Conformal Renormalization and Energy Functionals in AdS gravity

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HOLOGRAPHYCL

Outline

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Renormalization of bulk/surface functionals

Alternative Renormalization Scheme: Kounterterms

Counterterms and Kounterterms in AdS gravity

Beyond Kounterterms: Conformal Renormalization

Renormalization of Codimension-2 Functionals

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Black Hole Thermodynamics in AdS gravity

Euclidean static black hole metric

$$ds^2 = f^2(r)d\tau^2 + \frac{dr^2}{f^2(r)} + r^2 d\Omega_{D-2}^2\,, \quad f^2(r) = 1 - \frac{2\omega_D GM}{r^{D-3}} + \frac{r^2}{\ell^2}$$

Einstein-AdS gravity

$$I_{EH} = \frac{1}{16\pi G}\int\limits_M d^D x \sqrt{-g}\,(R - 2\Lambda)\,, \qquad \Lambda = -\frac{(D-1)(D-2)}{2\ell^2}$$

Gibbs free energy $G = TI^E$

$$TI_{bulk}^E = \frac{(D-3)}{(D-2)}M - TS + \lim_{r\to\infty}\frac{V(S^{D-2})}{8\pi G}\frac{r^{D-1}}{\ell^2}$$

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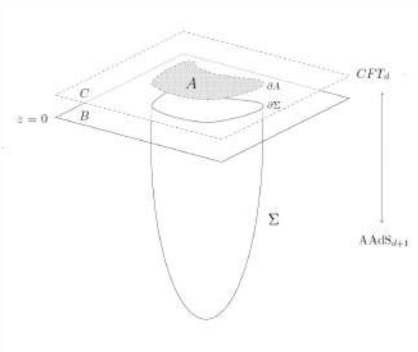
Energy functionals

Holographic Entanglement Entropy from Minimal Surface [Ryu-Takayanagi, 2006]



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$$S = -\partial_\alpha \lim_{\alpha\to 1} I^{(\alpha)}_{\text{grav}}$$

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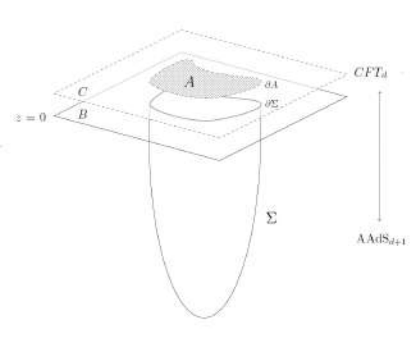

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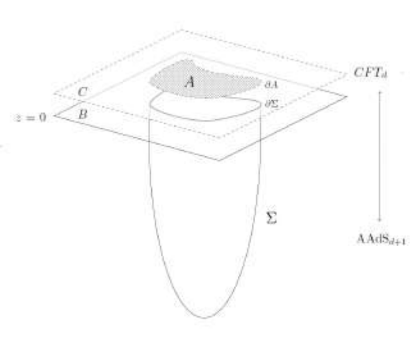

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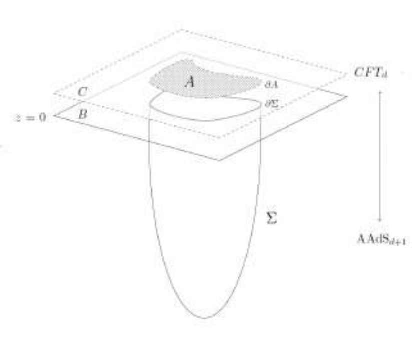

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Bulk gravity action evaluated in conical defects

$$I_{\mathsf{EH}}^{(\alpha)} = \frac{1}{16\pi G}\int\limits_{M^{(\alpha)}} d^4x\sqrt{g}R^{(\alpha)} = \frac{1}{16\pi G}\int\limits_{M} d^4x\sqrt{g}R + \frac{(1-\alpha)}{4G}\mathcal{A}\left[\Sigma\right]$$

Area functional (codimension-2)

$$\mathcal{A}\left[\Sigma\right] = \int\limits_{\Sigma} d^2y\sqrt{\gamma}$$

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Counterterms in AdS gravity

Holographic Renormalization [Henningson and Skenderis, 1998]

$$I_{\text{ren}} = I_{EH} - \frac{1}{8\pi G}\int_{\partial M} d^dx\,\sqrt{-h}\,K + \int_{\partial M} d^dx\,L_{ct}(h,\mathcal{R},\nabla\mathcal{R})$$

