An instability of black holes in Anti-de Sitter space

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Black Hole uniqueness theorems

- A 4-dimensional, asymptotically flat, static (electro-) vacuum space-time that is non-singular on and outside an event horizon is given by the Schwarzschild (Reissner-Nordström) black hole solution.

[Israel]

- A 4-dimensional, asymptotically flat, stationary (electro-)vacuum space-time that is non-singular on and outside an event horizon is given by the Kerr-Newman black hole solution (determined by M, Q, J).

[Carter, Hawking, Robinson, Wald, …]

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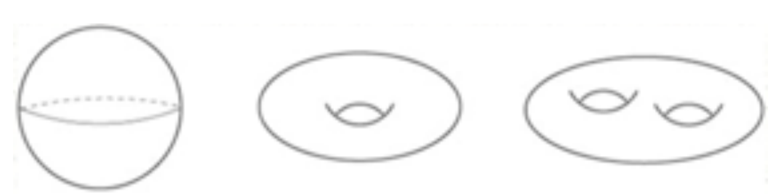
  [Carter, Hawking, Robinson, Wald, …]

Counterexamples

- Higher dimensions: Black ring solution in 5D, horizon has topology $S^2 \times S^1$ [Emparan, Reall]

- Other matter: ★ Yang-Mills fields [Volkov, Gal'tsov; …]
  ★ Scalar field $R - \nabla_\mu \Phi^* \nabla^\mu \Phi - \frac{1}{2} m^2 \Phi^* \Phi$ [Herdeiro, Radu]

- Different asymptotics: In 4D with anti-de Sitter asymptotics, horizon topology can be any Riemann-surface [Ammineborg, Bengtsson, Holst, Peldan]



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- Torus can be replaced by plane: *black brane*

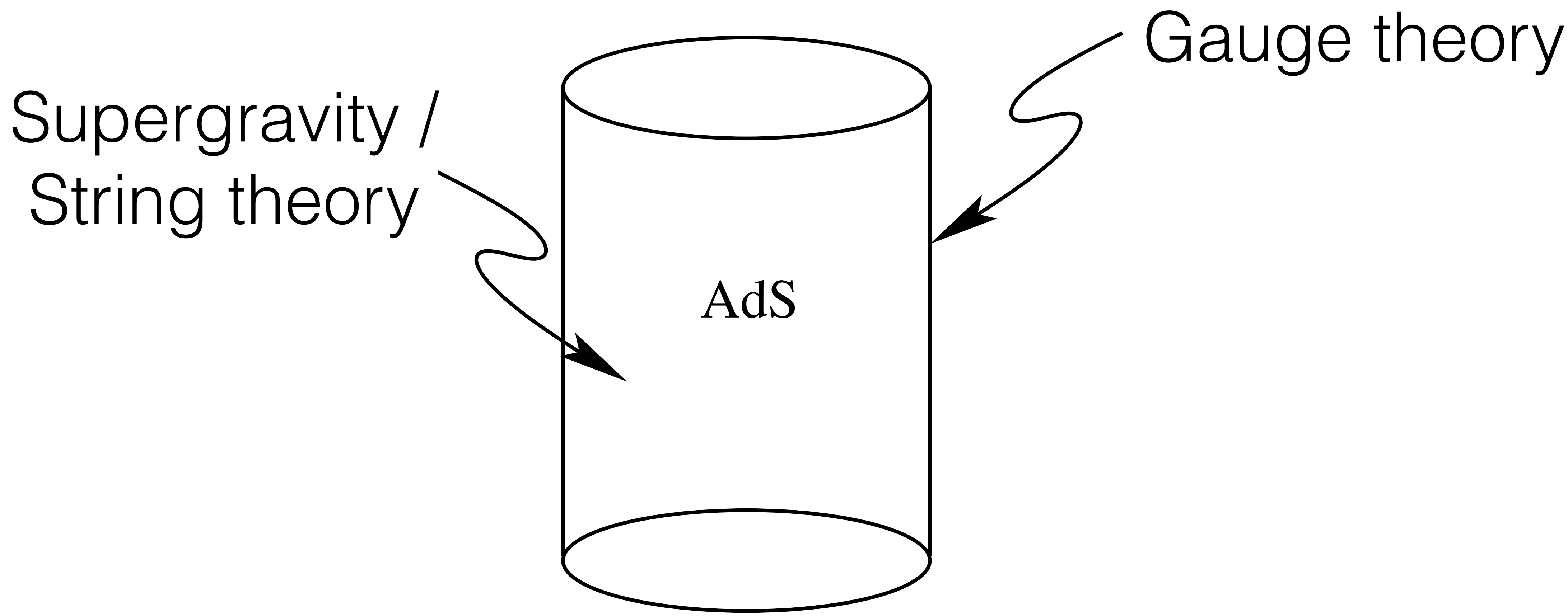
- There can be phase transitions between different black solutions when changing parameters, e.g. Reissner-Nordström (RN) black hole with AdS-asymptotics, can become unstable in the presence of

