# The Quantum Complex Systems Group at the IRB

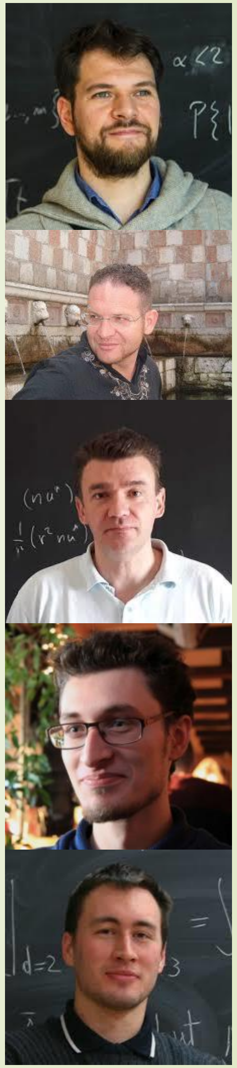
## People and Research activities

HRZZ
Hrvatska zaklada za znanost

IP 2016-6-3347
IP 2019-4-3321

# Current members of the group

- Fabio Franchini (Researcher)
- Salvatore Marco Giampaolo (Researcher)
- Domagoj Kuić (Post doc.)
- Jovan Odavic (Post doc.)
- Vanja Marić (Ph.D. student)



# Future members of the group

- A two years post doc. is expected to start at the end of march/beginning of April
- A three years post doc. is expected to start at the end of August/beginning of September
- A Ph.D. position is expected to be open from September/October. Possibility to follow the PhD course in Statistical Physics in Trieste (Italy)
- A second Ph.D position could open next year, to work on quantum technologies with training In Ljubljana and collaboration with several other European centers

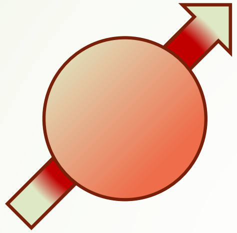
# Complex Systems: More is different

*CSs are made by many parts interacting with each other that give rise to behavior that cannot be explained by looking at the single components*

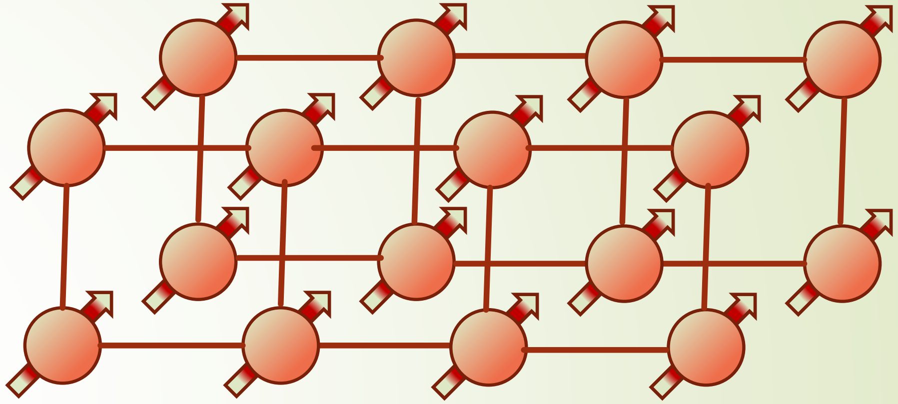
- Emergence: Traits of the system that cannot be explained looking at the single components
- Self-organization: Appearance of a coordination among the elements without an external action
- Non-Linearity: Different responses to the same input
- Networks: CSs form networks, i. e. collection of discrete objects and relationships between them

