial\phi)^2 \right)$$

Successfully reproduces properties of Wilson-Fisher fixed point.

GRAVITY+SCALAR

$$\Gamma_k[g,\phi] = \int d^d x \sqrt{g} \left(V(\phi^2) - F(\phi^2) R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right)$$

G. Narain, R.P., Class. and Quantum Grav. 27, 075001 (2010)
T. Henz, J. Pawlowski, A. Rodigast and C. Wetterich, Phys. Lett. B727 (2013) 298
D. Benedetti and F. Guarnieri, New J. of Phys. (2014) 053051

Functional flow of F, V

$$\partial_t V = \frac{k^4}{192\pi^2}\left\{6 + \frac{30\,V}{\Psi} + \frac{6(k^2\,\Psi + 24\,\phi^2\,k^2\,F'\,\Psi' + k^2\,F\Sigma_1)}{\Delta} + \left(\frac{4}{F} + \frac{5\,k^2}{\Psi} + \frac{k^2\,\Sigma_1}{\Delta}\right)\partial_t F + \frac{24\,\phi^2\,k^2\,\Psi'}{\Delta}\,\partial_t F'\right\}\,,$$

$$\partial_t F = \frac{k^2}{2304\pi^2}\left\{150 + \frac{120\,k^2\,F\,(3\,k^2 F - V)}{\Psi^2} - \frac{24}{\Delta}\left(24\,\phi^2\,k^2\,F'\,\Psi' + k^2\,\Psi + k^2\,F\Sigma_1\right)\right.$$

$$-\frac{36}{\Delta^2}\Big[-4\,\phi^2\,(6\,k^4\,F'^2 + \Psi'^2)\,\Delta + 4\,\phi^2\,\Psi\,\Psi'\,(7\,k^2\,F' - V')\,(\Sigma_1 - k^2) + 4\,\phi^2\Sigma_1\,(7\,k^2\,F' - V')\,(2\,\Psi\,V' - V\,\Psi')$$

$$+2\,k^4\,\Psi^2\,\Sigma_2 + 48\,k^4\,F'\,\phi^2\,\Psi\,\Psi'\,\Sigma_2 - 24\,k^4 F\,\phi^2\,\Psi'^2\,\Sigma_2\Big]$$

$$-\frac{\partial_t F}{F}\left[30 - \frac{10\,k^2 F\,(7\,\Psi + 4\,V)}{\Psi^2} + \frac{6}{\Delta^2}\Big(k^2\,F\,\Sigma_1\,\Delta + 4\,\phi^2\,V'\,\Psi'\,\Delta - 24\,k^4\,F\,\phi^2\,\Psi'^2\,\Sigma_2\right.$$

$$\left.\left.-4\,\phi^2\,k^2\,F\,\Psi'\,\Sigma_1(7\,k^2\,F' - V')\Big)\right] + \partial_t F'\,\frac{24\,k^2\,\phi^2}{\Delta^2}\Big[(k^2\,F' + 5\,V')\Delta - 12\,k^2\,\Psi\,\Psi'\,\Sigma_2 - 2\,(7\,k^2 F' - V')\,\Psi\,\Sigma_1\Big]\right\}$$

where we have defined the shorthands:

$$\Psi = k^2\,F - V\,;\quad \Sigma_1 = k^2 + 2\,V' + 4\,\phi^2\,V''\;;\quad \Sigma_2 = 2\,F' + 4\,\phi^2\,F''\;;\quad \Delta = \left(12\,\phi^2\,\Psi'^2 + \Psi\,\Sigma_1\right).$$

Polynomial truncations

$$\tilde{V}(\tilde{\phi}^2) = \tilde{\lambda}_0 + \tilde{\lambda}_2\tilde{\phi}^2 + \tilde{\lambda}_4\tilde{\phi}^4 + \ldots$$
$$\tilde{F}(\tilde{\phi}^2) = \tilde{\xi}_0 + \tilde{\xi}_2\tilde{\phi}^2 + \ldots$$

$$\partial_t\tilde{\lambda}_4 = \frac{9\lambda_4^2}{2\pi^2} + \frac{\tilde{G}\lambda_4}{\pi} + \ldots$$

[used in M. Shaposhnikov and C. Wetterich, Phys.Lett. B683, 196 (2010)]

OLD RESULTS

Only “Gaussian matter fixed point”

In $d = 3$ no trace of gravitationally coupled WF fixed point.
In polynomial expansion, all coefficients of ϕ^2 are negative.

New approach

Exponential parametrization:

$$g_{\mu\nu} = \bar{g}_{\mu\rho}(e^{h})^{\rho}{}_{\nu}$$

A priori theoretical motivation: functional integral over positive definite metrics.