Counterterms [Balasubramanian-Kraus, 1999], [Emparan, Johnson, Myers, 1999]

$$8\pi G\,L_{ct} = \frac{d-1}{\ell}\sqrt{-h} + \frac{\ell\sqrt{-h}}{2(d-2)}\mathcal{R} + \frac{\ell^3\sqrt{-h}}{2(d-2)^2(d-4)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{d}{4(d-1)}\mathcal{R}^2\right)$$
$$+\frac{\ell^5\sqrt{-h}}{(d-2)^3(d-4)(d-6)}\left(\frac{3d-2}{4(d-1)}\mathcal{R}\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{d(d+2)}{16(d-1)^2}\mathcal{R}^3\right.$$
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Black Hole Thermodynamics and Counterterms

Counterterm Method reproduces BH Thermo

$$G = U - TS$$

Internal Energy

$$U = M + E_0$$

Vacuum Energy in $D = 2n+1$ dimensions

$$E_0 = (-1)^n \frac{(2n-1)!!^2}{(2n)!} \frac{\mathrm{Vol}(S^{2n-1})}{8\pi G} \ell^{2n-2}$$

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Kounterterms in AdS gravity

Extrinsic counterterms

$$\tilde{I}_{ren} = I + c_d \int\limits_{\partial M} d^d x \, B_d(h, K, \mathcal{R})$$

Kounterterms = counterterms of unusual sort (depend on K_{ij} and $\mathcal{R}^{kl}_{ij}(h)$)

$D = 2n$ dimensions [R.O., hep-th/0504233]

$$B_{2n-1} = 2n\sqrt{-h} \int\limits_0^1 dt \; \delta^{[i_1 \cdots i_{2n-1}]}_{[j_1 \cdots j_{2n-1}]} K^{j_1}_{i_1} \left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3} - t^2 K^{j_2}_{i_2} K^{j_3}_{i_3} \right) \times \cdots$$

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Kounterterms in AdS gravity

Kounterterms in $D=2n+1$ [R.O., hep-th/0610230]

$$
\begin{aligned}
B_{2n} &= 2n\sqrt{-h}\int\limits_0^1 dt\int\limits_0^t ds\,\delta^{[j_1\cdots j_{2n}]}_{[i_1\cdots i_{2n}]}\,K^{i_1}_{j_1}\delta^{i_2}_{j_2}\left(\frac{1}{2}\mathcal{R}^{i_3i_4}_{j_3j_4}-t^2K^{i_3}_{j_3}K^{i_4}_{j_4}+\frac{s^2}{\ell^2}\delta^{i_3}_{j_3}\delta^{i_4}_{j_4}\right)\times\cdots\\
&\qquad\cdots\times\left(\frac{1}{2}\mathcal{R}^{i_{2n-1}i_{2n}}_{j_{2n-1}j_{2n}}-t^2K^{i_{2n-1}}_{j_{2n-1}}K^{i_{2n}}_{j_{2n}}+\frac{s^2}{\ell^2}\delta^{i_{2n-1}}_{j_{2n-1}}\delta^{i_{2n}}_{j_{2n}}\right).
\end{aligned}
$$

Kounterterms in AdS gravity

In $D = 2n$ dimensions

$$\begin{aligned} \text{tr}(F^n) &= dL_{2n-1}^{CS}(A) \\ F &= dA + A \wedge A \end{aligned}$$

Explicit realization of Chern-Simons forms

$$L_{2n-1}^{CS}(A) = n \int_0^1 dt \, \text{tr}\left[A F_t^{n-1} \right] \qquad F_t = t dA + t^2 A^2$$

Global issues (topology)

$$\int_{M_{2n}} (\text{Euler})_{2n} = (4\pi)^n n! \chi(M_{2n}) + \int_{\partial M_{2n}} B_{2n-1}$$

Kounterterms in AdS gravity

In $D = 2n$ dimensions

$$\begin{aligned} \mathrm{tr}(F^n) &= dL^{CS}_{2n-1}(A) \\ F &= dA + A \wedge A \end{aligned}$$

