Quantum Foundations II: Three no-go theorems

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Part 0.

MOTIVATION

Standard QM:
- The value a of an observable A is an eigenvalue: $A|a\rangle = a|a\rangle$.
$\Rightarrow$ The system has some value only when $|\psi\rangle$ is an eigenstate: $|\psi\rangle = |a\rangle$.
- But Schrödinger equation $\Rightarrow$ superposition $|\psi\rangle = \sum_{a'} c_{a'}|a'\rangle$.
$\Rightarrow$ To get a value we need a collapse $|\psi\rangle \to |a\rangle$.

Several problems with a collapse:
- Contextual (happens only when measurement is performed, not clear how exactly the system knows that it is measured).
- Nonlocal (happens in the whole universe at once).

Possible solution:
- Perhaps an observable (spin, position, ...) has some value v even when $|\psi\rangle$ is **not** an eigenstate $|a\rangle$?
- If so, then perhaps collapse is **not** needed to get a value!
Can it resolve all mysteries of QM?

- Such values v are often called “hidden variables”
because they are not a part of the standard quantum formalism.
- However, this name is misleading because v
are variables that we **actually observe** in experiments.
$\Rightarrow$ They may be “hidden” to theorists, not to experimentalists.
$\Rightarrow$ I will no longer call them “hidden variables”.

- Whatever those hypothetic values v are,
they must me compatible with existing experiments.
- But existing experiments are all compatible
with measurable predictions of standard QM.
$\Rightarrow$ The values must be compatible
with measurable predictions of standard QM.
$\Rightarrow$ This poses strong restrictions on possible physical laws for v.

- It turns out that v **cannot** obey some classical
properties that one would naively expect them to obey.
- Loosely speaking, v either must be $|\psi\rangle$ itself (!),
or something that has some properties similar to $|\psi\rangle$.
- Those restrictions have forms of various **no-go theorems**.
- In the rest I present those theorems in more detail.

The theorems in a nutshell:

Naive property 1:
- Measurement just reveals values v that existed before measurement.
No-go theorem 1. (contextuality) refutes it:
- The measurement must somehow **create** or **change** values v.

Naive property 2:
- Since we might get rid of nonlocal collapse, the creation/change of v might be governed by a local law.
No-go theorem 2. (nonlocality) refutes it:
- The creation/change of v must be governed by some **nonlocal** law.

Naive property 3:
- Since the actual value v might exist even without the eigenstate $|a\rangle$, the quantum state $|\psi\rangle$ might be just our subjective knowledge, it might not be objectively "real".
No-go theorem 3. (ψ-onticity) refutes it:
- If v is **objectively "real"**, then so is $|\psi\rangle$.

Part 1.

PROOF OF CONTEXTUALITY

The first proof: Bell (1966).

The most famous proof: Kochen-Specker (1967).

I present a much simpler proof: Mermin (1990), Peres (1990).

Multiplication table for 9 numbers:

a	b	c	abc
e	f	g	efg
i	j	k	ijk
aei	bfj	cgk	$aei \cdot bfj \cdot cgk = abc \cdot efg \cdot ijk$

Can we do the same for operators?

- Pauli matrices: $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1$, $\sigma_x \sigma_y \sigma_z = i$, $[\sigma_x, \sigma_y] = 2i\sigma_z$
- Eigenvalues of σ_x, σ_y, σ_z are ± 1.
- Spin operator: $S_x = \frac{\hbar}{2}\sigma_x$, ...