Physical gauge:

$$h_{\mu\nu} = h^{TT}_{\mu\nu} + \bar{\nabla}_{\nu}\xi_{\nu} + \bar{\nabla}_{\nu}\xi_{\mu} + \bar{\nabla}_{\mu}\bar{\nabla}_{\nu}\sigma - \frac{1}{d}\bar{g}_{\mu\nu}\bar{\nabla}^{2}\sigma + \frac{1}{d}\bar{g}_{\mu\nu}h$$

$h = constant$,

$\xi'^{\mu} \equiv \sqrt{-\bar{\nabla}^{2} - \frac{\bar{R}}{d-1}}\,\xi^{\mu} = 0$

R.P., G.P. Vacca, Eur.Phys.J. C75 (2015) 5, 188, arXiv:1501.00888 [hep-th]

FLOW EQUATIONS $d = 3$

R.P., G.P. Vacca, Eur.Phys.J. C75 (2015) 5, 188, arXiv:1501.00888 [hep-th]

$$\dot{v} = -3\,v + \frac{1}{2}\phi\, v' + \frac{f + 4f'^2}{6\pi^2\left(4f'^2 + f(1+v'')\right)} + O(\dot{f})$$

$$\dot{f} = -f + \frac{1}{2}\phi\, f' + \frac{25}{36\pi^2} + f\,\frac{(f+4f'^2)(1+3v''-2f'') + 2fv''^2}{12\pi^2(4f'^2 + f(1+v''))^2} + O(\dot{f})$$

Compare with equation for pure scalar in LPA

$$\dot{v} = -3\,v + \frac{1}{2}\phi\, v' + \frac{1}{6\pi^2(1+v'')}$$

Gravitationally dressed Wilson-Fisher fixed point

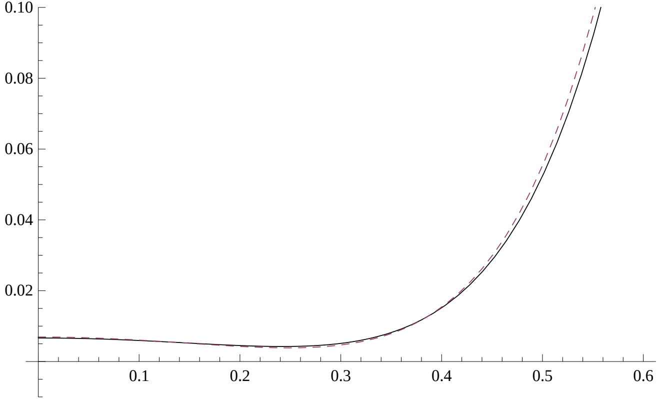


Figure: Solid curve: potential with gravity; dashed curve: LPA approximation of potential of Wilson-Fisher fixed point without gravity.

FLOW EQUATIONS $d = 4$

$$\dot{v} = -4v + \varphi v' + \frac{1}{16\pi^2} + \frac{f + 3f'^2}{32\pi^2\left(3f'^2 + f(1+v'')\right)} + O(\dot{f})$$

$$\dot{f} = -2f + \varphi f' + \frac{37}{384\pi^2} + f\,\frac{(f + 3f'^2)(1 - 3f'' + 3v'') + 2fv''^2}{96\pi^2(3f'^2 + f(1+v''))^2} + O(\dot{f})$$

ANALYTIC SOLUTIONS $d = 4$ $N = 1$

FP1

$$v_* = \frac{3}{128\pi^2} \approx 0.00237 \; ; \qquad f_* = \frac{41}{768\pi^2} \approx 0.00541$$

FP2

$$v_* = \frac{3}{128\pi^2} \approx 0.00237 \; ; \qquad f_* = \frac{37}{768\pi^2} + \frac{1}{6}\varphi^2 \approx 0.0049 + 0.167\varphi^2$$

FP3

$$v_* = \frac{3}{128\pi^2} \approx 0.002374 \; ; \qquad f_* = -\frac{41}{420\pi^2}\varphi^2 \approx -0.0976\varphi^2$$

Flow equations $O(N)$-invariant scalars

$$\begin{aligned} \dot{v} &= \ldots - c_d \frac{d(N-1)\rho}{12(\rho + v'(\rho))} \\ \dot{f} &= \ldots - c_d \frac{(N-1)\rho f'(\rho)}{(\rho + v'(\rho))^2} \end{aligned}$$