    - ★ Charged scalars („holographic superconductors") [Hartnoll, Herzog,Horowitz]
    - ★ Chern-Simons (CS) terms [Nakamura, Ooguri, Park]

  focus of this talk

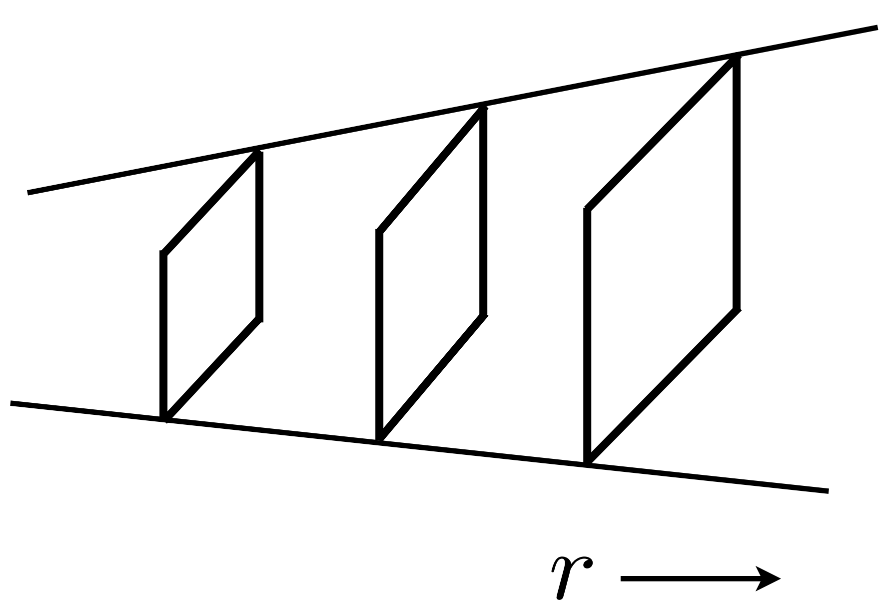
- Translates to phase transition in the dual field theory via AdS/CFT

AdS/CFT correspondence

- 

- Locally:

$$ds^2 \quad = \quad \frac{dr^2}{r^2} + r^2(-dt^2 + d\vec{x}^2)$$



- Original AdS/CFT correspondence: [Maldacena]