# Example: a ferromagnetic metal

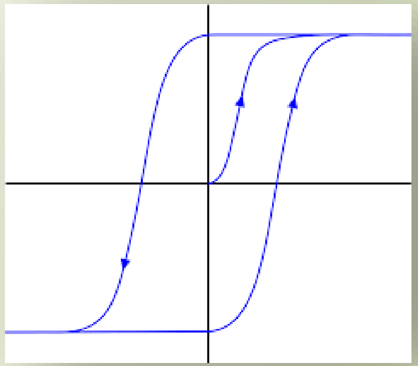
Single
Atom



Complex
System



- Networks: Lattice structure
- Self-organization: Ferromagnetic order
- Emergence: Hysteresis
- Non-Linearity: Response to external field



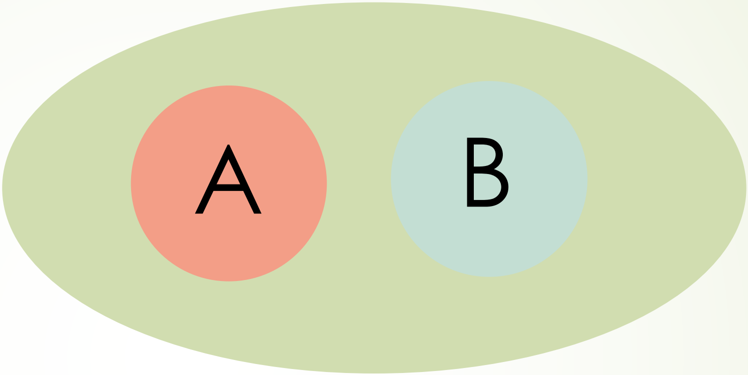
# Quantum Complex Systems:

## *CSs whose elements obey quantum mechanics*

- Can simulate different physical systems ranging from fundamental particle physics to complex materials
- Richer phase diagrams (emergence of new physics and new particles)
- More complicated to solve and study using analytical and numerical approaches
- Extreme Technological interest (quantum computation and information)
- Entanglement

# Entanglement:

*A characteristic feature of quantum mechanics*



## Non-Entangled System

$$|\varphi_{A\mathrm{U}B}\rangle = |\varphi_A\rangle \otimes |\varphi_B\rangle$$

## Entangled System

$$|\varphi_{A\mathrm{U}B}\rangle \neq |\varphi_A\rangle \otimes |\varphi_B\rangle$$

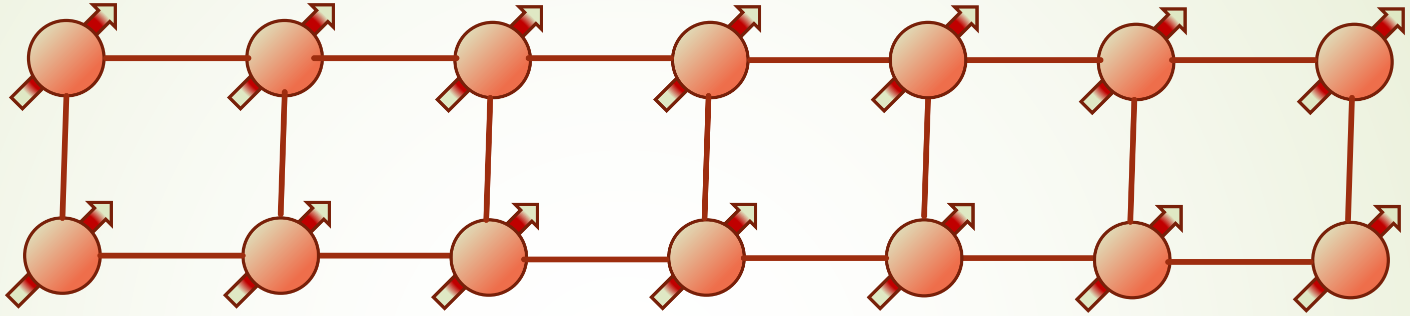
(Measures on A affect B)

- Entanglement can be quantified and its analysis is actually used to characterize different behaviors of quantum complex systems

# Active Fields of Research of the group

- Low Dimensional Quantum Systems
- Interaction and entanglement in particles physics
- Localization, Glasses & Random matrices