Consider a composite system of two spin-$\frac{1}{2}$ particles.
- In this system consider the following 9 observables:

$$
\begin{array}{ccc|l}
\sigma_x \otimes 1 & 1 \otimes \sigma_x & \sigma_x \otimes \sigma_x & 1 \\
1 \otimes \sigma_y & \sigma_y \otimes 1 & \sigma_y \otimes \sigma_y & 1 \\
\sigma_x \otimes \sigma_y & \sigma_y \otimes \sigma_x & \sigma_z \otimes \sigma_z & 1 \\
\hline
1 & 1 & -1 & 1 \cdot 1 \cdot (-1) \neq 1 \cdot 1 \cdot 1
\end{array}
$$

- operators in the same row commute (can be simultaneously measured)
- operators in the same column commute

However, $-1 \neq 1$
$\Rightarrow$ Impossible to simultaneously assign values to all 9 observables.

$\Rightarrow$ Contextuality:
At least some of the values are created or changed by the measurement.

Note: We tried (and failed) to associate values with
composite operators ($\sigma_x \otimes \sigma_y$, ...),
not with **local** operators (σ_x, σ_y, ...)

Part 2.

PROOF OF NONLOCALITY

- In the proof of contextuality we proved that one cannot associate values with **composite** observables ($\sigma_x \otimes \sigma_y$, ...).
- We said nothing about values associated with **local** observables (σ_x, σ_y, ...).

- To prove nonlocality, we need to prove that one cannot associate values with **local** observables.
- However, nonlocality cannot be proved for any state.
- Nonlocality is state-dependent, it appears only for **entangled** states.

The first and most famous proof: Bell (1964).
- Uses $|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle$, easy to prepare in experiments.
- However, the proof is not simple mathematically.

I present the GHZ proof: Greenberger, Horne and Zeilinger (1989).
- simple mathematically
- uses a state of 3 entangled particles:

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle - |\downarrow\rangle|\downarrow\rangle|\downarrow\rangle)$$

- not easy to prepare in experiments

Here the following notation is used:
- eigenstates of operator σ_z: $\sigma_z|\uparrow\rangle = +|\uparrow\rangle$, $\sigma_z|\downarrow\rangle = -|\downarrow\rangle$
- tensor product $\otimes$ is understood
- for composite operators the following notation is used

$$\sigma_x \otimes \sigma_y \otimes \sigma_z \equiv \sigma_x^{(1)}\sigma_y^{(2)}\sigma_z^{(3)}$$

The state $|\text{GHZ}\rangle$ satisfies

$$\sigma_x^{(1)}\sigma_x^{(2)}\sigma_y^{(3)}|\text{GHZ}\rangle = \sigma_x^{(1)}\sigma_y^{(2)}\sigma_x^{(3)}|\text{GHZ}\rangle = \sigma_y^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}|\text{GHZ}\rangle = +|\text{GHZ}\rangle$$

Now **assume**(!) that, for $|\text{GHZ}\rangle$, we can associate a value (number) s with each local observable, e.g. $\sigma_x^{(1)} \to s_x^{(1)}$
$\Rightarrow$ The numbers must satisfy

$$s_x^{(1)}s_x^{(2)}s_y^{(3)} = s_x^{(1)}s_y^{(2)}s_x^{(3)} = s_y^{(1)}s_x^{(2)}s_x^{(3)} = +1$$

Multiply all three $\Rightarrow$

$$\left(s_x^{(1)}s_x^{(2)}s_x^{(3)}\right)^2 s_y^{(1)}s_y^{(2)}s_y^{(3)} = +1 \;\Rightarrow\; s_y^{(1)}s_y^{(2)}s_y^{(3)} = +1$$

But QM tells us that

$$\sigma_y^{(1)}\sigma_y^{(2)}\sigma_y^{(3)}|\text{GHZ}\rangle = -|\text{GHZ}\rangle$$

which is a contradiction!

$\Rightarrow$ The assumption was wrong, it is not possible
to simultaneously associate values with all local observables.

- Contradiction $\Rightarrow$ s-values could not have been preexisting before measurement.
$\Rightarrow$ Measurement somehow creates or changes them (**contextuality**).