$\mathcal{N} = 4, SU(N)$ Yang-Mills in 't Hooft limit,

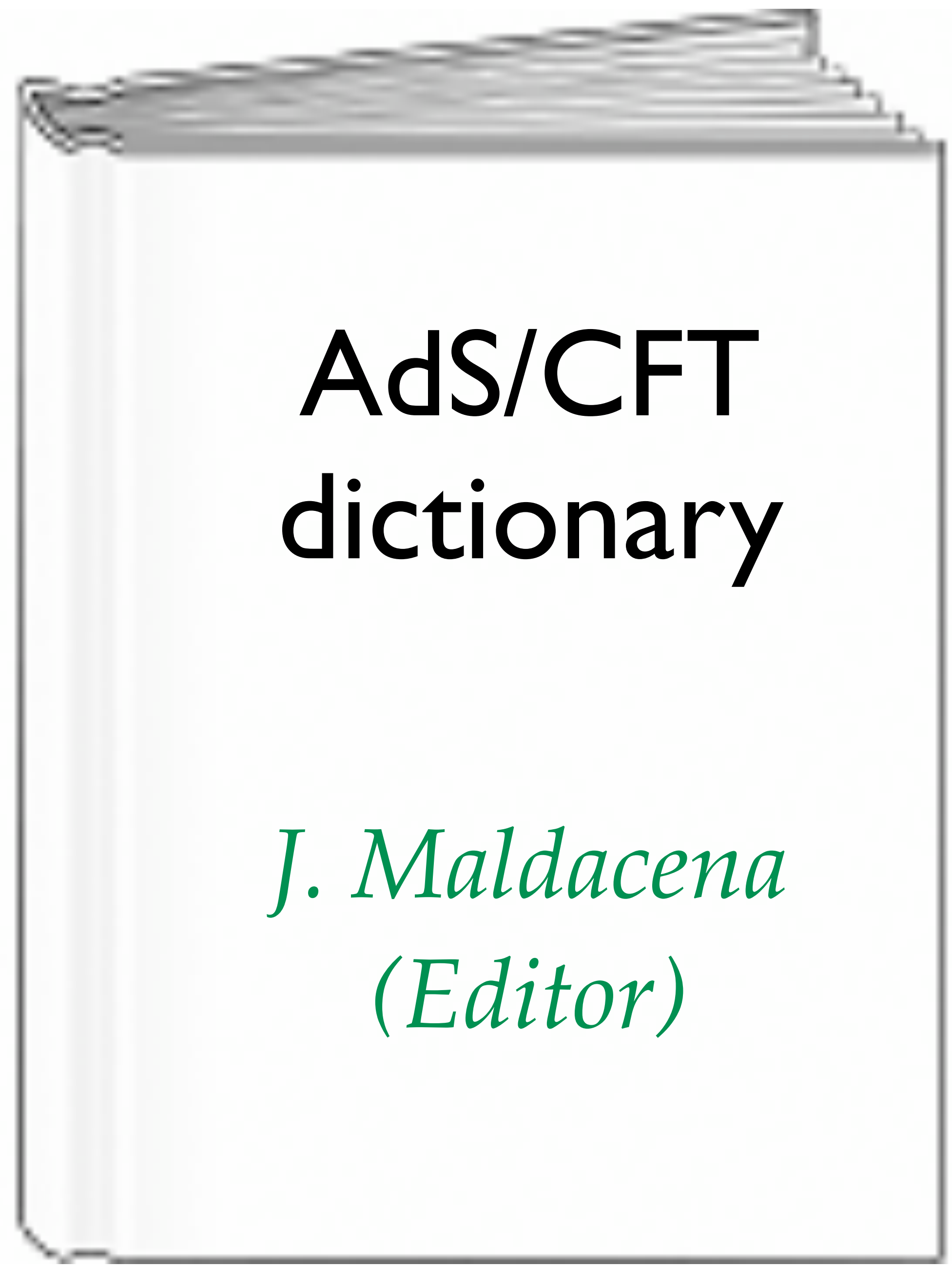
i.e. for large N, and large $\lambda_{\text{'tHooft}} = g_{YM}^2 N$

$$\equiv$$

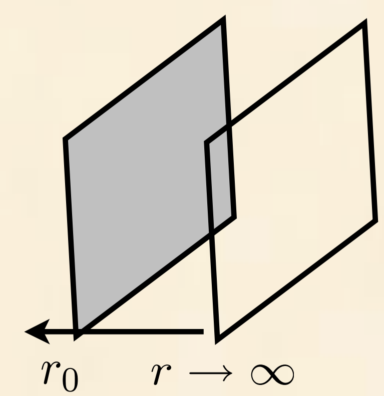
Supergravity in the space AdS_5

- Generalizes to other dimensions and other gauge theories

Short review of AdS/CFT at finite T and charge density



CFT	AdS
Vacuum	Empty AdS
	$$ds^2 = \frac{dr^2}{r^2} + r^2(-dt^2 + d\vec{x}^2)$$
Thermal ensemble at temperature T	AdS black brane



CFT	AdS
Vacuum	Empty AdS
	$$ds^2 = \frac{dr^2}{r^2} + r^2(-dt^2 + d\vec{x}^2)$$
Thermal ensemble at temperature T	$$ds^2 = \frac{dr^2}{r^2 f} + r^2(-f dt^2 + d\vec{x}^2)$$
	$$f = 1 - \frac{2M}{r^{D-1}}$$
	$$T = T_H \sim r_0 \sim M^{1/(D-1)}$$

CFT

Thermal ensemble at temperature T and charge density ρ

AdS

Charged black brane with gauge field

$$A_t \sim \frac{\rho}{r^{D-3}}$$

Reissner-Nordström (RN):

$$f = 1 - \frac{2M}{r^{D-1}} + \frac{Q^2}{r^{2(D-2)}}$$

with $Q \sim \rho$

CFT	AdS
Source and VEV of gauge invariant operator $\mathcal{O}_\phi$	$\phi(t, \vec{x}, r \to \infty)$
Examples:	
Energy-momentum tensor T_{ab}	Metric g_{ab}
Current J_a	Gauge field A_a

RN black hole

(4D, asymptotically flat, spherical horizon)