# Low Dimensional Quantum systems



- The dynamic of the system is described by an Hamiltonian whose terms do not commute with each other
- Quantum complex systems usually undergoe a quantum phase transition.
- Different phases can be characterized both by order parameters and/or by entanglement properties

# Landau’s Theory

- LT as many other theories in modern physics follow a reductionist approach

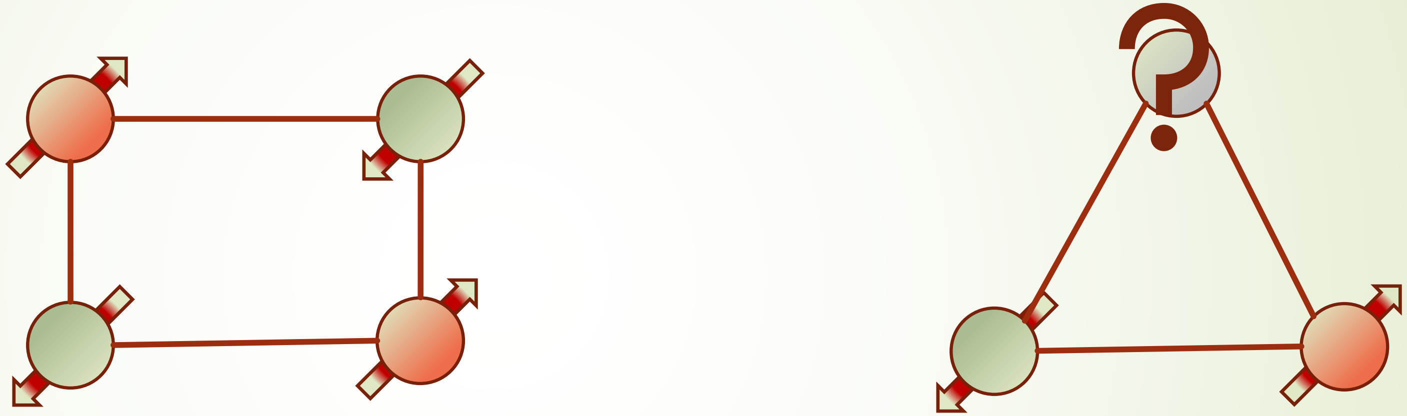
- Phenomena and laws governing CSs are interpreted as effects of the symmetries of a system (some quite intuitive, some highly abstract)

- Other aspects, such as boundary conditions, are assumed as not affecting the main behavior of the system

- Quantum Systems can violate LT (non local effects)

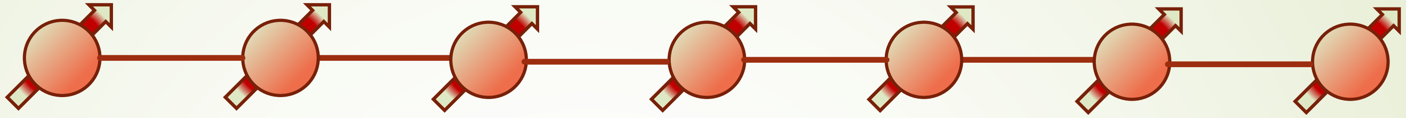
# Frustration:

*The impossibility to satisfy simultaneously all the constrains of a complex system*



- Closed loop made by an odd number of elements interacting antiferromagnetically induces frustration in the system
- Frustration depends on boundary conditions

# Frustration vs Landau’s Theory



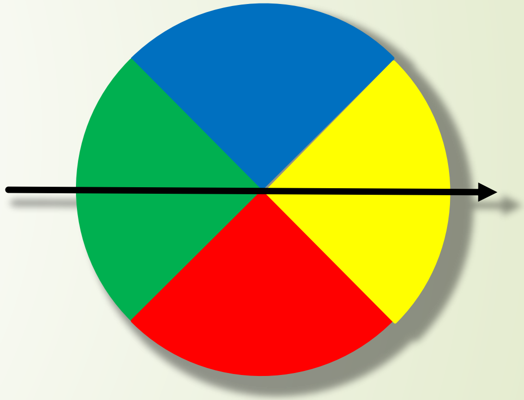
$$H=\sum_{i=1}^{N-1}\left(-\cos\theta\,\sigma_i^x\sigma_{i+1}^x+\sin\theta\,\sigma_i^y\sigma_{i+1}^y\right)$$
$$+\Delta\left(-\cos\theta\,\sigma_1^x\sigma_N^x+\sin\theta\,\sigma_1^y\sigma_N^y\right)$$