- Perfect **correlation**, e.g. $s_x^{(1)} s_x^{(2)} s_y^{(3)} = 1$.

correlation + contextuality $\Rightarrow$ correlation could not have been prearranged.
$\Rightarrow$ Correlation must somehow be arranged at the time of measurement.
$\Rightarrow$ The 3 measurement apparatuses must somehow communicate with each other.

- But measurement apparatuses can be far away from each other.
$\Rightarrow$ Somehow they must communicate instantaneously
(or faster than c).

That's the proof of **nonlocality**!

Why can't nonlocality be used to send signals faster than light?

- **Signal** is an anthropomorphic concept:
Array of symbols (e.g. $01101\ldots$) **freely chosen** by a human agent, not randomly picked by nature.

- A human can prepare any $|\psi\rangle$ at will.
- E.g. $|\psi\rangle = |\uparrow\rangle|\uparrow\rangle + |\downarrow\rangle|\downarrow\rangle$ guarantees the correlation $s_z^{(1)} s_z^{(2)} = 1$.
- But a human cannot choose in advance whether it will be $(s_z^{(1)}, s_z^{(2)}) = (1, 1)$ or $(s_z^{(1)}, s_z^{(2)}) = (-1, -1)$.
- Instead, nature picks one of those **randomly**.
(Or perhaps pseudo-randomly, but we cannot distinguish between random and pseudo-random in practice.)

Some interpretations of QM still deny nonlocality.
- Different interpretations use different arguments.

The most frequent argument for locality: Signal locality.
- Nonlocality proved by the theorem cannot be used to send signals.
$\Rightarrow$ Nonlocality doesn't have practical consequences.
$\Rightarrow$ It's metaphysics, not physics.

Counterargument:
- If physics was **only** an empirical science,
nonlocality would be metaphysics.
- But physics is based on empirical data **and** logic.
- Proof of nonlocality is a combination of both
(correlation is an empirical fact, the rest is logic).
$\Rightarrow$ Proof of nonlocality is physical.

Which argument is more convincing?
- No consensus among experts.
$\Rightarrow$ I leave the decision to you!

Part 3.

PROOF OF ψ-onticity

What is the meaning of the quantum state $|\psi\rangle$?
- Is it an objective property of a **single** system? (ψ-ontology)
- Or is it only a tool to calculate **probability**? (ψ-epistemology)

PBR theorem:
M.F. Pusey, J. Barrett, T. Rudolph, Nature Phys. **8**, 476 (2012).

Consists of
1. Mathematical definition of the difference between "ontological" and "epistemological".
2. Technical proof (with the aid some auxiliary assumptions)
   QM $\Rightarrow$ $|\psi\rangle$ is ontological (objectively real)!

Main assumption in the proof:
- Some objective properties λ exist.

Example from classical probability - coin flipping:

To make it non-trivial, assume **unfair** coin flipping

$$p(\text{head}) \neq p(\text{tail})$$

- Are $p(\text{head})$ and $p(\text{tail})$ **intrinsic** properties of a **single** coin?
- If they are, we shall say that $p(\text{head})$ and $p(\text{tail})$ are **real** (objective) properties of the coin.

Two possibilities:

1. Unfair **coin**
- $p(\text{head}) \neq p(\text{tail})$ because the distribution of the coin-mass is not uniform
- this is a property of the **coin itself**
- from the knowledge of $\lambda = \text{mass distribution}$ $\Rightarrow p(\text{head}), p(\text{tail})$ can be determined **uniquely**

2. Unfair **flipping**
- $p(\text{head}) \neq p(\text{tail})$ because the act of **flipping is unfair**
- this is **not** a property of the coin
- from the knowledge of $\lambda = \text{mass distribution}$ $\Rightarrow p(\text{head}), p(\text{tail})$ can **not** be determined uniquely

This motivates the general **definition**:

A probability distribution $\mu(\lambda)$ is *ontic*
(i.e., corresponds to something real)
iff it can be determined uniquely from the fundamental λ.