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Gauge-invariant extension of CS forms $L^{TF}_{2n+1}(A,\bar{A}) = L^{CS}_{2n+1}(A) - L^{CS}_{2n+1}(\bar{A}) + d\beta_{2n}(A,\bar{A})$

Contact term

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Renormalized Action = Renormalized Volume

Black Hole Thermodynamics

$$TI_{bulk}^{E} = \frac{(D-3)}{(D-2)}M - TS + \lim_{r\to\infty}\frac{V(S^{D-2})}{8\pi G}\frac{r^{D-1}}{\ell^2}$$

Euclidean Kounterterms

$$T\,c_d\int_{\partial M} B_d = \frac{M}{(D-2)} + E_0 - \lim_{r\to\infty}\frac{V(S^{D-2})}{8\pi G}\frac{r^{D-1}}{\ell^2}$$

Correct Black Hole Thermo with $U = M + E_0$

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Boundary conditions in AdS gravity

Fefferman-Graham expansion for AAdS Einstein spaces

$$ds^2 = \frac{\ell^2}{z^2}\,dz^2 + \frac{1}{z^2}\,g_{ij}(x,z)\,dx^i dx^j\,, \qquad g_{ij}(x,\rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \cdots$$

Dirichlet b.c. $\delta h_{ij} = 0$ does not make sense in AAdS spaces [Papadimitriou and Skenderis, 2004]

$$h_{ij} = \frac{g_{(0)ij}}{z^2} + \ldots$$

Renormalization — variational problem in $g_{(0)ij}$

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Kounterterms and Holography

Asymptotic form of the extrinsic curvature

$$K_{ij} = \frac{1}{\ell}\frac{g_{(0)ij}}{z^2} + \ldots$$

Counterterms of a different sort ...

$$\tilde{I}_{ren} = I_{EH} + c_d \int\limits_{\partial M} d^d x\, B(f(h), K)$$

... as long as the theory is *holographic*

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From extrinsic to intrinsic renormalization in 4D

AdS gravity action + KTs

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Adding zero...

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Kounterterms turn into counterterms [O.Miskovic and R.O., 0902.2082]

$$L_{ct} = \frac{1}{8\pi G}\frac{\sqrt{-g}}{z^3}\left(\frac{2}{\ell} + \frac{\ell}{2}z^2\mathcal{R}(g)\right) + \mathcal{O}(z)$$
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$$\times\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}\left(\frac{\delta^{j_2}_{i_2}}{\ell} - \ell S^{i_2}_{j_2}\right)\left(\frac{\delta^{j_3}_{i_3}}{\ell} - \ell S^{i_3}_{j_3}\right) + \frac{1}{\ell^2}\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}\right) + \ldots$$

Kounterterms turn into counterterms [O Miskovic and R.O., 0902.2082]

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EH-AdS gravity +KTs

$$\mathrm{B}_{2n-1} = 2n\sqrt{-h}\int\limits_0^1 dt\ \delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]} K^{j_1}_{i_1}\left(\frac{1}{2}\mathcal{R}^{j_2j_3}_{i_2i_3} - t^2 K^{j_2}_{i_2}K^{j_3}_{i_3}\right)\times\cdots$$
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Expanding and collecting...

$$L_{ct} = \frac{\sqrt{-h}}{8\pi G}\Bigg[\frac{(2n-2)}{\ell} + \frac{\ell}{2(2n-3)}\mathcal{R} + \\ + \frac{\ell^3}{2(2n-3)^2(2n-5)}\left(2\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{(2n+1)}{4(2n-2)}\mathcal{R}^2 - \frac{(2n-3)}{4}\mathcal{R}^{ijkl}\mathcal{R}_{ijkl}\right) + \cdots\Bigg].$$

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Holographic Renormalization = Kounterterms?

Well...almost. [G.Anastasiou, O.Miskovic, R.O. and I.Papadimitriou, 2003.06425]

$$\tilde{I}_{\text{ren}} = I_{\text{HR}} - \frac{\ell^3}{64\pi G(2n-3)(2n-5)} \int_{\partial M} \sqrt{-h}\, \mathcal{W}^{ijkl} \mathcal{W}_{ijkl} + \ldots$$

A similar result in $D = 2n+1$ dimensions.