- Odd N
- Assuming Δ=0 (Open boundary condition) there is no close loop: impossibility to have frustration
- Assuming Δ=1 (Periodic boundary condition) there is a close loop (presence of frustration depending on $\theta$)

# Open Boundary: Thermodynamic limit

$$H = \sum_{i=1}^{N-1} \left(-\cos\theta\, \sigma_i^x \sigma_{i+1}^x + \sin\theta\, \sigma_i^y \sigma_{i+1}^y\right)$$

- 🔵 Antiferromagnetic order on y
- 🟢 Antiferromagnetic order on x
- 🔴 Ferromagnetic order on y
- 🟡 Ferromagnetic order on x

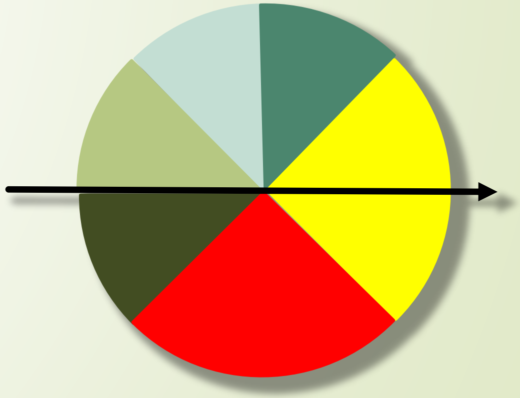


- According with LT the same diagram has to be valid also for other BC

# Periodic boundary

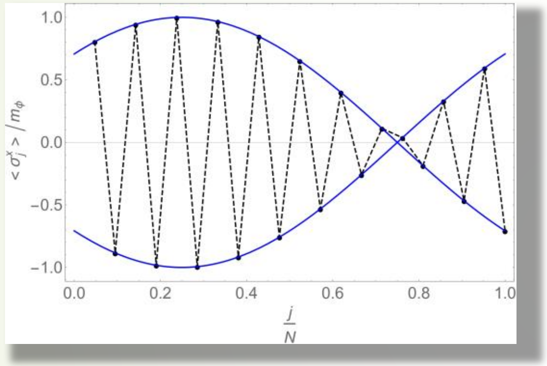
$$H = \sum_{i=1}^{N-1} \left( -\cos\theta\, \sigma_i^x \sigma_{i+1}^x + \sin\theta\, \sigma_i^y \sigma_{i+1}^y \right) + \left( -\cos\theta\, \sigma_1^x \sigma_N^x + \sin\theta\, \sigma_1^y \sigma_N^y \right)$$

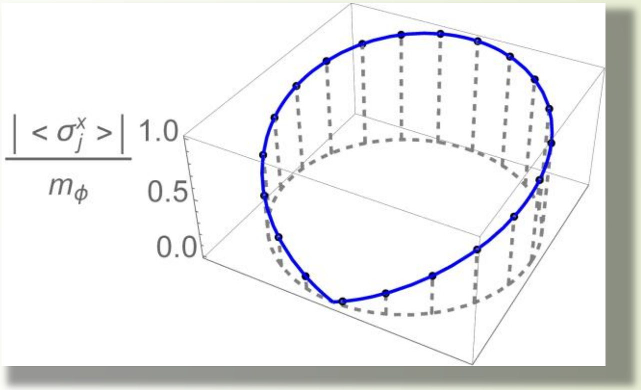
- ● Mesoscopic ferromagnetism on y
- ● Incommensurate antiferromagnetism on y
- ● Mesoscopic ferromagnetism on x
- ● Incommensurate antiferromagnetism on x
- ● Ferromagnetic order on y
- ● Ferromagnetic order on x