$c_d^{-1} = (4\pi)^{d/2}\Gamma(d/2+1)$
$\rho = \phi^2$

P. Labus, R.P., G.P. Vacca, Phys.Lett. B753 (2016) 274-281
arXiv:1505.05393 [hep-th]

ANALYTIC SOLUTIONS $O(N)$ MODEL, ONE LOOP

FP1

$$\begin{aligned} v_* &= c_d\left[\frac{(d-1)(d-2)}{2d}+\frac{N-1}{d}\right] , \\ f_* &= -c_d\frac{d^5-4d^4-7d^3-50d^2+60d+24}{24d(d-1)(d-2)}-c_d\frac{(N-1)d}{12(d-2)} \end{aligned}$$

$f_* > 0$
$N = 1 \Rightarrow d < 6.17$
$d = 4 \Rightarrow N \leq 11$

Analytic solutions $O(N)$ model, RG improved

FP1

$$
\begin{aligned}
v_* &= c_d \left[\frac{(d^2-1)(d-2)}{d(d+2)} + \frac{N-1}{d} \right] , \\
f_* &= -c_d \frac{d^6 - 2d^5 - 15d^4 - 46d^3 + 38d^2 + 96d - 24}{12d(d-1)(d^2-4)} \\
&\qquad - c_d \frac{(N-1)d}{12(d-2)}
\end{aligned}
$$

$f_* > 0$

$N = 1 \;\Rightarrow d < 5.73$

$d = 4 \;\Rightarrow N \leq 14$

Analytic solutions $O(N)$ model, one loop

FP2

$$
\begin{aligned}
v_* &= c_d\left[\frac{(d-1)(d-2)}{2d}+\frac{N-1}{d}\right] \\
f_* &= c_d\left[\frac{d^5-4d^4-7d^3-50d^2+84d+24}{24d(d-1)(d-2)}\right. \\
&\qquad \left.-\frac{(N-1)(d^2-d+12)}{12(d-1)(d-2)}\right]+\frac{\varphi^2}{2(d-1)}
\end{aligned}
$$

$f_* > 0$
$N = 2 \;\Rightarrow d < 5.8$
$d = 4 \;\Rightarrow N < 5.6$
Not present in improved eqs.

ANALYTIC SOLUTIONS $O(N)$ MODEL

Same in one-loop and improved eqs.
FP3 (upper sign), FP4 (lower sign)

$d = 3$,

$$v_* = \frac{N}{18\pi^2}\,, \qquad f_* = -\frac{9N - 80 \pm \sqrt{9N^2 - 264N + 5296}}{96(N-1)}\varphi^2$$

$d = 4$

$$v_* = \frac{2+N}{128\pi^2}\,, \qquad f_* = -\frac{6N - 41 \pm \sqrt{4N^2 - 100N + 1321}}{48(N-1)}\varphi^2$$

FP3 also exists for $N = 1$ but $f_* < 0$ always
FP4 exists and acceptable for $0 < N < 15.3$ ($d = 3$) and $1 < N < 11.2$ ($d = 4$)

Large N

$\rho = \varphi^2/2$, $w = u'$, $u \to Nu$ etc

$$\dot{u} = -du + (d-2)\rho\, u' + \frac{c_d}{1+u'}$$

$$\dot{f} = -(d-2)f + \left((d-2)\rho - \frac{c_d}{(1+u')^2}\right) f' - \frac{d}{12}\frac{c_d}{1+u'}$$

$$\rho = c_d \frac{d}{4}\, {}_2F_1\left(2, 1-\frac{d}{2}; 2-\frac{d}{2}; -w\right)$$

$$f(\rho) = g(w) = -\frac{d\, c_d}{12(d-2)}\, {}_2F_1\left(1, 1-\frac{d}{2}; 2-\frac{d}{2}; -w\right) < 0$$

$f(R)$ GRAVITY, FUNCTIONAL TREATMENT

Do not expand $f(R)$ but write flow equation for f

$$\partial_t \tilde{f}(\tilde{R}) = \beta(\tilde{f}, \tilde{f}', \tilde{f}'', \tilde{f}''')$$

where $\tilde{R} = R/k^2$, $\tilde{f} = f/k^4$.
For large $\tilde{R}$

$$\tilde{f}(\tilde{R}) = A\tilde{R}^2 \left(1 + \sum_{n>0} d_n \tilde{R}^{-n}\right)$$

Dario Benedetti, Francesco Caravelli, JHEP 1206 (2012) 017, Erratum-ibid. 1210 (2012) 157 arXiv:1204.3541 [hep-th]
Juergen A. Dietz, Tim R. Morris,JHEP 1301 (2013) 108 arXiv:1211.0955 [hep-th]

$f(R)$ GRAVITY, FUNCTIONAL TREATMENT

Theorem 1: $\Gamma_*(g_{\mu\nu}) = A_* \int d^4x \sqrt{g} R^2$, $A_* \neq 0$

Theorem 2: if $\tilde{f}_*$ exists, the spectrum of perturbations is discrete, real, and there are at most finitely many relevant direction.