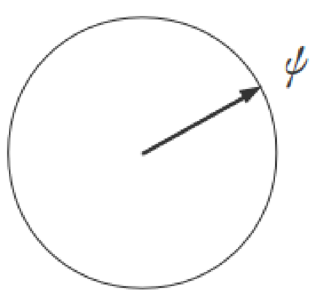
Otherwise, $\mu(\lambda)$ is called **epistemic**.

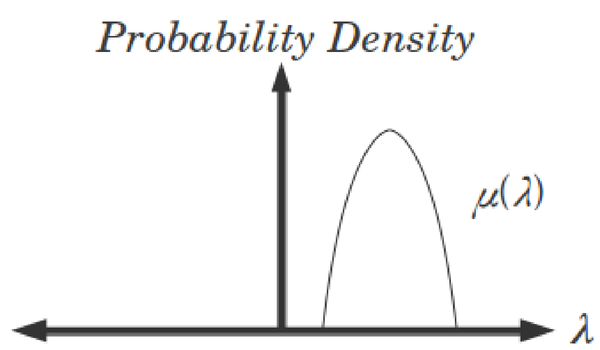
Now apply to QM:
- QM is an unfair game (not all probabilities are equal).
- Is QM an unfair "coin" or an unfair "flipping"?

Analogy: coin flipping $\leftrightarrow$ QM
set $\{\text{head}, \text{tail}\}$ $\leftrightarrow$ set of all different states in the Hilbert space $\{|\psi\rangle\}$

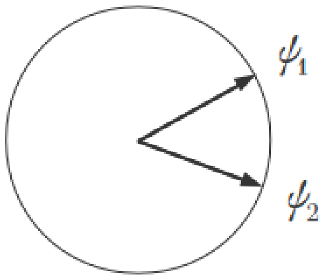
($|\psi\rangle$ is different from $|\psi'\rangle$ iff $|\psi'\rangle \neq c|\psi\rangle$.)

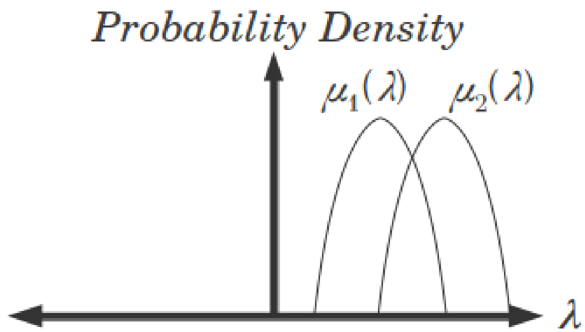
- In general, experimentalists do **not** have a full control
over all fundamental degrees of freedom λ.
$\Rightarrow$ When they prepare $|\psi\rangle$ in the laboratory, this actually means
that they have prepared some **probability distribution** $\mu(\lambda)$:





Assume that for states $|\psi_1\rangle$, $|\psi_2\rangle$ their λ-distributions overlap:





- For $\lambda \in$ overlap, one cannot know whether λ belongs to $\mu_1(\lambda)$ or $\mu_2(\lambda)$

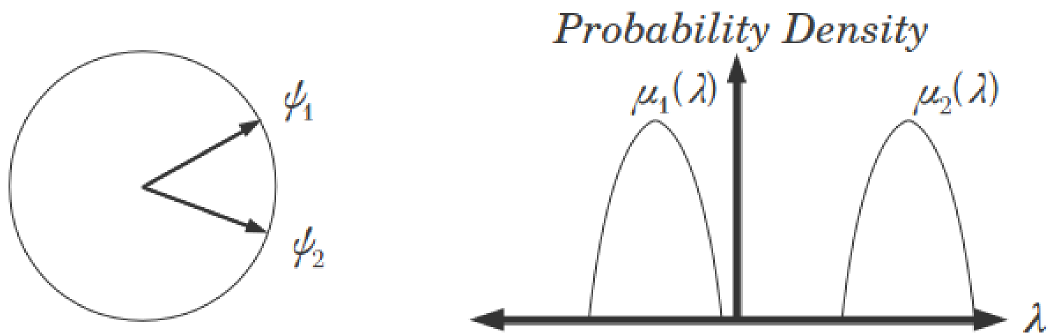
$\Rightarrow$ λ does not uniquely determine $\mu(\lambda)$