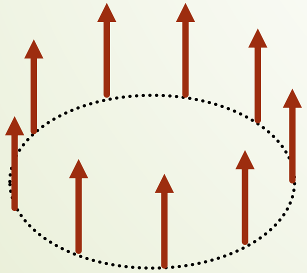
# The two new kinds of order

## Incommensurate antiferromagnetism

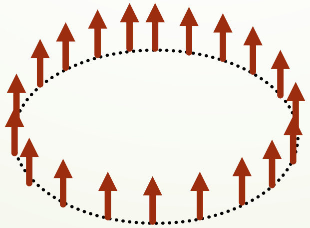




## Mesoscopic ferromagnetism



**N=9**



**N=19**

**N**

**Magnetic Moment**

# Interaction in particle physics

- In the standard approach, fundamental particles are considered free
- Weak interactions are introduced through scattering
- The approach fails in regimes where the interaction cannot be considered as weak
- Interactions open the possibility for new approach to detect still unobserved particles

# Example: Entanglement to detect axions

- Exploit the fermion-axion interaction that generate an indirect fermion-fermion interaction induced by the axions
- We consider two neutrons at rest at distance d

$$H = -\frac{A}{r^3}[Q - Be^{-mr}(m^2r^2\sigma_1^z\sigma_2^z + Q(mr+1)]$$

$$A = \frac{g^2q_e^2}{64\,\pi M^2} \qquad B = \frac{4g_p^2}{g^2q_e^2} \qquad Q = 2\sigma_1^z\sigma_2^z - \sigma_1^x\sigma_2^x - \sigma_1^y\sigma_2^y$$

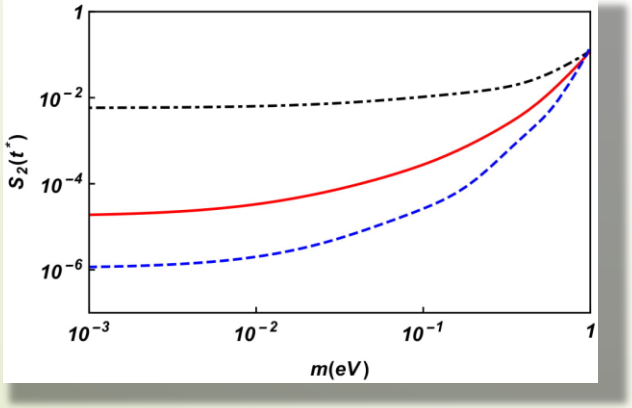
# Example: Entanglement to detect axions

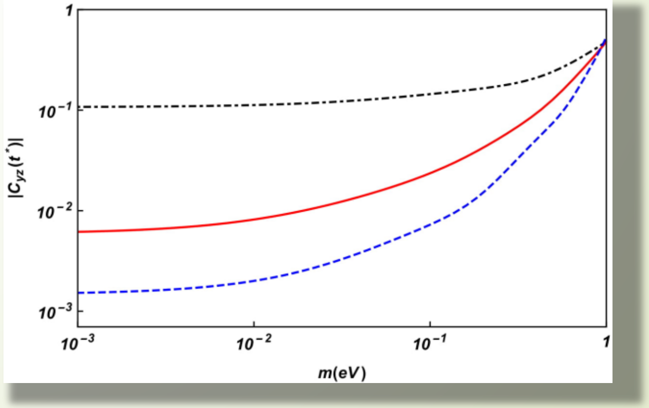
$$S_2 = -Ln(1 - sin^4(2\theta)\, sin^2(\Gamma t))$$

$$\Gamma = \frac{6A}{r^3}[1 - Be^{-mr}(m^2r^2 + 3mr + 3)]$$

$$t^* = \frac{6A}{\pi\, r^3}$$

Entanglement is zero if axions does not exists





# Mixing particles (Pontecorvo Theory)

Mass states

$|\nu_1\rangle, |\nu_2\rangle \qquad m_1, m_2$

Flavor states

$$|\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$
$$|\nu_\mu\rangle = \cos\theta\,|\nu_2\rangle - \sin\theta\,|\nu_1\rangle$$