D. Benedetti, arXiv:1301.4422 [hep-th]

Several studies have failed to find a satisfactory fixed point solution.

Split symmetry

Because of

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

the bare action is invariant under

$$\begin{aligned} \delta\bar{g}_{\mu\nu} &= \epsilon_{\mu\nu} \ , \\ \delta h_{\mu\nu} &= -\epsilon_{\mu\nu} \ . \end{aligned}$$

but the EAA $\Gamma_k(\mathbf{h}; \bar{\mathbf{g}})$ is not.

$$\frac{\delta^{(n)}\Gamma_k(h;\bar{g})}{\delta h^n} \neq \frac{\delta^{(n)}\Gamma_k(h;\bar{g})}{\delta \bar{g}^n}$$

Problem

Suppose we start from Hilbert action and expand

$$S(g) = S(\bar{g}) + \int S'(\bar{g})h + \int S''(\bar{g})h^2 + \int S'''(\bar{g})h^3 + \int S''''(\bar{g})h^4 + \dots$$

all contain the same Newton constant.
However if we apply the ERGE

$$\begin{aligned} \dot{\Gamma}(h;\bar{g}) &= \dot{\Gamma}(0;\bar{g}) + \int \dot{\Gamma}'(0;\bar{g})h + \int \dot{\Gamma}''(0;\bar{g})h^2 \\ &+ \int \dot{\Gamma}'''(0;\bar{g})h^3 + \int \dot{\Gamma}''''(0;\bar{g})h^4 + \dots \end{aligned}$$

each term gives a different “beta function”

Plan

- Write the anomalous Ward identity for the split symmetry or a subgroup thereof
- Solve it to eliminate from the EAA a number of fields equal to the number of parameters of the transformation
- Write the flow equation for the EAA depending on the remaining variables

Only carried through for $\epsilon_{\mu\nu} = \epsilon \bar{g}_{\mu\nu}$

The msWI

$$\delta_\epsilon \Gamma_k = \epsilon \partial_t \Gamma_k$$

Under finite transformations

$$\Gamma_k(h^{T\mu}{}_\nu, h^\perp, \bar{h}, C^*_\mu, C^\mu; \bar{g}_{\mu\nu}) = \Gamma_{\Omega^{-1}k}(h^{T\mu}{}_\nu, h^\perp, \bar{h} - 2d \log \Omega, C^*_\mu, C^\mu; \Omega^2 \bar{g}_{\mu\nu})$$

Solving the msWI

$$\Gamma_k(h^{T\mu}{}_\nu, h^\perp, \bar{h}, C^*_\mu, C^\mu; \bar{g}_{\mu\nu}) = \hat{\Gamma}_{\hat{k}}(h^{T\mu}{}_\nu, h^\perp, C^*_\mu, C^\mu; \hat{g}_{\mu\nu})$$

where e.g.

$$\hat{k} = \bar{V}^{1/d}k \; ; \qquad \hat{g}_{\mu\nu} = \bar{V}^{-2/d}\bar{g}_{\mu\nu}$$

We have eliminated *one* degree of freedom.

In linear parametrization
T. R. Morris, JHEP **1611** (2016) 160, arXiv:1610.03081 [hep-th]
N. Ohta, arXiv:1701.01506 [hep-th]

Continuum, covariant approach to QG

- perturbation theory fails at m_P, however....
- EFT approach useful at $E \ll m_P$
- can compute unambiguously non-local (IR) part of EA
- local terms not calculable in perturbation theory
- possible QFT completion requires another FP.

AS: STRENGTHS AND WEAKNESSES

- use powerful QFT tools
- bottom up approach
- guaranteed to give correct low energy limit
- highly predictive: only few parameters undetermined
- inclusion of matter relatively easy
- BSM physics constrained

but

- strong coupling problem
- scheme dependence
- off shell: gauge dependence (on its way to being solved)
- no observables computed yet

Current frontier I

Truly functional truncations can be studied, but are hard. (Less tolerant of bad approximations.)
In scalar-tensor gravity, using exponential parametrization and physical gauge the equations can be simplified to the point where one can find some nontrivial analytic solutions.

No such success in $f(R)$ gravity.

CURRENT FRONTIER II

Use of background field method makes effective action depend on two fields.
Split-symmetry Ward identities need to be taken into account.
This work has only begun.

OUTLOOK

Many open technical and conceptual problems.

AS may play role in BSM physics even aside from gravity.

Other techniques could play a role, e.g. two loop calculations, ϵ expansion, large N expansion etc.

Hvala na pozornosti