- Charged black hole solution of

$$\int d^4x\sqrt{-g}[R - F_{\mu\nu}F^{\mu\nu}]$$

- $$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)} + r^2 d\Omega^2$$

- $$A = \frac{Q}{r}dt$$

- RN metric can be written as

$$ds^2 = -\frac{\Delta}{r^2}dt^2 + \frac{r^2}{\Delta}dr^2 + r^2 d\Omega^2$$

with

$$\Delta = (r - r_+)(r - r_-)$$

where

$$r_\pm = M \pm \sqrt{M^2 - Q^2}$$

- r_+ : event horizon
- Black hole for $M \geq |Q|$
- $T_H = \frac{\sqrt{M^2 - Q^2}}{2Mr_+^2} \xrightarrow{|Q| \to M} 0$

Extremal RN black holes

- $|Q| = M$

- $T_H = 0$
- $r_+ = r_- = M$
- $ds^2 = -\left(1 - \frac{M}{r}\right)^2 dt^2 + \frac{dr^2}{\left(1 - \frac{M}{r}\right)^2} + r^2 d\Omega^2$

- Near horizon geometry: Introduce $r = M(1+\lambda)$

$$ds^2 \sim (-\lambda^2 dt^2 + M^2\lambda^{-2}d\lambda^2) + M^2 d\Omega^2$$

to lowest order in λ

$AdS_2 \times S^2$

Overview of remainder

- Review instability of 5D asymptotically AdS RN black brane [Nakamura, Ooguri, Park]
- Motivation to include higher derivative corrections
- Instability with higher derivative corrections [work in progress with Danny Brattan, Abhiram Kidambi and Amos Yarom]
- Outlook

Instability of Nakamura, Ooguri & Park

5D RN black holes in AdS unstable to helical phase if CS-term in

$$S = \int d^5x\sqrt{-g}[(R+12) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha}{6}\epsilon^{\mu\nu\rho\sigma\tau}A_\mu F_{\nu\rho}F_{\sigma\tau}]$$

large enough, i.e. $\alpha > \alpha_c \approx 0.2896$ [Nakamura, Ooguri, Park]

- In dual CFT, α determines the anomaly of the current dual to A

$$\partial_a J^a \sim \alpha\epsilon^{abcd}F_{ab}F_{cd}$$

3D CS-term

In 3D CS term leads to massive gauge fields:

[Deser, Jackiw, Templeton]

- $S = \int d^3x \left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{\kappa}{4}\epsilon^{\mu\nu\rho}F_{\mu\nu}A_\rho\right)$
- Equation of motion: $\partial_\mu F^{\mu\nu} + \frac{\kappa}{2}\epsilon^{\nu\rho\sigma}F_{\rho\sigma} = 0$
- Dual field strength: $\tilde{F}^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho}F_{\nu\rho}$ i.e. $F^{\mu\nu} = \epsilon^{\mu\nu\rho}\tilde{F}_\rho$
- Equation of motion and $\epsilon_{\nu\mu\rho}\epsilon^{\nu\alpha\beta} = (\delta^\alpha_\mu\delta^\beta_\rho - \delta^\beta_\mu\delta^\alpha_\rho)$

$$\Longrightarrow \partial_\rho\tilde{F}_\sigma - \partial_\sigma\tilde{F}_\rho - \kappa F_{\rho\sigma} = 0$$

- $$\partial_\rho \tilde{F}_\sigma - \partial_\sigma \tilde{F}_\rho - \kappa F_{\rho\sigma} = 0$$

$$\Longrightarrow \partial^\rho \partial_\rho \tilde{F}_\sigma - \partial_\sigma \underbrace{\partial^\rho \tilde{F}_\rho}_{=0} - \kappa \underbrace{\partial^\rho F_{\rho\sigma}}_{= -\kappa \tilde{F}_\sigma} = 0$$

- $$\Box \tilde{F}_\sigma + \kappa^2 \tilde{F}_\sigma = 0$$

  CS-term in 3D leads to mass term $\sim \kappa^2$

- [Nakamura, Ooguri, Park]: CS-term in 5D can turn the Maxwell theory tachyonic

Instability in 5D

- Instability most likely to occur at $T = 0$

  $\Rightarrow$ consider extremal RN

- Near horizon geometry $AdS_2 \times \mathbb{R}^3$

  $\Rightarrow$ look for modes with $m^2_{AdS_2} < m^2_{BF} = -\frac{1}{4r_2^2}$

  r_2 : AdS_2-radius

The BF bound

[Breitenlohner, Freedman]

- AdS_D -spacetime is stable even in presence of scalars with negative mass-square in the Lagrangian if

$$m^2 r_D^2 \geq -(D-1)^2/4$$

AdS_D-radius

- Positive contribution to energy from gradient terms dominates over negative potential energy.