Hamiltonian of a traveling particle

$$H = \omega_0\sigma^z + cost \qquad \omega_0 = \frac{c^2}{4E}(m_1^2 - m_2^2)$$

Probabilty of transitions

$$P_{v_e\to v_\mu} = P_{v_\mu\to v_e} = sin^2(2\theta)sin^2(\omega_0 t)$$

**CP and T preserved**

# Interacting Mixing particles

Hamiltonian of two traveling particles interacting gravitationally

$$\Omega = g(m_1 - m_2)^2$$

$$H = \omega(\sigma_1^z + \sigma_2^z) + \Omega\sigma_1^z\sigma_2^z + cost$$

$$\omega = \omega_0 + g(m_1^2 - m_2^2)$$

Interaction induces
Entanglement

$$exp\big(S_2(t)\big) = 1 - \frac{1}{2}(sin^4(2\theta)sin^2(2\Omega t))$$

Probabilty of transitions

$$P_{v_e \to v_\mu} - P_{v_\mu \to v_e} = sin^2(2\theta)cos(2\theta)sin(2\omega t)sin(2\Omega t)$$

**T symmetry violated**

# Random matrices and localization

- Statistical approach with matrices instead of scalars
- Generate matrices from rotational invariant weight

$$\mathcal{Z} = \int \mathcal{D}\mathbf{M} e^{-\mathrm{Tr} V(\mathbf{M})} \qquad \mathcal{F} = -\ln \mathcal{Z}$$

- Captures the universality of many degrees of freedom with "quantum chaotic" interactions: spectra of heavy nuclei, quantum transport of electrons, string theory, financial markets, big data, and mathematics
- Extremely versatile tool, but very abstract…

# Random matrices and localization

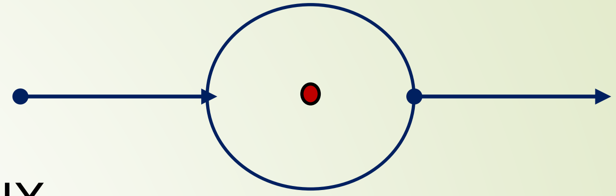
$$\mathcal{Z} = \int \mathcal{D}\mathbf{M} e^{-\mathrm{Tr}V(\mathbf{M})} \qquad \mathcal{F} = -\ln \mathcal{Z}$$

- Equilibrium configuration governed by minimum of $\mathcal{F}$
- **New Breakthrough**: system can choose preferred basis, despite rotational invariant weight
- New type of Spontaneous Symmetry Breaking
- Breaks a priori-invariance between degrees of freedom
  $\Rightarrow$ Loss of ergodicity

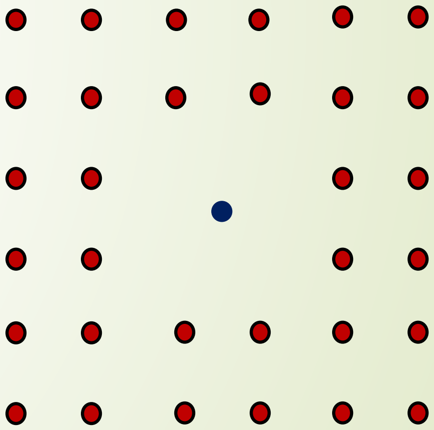
# Random matrices and localization: Applications

- Conduction in disordered systems (Anderson and Many-Body localization, metal/insulator transition)
- Clusterization in networks
- Glasses and spin-glasses
- New, powerful models for Replica Symmetry Breaking
- New objects for mathematics research
- Fundamental theories (string theory, GUT,…)

# Aharonov-Bohm Localization



- An electron circling a magnetic flux acquire a phase proportional to the flux



- An electron surrounded by fluxes needs to satisfy several phase requirements to escape the fluxes
- Low energy electrons become trapped, but there is not classical potential
  ⇒ quantum, topological trapping!