$\Rightarrow$ (by definition) $\mu(\lambda)$ is not ontic

$\Rightarrow$ $|\psi\rangle$ is not ontic

$\Rightarrow$ To prove that $|\psi\rangle$ is **not** ontic, it is sufficient to prove that there is **at least one pair** $|\psi_1\rangle$, $|\psi_2\rangle$ ($|\psi_1\rangle \neq c|\psi_2\rangle$) for which $\mu_1(\lambda)$ and $\mu_2(\lambda)$ do overlap.

$\Rightarrow$ The converse (that $|\psi\rangle$ is ontic)
is much more difficult to prove:
- One needs to prove that for **any pair** $|\psi_1\rangle$, $|\psi_2\rangle$ ($|\psi_1\rangle \neq c|\psi_2\rangle$)
the overlap does not exist:



- Yet, the PBR theorem proves exactly this!

This is not only difficult to prove (sketch in the next section),
but also very surprising:

- The absence of overlap $\mu_1(\lambda)\mu_2(\lambda) = 0 \quad \forall\, \lambda$
is not surprising when $\langle\psi_1|\psi_2\rangle = 0$.

- What is surprising is that $\mu_1(\lambda)\mu_2(\lambda) = 0 \quad \forall\, \lambda$
even when $\langle\psi_1|\psi_2\rangle \neq 0$.

- Why is that surprising? Because

$$
\begin{aligned}
& & & \langle\psi_1|\psi_2\rangle \neq 0 \\
\left[1 = \int da|a\rangle\langle a|\right] & \Rightarrow & & \int da\langle\psi_1|a\rangle\langle a|\psi_2\rangle \neq 0 \\
& \Rightarrow & & \langle\psi_1|a\rangle\langle a|\psi_2\rangle \neq 0 \ \text{ for some } \ a \\
\left[\rho_i(a) = |\langle a|\psi_i\rangle|^2\right] & \Rightarrow & & \rho_1(a)\rho_2(a) \neq 0 \ \text{ for some } \ a
\end{aligned}
$$

and yet $\mu_1(\lambda)\mu_2(\lambda) = 0 \ \forall\lambda$.

- In other words, QM-distributions overlap,
but corresponding λ-distributions do not overlap!

Sketch of the proof:

- Here I present the proof of the absence of overlap for a simple example of a pair of non-orthogonal states. (PBR also generalize it to an arbitrary pair.)

2-dimensional Hilbert space with orthogonal basis $|0\rangle$, $|1\rangle$.
Another orthogonal basis $|+\rangle$, $|-\rangle$

$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

Non-orthogonal pair $|0\rangle$, $|+\rangle$:

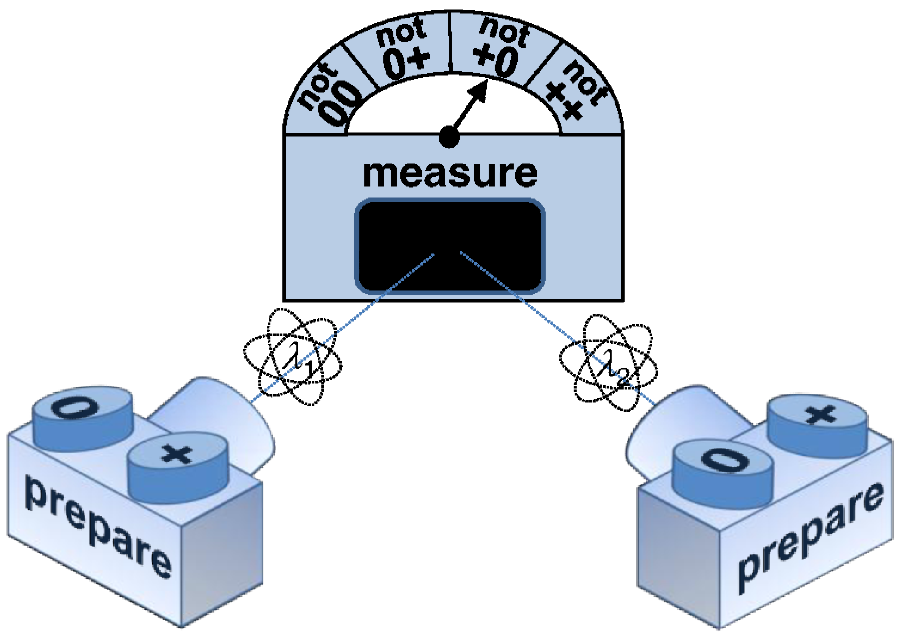
$$\langle 0|+\rangle = 1/\sqrt{2}$$

Goal: prove that $\mu_0(\lambda)\mu_+(\lambda) = 0 \ \forall\lambda$
Strategy: assume the opposite and derive a contradiction!

- Assume overlap $\Rightarrow$ finite probability p that $\lambda \in$ overlap.

Consider two similar systems:

- Each prepared either in $|0\rangle$ or $|+\rangle$
  (but experimentalist does not know in which one it is prepared).



- Probability of overlap in each is p.
- Assume they are statistically independent.

$\Rightarrow$ Probability of overlap in both is $P_{\text{joint}} = p \cdot p > 0$.

$\Rightarrow$ Consequence of the assumed overlap:

There is a probability $P_{\text{joint}} > 0$ that the outcome will be consistent with **all four** possibilities for the initial preparation ($|0\rangle|0\rangle$, $|0\rangle|+\rangle$, $|+\rangle|0\rangle$, and $|+\rangle|+\rangle$)

Now compare it with predictions of QM:
- Measure the joint system in a specially chosen complete orthogonal basis:

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}[|0\rangle|1\rangle + |1\rangle|0\rangle], \qquad |\phi_2\rangle = \frac{1}{\sqrt{2}}[|0\rangle|-\rangle + |1\rangle|+\rangle],$$
$$|\phi_3\rangle = \frac{1}{\sqrt{2}}[|+\rangle|1\rangle + |-\rangle|0\rangle], \qquad |\phi_4\rangle = \frac{1}{\sqrt{2}}[|+\rangle|-\rangle + |-\rangle|+\rangle]$$

- This basis has the property (notation: $|ab\rangle \equiv |a\rangle|b\rangle$)

$$\langle\phi_1|00\rangle = 0, \quad \langle\phi_2|0+\rangle = 0, \quad \langle\phi_3|+0\rangle = 0, \quad \langle\phi_4|++\rangle = 0$$

$\Rightarrow$ Whatever the outcome of a single measurement will be ($|\phi_1\rangle$, $|\phi_2\rangle$, $|\phi_3\rangle$, or $|\phi_4\rangle$), it is *certain* that it will **eliminate** one of the possibilities ($|00\rangle$, $|0+\rangle$, $|+0\rangle$, or $|++\rangle$).
$\Rightarrow$ $P_{\text{joint}} = 0$
$\Rightarrow$ Contradiction with the Consequence above! *Q.E.D.*

Forthcoming talks:

Quantum Foundations III:
Decoherence

Quantum Foundations IV:
Quantum theory of measurement and Bohmian mechanics

Quantum Foundations V:
Relativistic QFT from a Bohmian perspective: A proof of concept

Quantum Foundations VI:
Suggestions welcome