- Reminder: $ds^2_{AdS_D} = \frac{r_D^2}{r^2} dr^2 + \frac{r^2}{r_D^2}(-dt^2 + d\vec{x}^2)$

- Energy

further non-negative terms

$$E = \int d^{D-1}x \sqrt{-g}\,[g^{rr}\partial_r\phi\partial_r\phi + m^2\phi^2 + \ldots]$$

$\sim r^{D-2}$ $\sim r^{-2\lambda}$

convergent if $\phi \sim r^{-\lambda}$ for $\lambda > \left(\frac{D-1}{2}\right)$
(i.e. $D - 1 - 2\lambda < 0$)

- Thus $g^{rr}\partial_r\phi\partial_r\phi + m^2\phi^2 \sim \left(\frac{\lambda^2}{r_D^2} + m^2\right)\phi^2$

i.e. $E > 0$ if $m^2 r_D^2 \geq -(D-1)^2/4$

Near horizon analysis

- Ansatz: [Nakamura, Ooguri, Park; Donos, Gauntlett]

$\mathbb{R}^3$

$$ds^2 = \frac{-dt^2 + dr^2}{12 r^2} + d\vec{x}^2 + Q(t,r)^2 dt^2 + 2Q(t,r)\omega dt$$

AdS_2 with $r \to \frac{r_2^2}{r}$ and $r_2^2 = \frac{1}{12}$

with

$$\omega = \cos(kx_1)dx_2 - \sin(kx_1)dx_3$$

- Killing vectors: $\partial_{x_2}, \ \partial_{x_3}$

$$\partial_{x_1} - k(x_2\partial_{x_3} - x_3\partial_{x_2})$$

- $A = \frac{E}{12r}dt + b(t,r)\omega$ with $E = 2\sqrt{6}$

near horizon electrical field

- To linear order in b, Q :

$$(\Box_{AdS_2} - k^2)\psi + E\,\Box_{AdS_2} b = 0$$

$$(\Box_{AdS_2} - k^2)b - 4\alpha E k b + E\psi = 0$$

with $\psi = -12r^2 Q'$

- Strategy: (1) Determine effective mass $m^2(k, \alpha)$

(2) Determine $k_0(\alpha)$ minimizing $m^2(k, \alpha)$ for fixed $\alpha \Rightarrow m^2_{\min}(\alpha) = m^2(k_0(\alpha), \alpha)$

(3) Find α_c for which $m^2_{\min}(\alpha) < m^2_{BF}$ for $\alpha > \alpha_c$

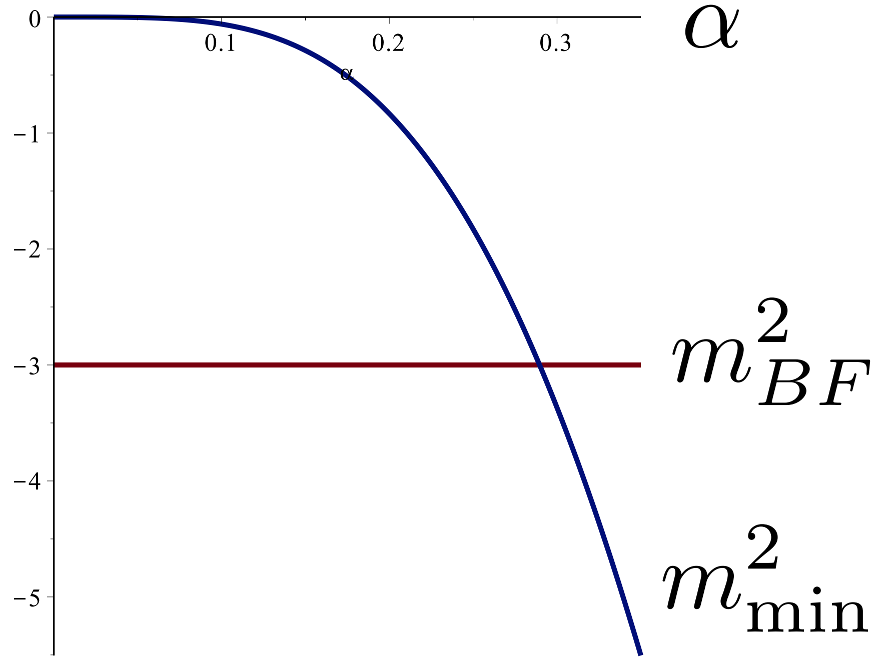
Concretely:

(1) $$\det\begin{pmatrix} m^2-k^2 & Em^2 \\ E & m^2-k^2-4\alpha Ek \end{pmatrix}=0$$

$$\Rightarrow m^2=\frac{1}{2}\left(2k^2+E^2+4\alpha Ek-E\sqrt{E^2+8\alpha Ek+4k^2(1+4\alpha^2)}\right)$$

(2) $$k_0=E\frac{2\alpha+4\alpha^3+\alpha\sqrt{1+4\alpha^2+16\alpha^4}}{1+4\alpha^2}$$

(3) $\alpha_c=0.2896$

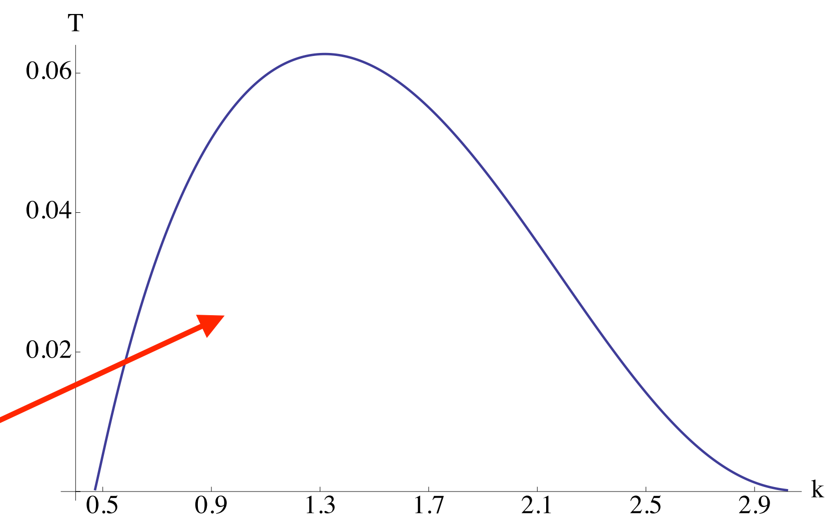


- α_c coincides with value obtained by looking for normalizable fluctuations in full geometry, which grow in time.

[Nakamura, Ooguri, Park]

- For $\alpha > \alpha_c$ instability appears for range of k and T
e.g. $\frac{\alpha}{\alpha_c} \approx 1.47$

Instability occurs in this region



[Taken from: Donos, Gauntlett]

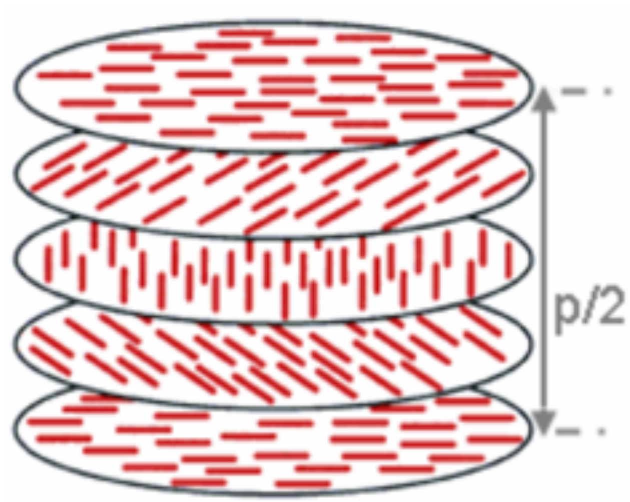
- Solution with particular $k(T)$ minimizes free energy at fixed T

[Donos, Gauntlett]

- End point of instability: Black brane with helical order found numerically by Donos & Gauntlett

- Holographic interpretation: Gauge field A dual to a current J in CFT; J acquires helical order.

- Ground states with helical order exist in condensed matter physics, e.g. in some displays with liquid crystals



[Wikipedia:Cholesteric liquid crystals]

- 5D minimal $\mathcal{N} = 2$ SUGRA:

$$\alpha_s = \frac{1}{2\sqrt{3}} \approx 0.2887 < \alpha_c$$

- But $\dfrac{\alpha_c - \alpha_s}{\alpha_c} \approx 0.003$

- Could higher derivative corrections to α_s and α_c lead to $\alpha_s > \alpha_c$, i.e. an instability?

- Minimal 5D SUGRA relevant: Every $\mathcal{N} = 2$ super-symmetric compactification to AdS_5 can be truncated to minimal $\mathcal{N} = 2$ SUGRA.