# We are offering 4 diploma projects

- **“Towards quantum glasses: Disorder in topological frustrated quantum systems”** (Mentor Dr. Giampaolo)
- **“Frustrating dynamics”** (Mentor Dr. Franchini)
- **“Statistical behavior of concurrence in the dynamics of systems of interacting particles”** (Mentor Dr. Giampaolo)
- **“Confinement without potential”** (Mentor Dr. Franchini)

# First Thesis on Frustrated systems

- **Title: “Towards quantum glasses: Disorder in topological frustrated quantum systems”**

Classical and Quantum complex systems were analyzed in the framework of Landau theory in which order parameters are believed not to depend on boundary conditions. However, when one introduces in the system the topological frustration, the Landau theory cannot explain the different boundary dependent phases that characterize the systems also in the thermodynamic limit. This result is of huge relevance taking into account that the frustration is one of the two fundamental ingredients of the glassy systems. In this work the student will make the first steps towards the quantum spin glass, analyzing systems with site dependent interactions. She/he will study both classical quantities, as the order parameter, as well as quantum ones, as the analysis of the different entanglement measures to determine the onset of glassy behaviors.

Mentor Dr. Giampaolo

# Second Thesis on Frustrated systems

- **Title: “Frustrating dynamics”**

The past century has taught us a great deal on the equilibrium of strongly correlated systems, that it, systems whose properties differ significantly from those of their elementary constituents, due to interactions. In the new century, guided by experimental breakthroughs, the community has moved toward understanding different types of non-equilibrium behaviors. In this work, the student will solve and study the dynamics of frustrated quantum spin chains, that is, one-dimensional systems which, already at equilibrium, show different properties due to a global constraint which is not present without frustration. The student will learn how these systems are solved through analytical techniques and use these solutions to follow their evolution numerically.

Mentor Dr. Franchini

# Thesis on particles physics

- **Title: "Statistical behavior of concurrence in the dynamics of systems of interacting particles"**

Entanglement is usually quantified by entropic measurements, but its properties are much more complex than what can be expressed with a single number. This fact plays a key role in the dynamics of complex systems that may present three distinct dynamical phases, known as thermalization, Anderson localization, and many-body localization. It was shown that such phases are marked by different patterns of the spectrum of the reduced density matrix. While the entanglement spectrum displays Poisson statistics in case of Anderson localization, it shows Wigner-Dyson statistics for both many-body localization and thermalization, albeit the distribution is asymptotically reached within different time scales In this work the student will analyze the hypothesis that this type of trend has a correspondence also for other measures of entanglement, using a setup inspired either by condensed matter physics or by high energy physics, focusing on interacting particles.

Mentor Dr. Giampaolo

# Thesis on electronic systems

- **Title: “Confinement without potential”**

An electron circling a magnetic flux acquires a phase, which is topological in nature and has no classical counterpart. If several magnetic fluxes surround a certain region of space containing one electron, the phase requirement for the electron to travel across the fluxes will increase its energy, effectively trapping a low energy electron in a region of space, even in absence of a classical potential, giving thus rise to a pure quantum, topological trapping, which can be simulated with a mixture of analytical and numerical work. Alternatively, the student can study a different type of confinement, due to a spontaneous breaking of rotational symmetry in a random matrix system. In this case, the student will study the metropolis evolution of a large matrix under a statistical weight that tends to concentrate the matrix eigenvalues around two different given values, to show that the eigenvectors of the matrix span two spaces that do not mix under the evolution.

Mentor Dr. Franchini