Last term is zero for most AAdS spaces, but not for gravitational instantons.

Patching up the theory

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Renormalized AdS Action

Euler-Gauss-Bonnet Theorem in 4D

$$\int_{\partial M} d^3x\, B_3(K,\mathcal{R}) = \int_M d^4x\, GB - 32\pi^2\chi(M)$$

4D Renormalized AdS action [R. Aros et al., gr-qc/9909015]

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GB coupling is also singled out by SUSY [Andrianopoli and D'Auria, arXiv:1405.2010]

Renormalized AdS action = MacDowell-Mansouri action (1977)

$$I_{\text{ren}} = \frac{\ell^2}{256\pi G} \int\limits_M d^4x \sqrt{-g}\, \delta^{[\nu_1 \cdots \nu_4]}_{[\mu_1 \cdots \mu_4]} \left[R^{\mu_1\mu_2}_{\nu_1\nu_2} + \frac{\delta^{\mu_1\mu_2}_{\nu_1\nu_2}}{\ell^2} \right] \left[R^{\mu_3\mu_4}_{\nu_3\nu_4} + \frac{\delta^{\mu_3\mu_4}_{\nu_3\nu_4}}{\ell^2} \right] .$$

Weyl tensor

$$W^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} - 4 S^{[\alpha}_{[\mu} \delta^{\beta]}_{\nu]} , \qquad \text{Schouten } S^{\alpha}_{\mu} = \frac{1}{D-2} (R^{\alpha}_{\mu} - \frac{1}{2(D-1)} \delta^{\alpha}_{\mu} R)$$

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Renormalized action for Einstein spaces

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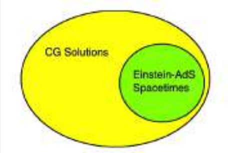
Conformal Renormalization

Embedding Einstein theory in Conformal Gravity



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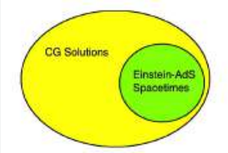

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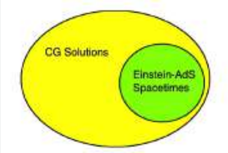

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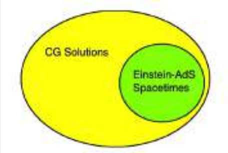

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Einstein Gravity from Conformal Gravity in 4D

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Einstein gravity from CG with Neumann bc's [Maldacena, 2011]

$$I_{CG} = \alpha_{CG} \int\limits_M d^4x \sqrt{-g}\, W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta}$$

Fefferman-Graham expansion for AAdS spaces in CG

$$ds^2 = \frac{\ell^2}{z^2}\, dz^2 + \frac{1}{z^2}\, g_{ij}(x,z)\, dx^i dx^j \,, \quad g_{ij}(x,\rho) = g_{(0)ij}(x) + z^2 g_{(2)ij}(x) + \cdots$$
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EOM for Conformal Gravity

$$B_{\mu\nu} = \nabla^\lambda C_{\mu\nu\lambda} + S^{\lambda\sigma} W_{\lambda\mu\sigma\nu} = 0 \,, \qquad C^\mu_{\nu\lambda} = \nabla_\nu S^\mu_\lambda - \nabla_\lambda S^\mu_\nu$$

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Einstein-AdS spaces

$$S^{\mu}_{\nu} = -\frac{1}{2\ell^2}\delta^{\mu}_{\nu}, \quad C_{\mu\nu\lambda} = 0, \quad B_{\mu\nu} = 0$$

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AdS gravity in 6D

EH Action+Euler term

$$\tilde{I}_{\text{ren}} = \frac{1}{16\pi G} \int\limits_M d^6x \sqrt{-g} \left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72} (Euler)_6 \right) ,$$

In terms of fully-antisymmetric objects

$$\begin{aligned} \tilde{I}_{\text{ren}} &= \frac{1}{16\pi G \times 192} \int\limits_M d^6x \sqrt{-g}\, \delta^{[\nu_1 \cdots \nu_6]}_{[\mu_1 \cdots \mu_6]} \left[R^{\mu_1\mu_2}_{\nu_1\nu_2} \delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]} \delta^{[\mu_5\mu_6]}_{[\nu_5\nu_6]} \right. \\ &\quad \left. + \frac{2}{3\ell^2} \delta^{[\mu_1\mu_2]}_{[\nu_1\nu_2]} \delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]} \delta^{[\mu_5\mu_6]}_{[\nu_5\nu_6]} - \frac{\ell^4}{3} R^{\mu_1\mu_2}_{\nu_1\nu_2} R^{\mu_3\mu_4}_{\nu_3\nu_4} R^{\mu_5\mu_6}_{\nu_5\nu_6} \right] , \end{aligned}$$