[Gauntlett, Varela]

Higher derivative terms

- Most general form up to 4 derivatives (modulo field redefinitions and partial integration) [Myers, Paulos, Sinha]

$$S = S_0 + \int d^5x\sqrt{-g}\,[c_1 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + c_2 R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$
$$+c_3(F^2)^2 + c_4 F^4 + c_5 \epsilon^{\mu\nu\rho\sigma\tau}A_\mu R_{\nu\rho\alpha\beta}R_{\sigma\tau}{}^{\alpha\beta}]$$

- $c_1 = \frac{1}{8}\frac{c-a}{c}$

$$T^a_a = \frac{c}{16\pi^2}W_{abcd}W^{abcd} - \frac{a}{16\pi^2}(R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2)$$

Weyl-tensor

4D Euler density

- If A vector in $\mathcal{N}=2$ gravity supermultiplet:

$$c_2 = -\frac{c_1}{2}\,, \quad c_3 = \frac{c_1}{24}\,, \quad c_4 = -\frac{5c_1}{24}\,, \quad c_5 = \frac{c_1}{2\sqrt{3}}\,, \quad \alpha_s = \frac{1-288c_1}{2\sqrt{3}}$$

- Leave c_i arbitrary for moment, but for sensible derivative expansion need $\forall_i : c_i \ll 1$

- Following above strategy, need to take into account

[Myers, Paulos, Sinha]

★ Correction to condition for extremality:

$$\frac{q^2}{r_0^6} = 2[1 - 48(c_1 - 2(2c_3 + c_4))]$$

★ Correction to AdS_2-radius: $r_2^2 = \frac{1}{12} + (4c_2 + 16c_3 + 8c_4)$

$$\Rightarrow m_{BF}^2 = -\frac{1}{4r_2^2} = -(3 - 144c_2 - 576c_3 - 288c_4)$$

Ansatz (corrections to background from [Myers, Paulos, Sinha]):

$$ds^2 = \frac{-dt^2 + dr^2}{(12 - 576c_2 - 2304c_3 - 1152c_4)r^2} + d\vec{x}^2 + Q^2 dt^2 + 2Q dt \omega_2$$

$$A = \left(\frac{2\sqrt{6}}{12} - 4\sqrt{6}(c_1 + 2c_2 + 4c_3 + 2c_4) \right) r^{-1} dt + b\omega_2$$

Plug this into Einstein & Maxwell eqs.

Result:

$$\alpha_c = \alpha_c^{(0)} + 11.82 c_1 + 37.06 c_2 + 183.67 c_3 + 55.00 c_4 - 12.61 c_5$$

[cf. also Takeuchi]

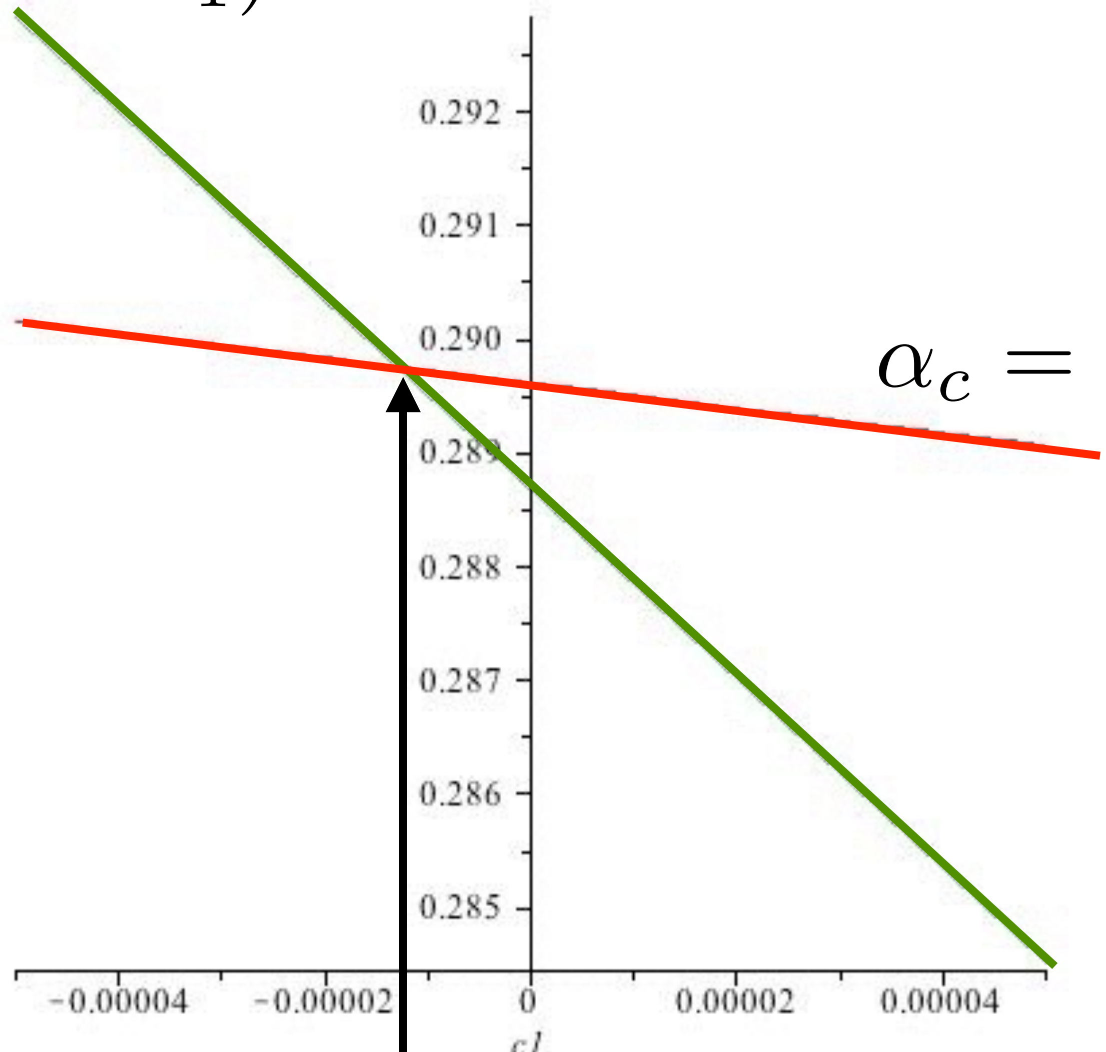
i.e. in supersymmetric case:

$$\alpha_c = \alpha_c^{(0)} - 14.16 c_1$$

$$\frac{1}{8}\frac{c-a}{c}$$

But α_s also decreases with positive c_1

$$\alpha_s = \alpha_s^{(0)}(1 - 288c_1)$$



$$\alpha_c = \alpha_c^{(0)} - 14.16c_1$$

$$c_1 = \frac{1}{8}\frac{c-a}{c} = -1.30027 \times 10^{-5}$$

Results on $c - a$

- In $\mathcal{N} = 1$ SCFT: $\frac{1}{2} \leq \frac{a}{c} \leq \frac{3}{2}$ [Hofmann, Maldacena]

  with $\frac{a}{c} = \frac{3}{2}$ for free theory with only vector multiplets.

  However, $\frac{a-c}{c} = \frac{1}{2} \not\ll 1$

- „Normal“ large N CFTs (with $SU(N), SO(N), Sp(N)$ gauge group) have $c > a$ [Buchel, Myers, Sinha]

- Certain non-Lagrangian theories constructed by [Gaiotto; Gaiotto, Maldacena] can have $c_1 < 0$

$c - a$

- Mixed current-gravitational anomaly: [Anselmi, Freedman, Grisaru, Johansen]

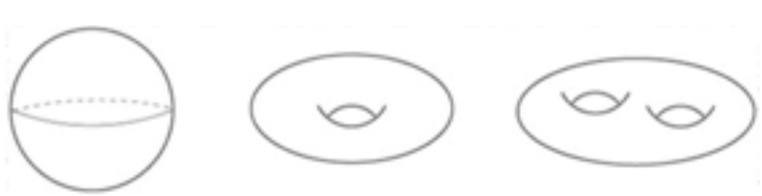
$$D_a J^a = \frac{c-a}{24\pi^2} R_{abcd} \tilde{R}^{abcd}$$

- Violations of the KSS bound: $\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{c-a}{c} + \ldots\right)$

[Brigante, Liu, Myers, Shenker, Yaida; Buchel, Myers, Sinha]

- In 4D CFTs with $a > c$, universal term in entanglement entropy can become negative for certain higher genus entangling surfaces

[Perlmutter, Rangamani, Rota]



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Summary

- Black hole uniqueness theorems less stringent for AdS-asymptotics
- Phase transitions possible between different black solutions when changing parameters (e.g. T)
- Holographic interpretation: Phase transition in the dual CFT
- Discussed 1 example, instability of 5D RN black brane in the presence of a CS-term

Outlook

- Find unstable mode in full geometry, including higher derivative terms (not just near horizon)
- What is the actual ground state of a given theory (i.e. are there other instabilities when coupling to further fields)?
- Fruitful cross-fertilization between black holes with AdS asymptotics and field theory / condensed matter physics (e.g. discovery of new black branes with helical structure)

[For more details: Sean Hartnoll, „Lectures on holographic methods for condensed matter physics“, arxiv: 0903.3246]