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$$\begin{aligned}\tilde{I}_{\text{ren}} &= \frac{1}{16\pi G\times 192}\int\limits_{M} d^6x\sqrt{-g}\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}\left[R^{\mu_1\mu_2}_{\nu_1\nu_2}\delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]}\delta^{[\mu_5\mu_6]}_{[\nu_5\nu_6]}\right.\\ &\left.+\frac{2}{3\ell^2}\delta^{[\mu_1\mu_2]}_{[\nu_1\nu_2]}\delta^{[\mu_3\mu_4]}_{[\nu_3\nu_4]}\delta^{[\mu_5\mu_6]}_{[\nu_5\nu_6]}-\frac{\ell^4}{3}R^{\mu_1\mu_2}_{\nu_1\nu_2}R^{\mu_3\mu_4}_{\nu_3\nu_4}R^{\mu_5\mu_6}_{\nu_5\nu_6}\right],\end{aligned}$$

AdS gravity in 6D

Polynomial of $W_{(E)}$

$$\tilde{I}_{\text{ren}} = \frac{\ell^4}{16\pi G\times 4!}\int\limits_M d^6x\sqrt{-g}\Big[\frac{1}{2\ell^2}\delta^{[\nu_1\cdots\nu_4]}_{[\mu_1\cdots\mu_4]}W^{\mu_1\mu_2}_{(E)\nu_1\nu_2}W^{\mu_3\mu_4}_{(E)\nu_3\nu_4} - \frac{1}{4!}\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}W^{\mu_1\mu_2}_{(E)\nu_1\nu_2}W^{\mu_3\mu_4}_{(E)\nu_3\nu_4}W^{\mu_5\mu_6}_{(E)\nu_5\nu_6}\Big],$$

Conformal Gravity in 6D

There are three Conformal Invariants in 6D

$$
\begin{aligned}
I_1 &= W_{\alpha\beta\mu\nu} W^{\alpha\sigma\lambda\nu} W_{\sigma}{}^{\beta\mu}{}_{\lambda} \,, \\
I_2 &= W_{\mu\nu\alpha\beta} W^{\alpha\beta\sigma\lambda} W_{\sigma\lambda}{}^{\mu\nu} \,, \\
I_3 &= W_{\mu\rho\sigma\lambda} \left(\delta^{\mu}_{\nu} \Box + 4R^{\mu}_{\nu} - \frac{6}{5} R \delta^{\mu}_{\nu} \right) W^{\nu\rho\sigma\lambda} + \nabla_{\mu} J^{\mu} \,,
\end{aligned}
$$

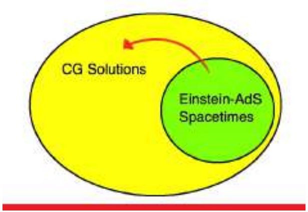
with

$$
\begin{aligned}
J_{\mu} \quad &= \quad 4R_{\mu}{}^{\lambda\rho\sigma} \nabla^{\nu} R_{\nu\lambda\rho\sigma} + 3R^{\nu\lambda\rho\sigma} \nabla_{\mu} R_{\nu\lambda\rho\sigma} - 5R^{\nu\lambda} \nabla_{\mu} R_{\nu\lambda} \\
&\quad + \frac{1}{2} R \nabla_{\mu} R - R^{\nu}_{\mu} \nabla_{\nu} R + 2R^{\nu\lambda} \nabla_{\nu} R_{\lambda\mu} .
\end{aligned}
$$

Conformal Covariantization

Einstein	$\rightarrow$	Conformal	CI's
$\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}\mathbf{W}^{\mu_1\mu_2}_{(E)\nu_1\nu_2}\mathbf{W}^{\mu_3\mu_4}_{(E)\nu_3\nu_4}\mathbf{W}^{\mu_5\mu_6}_{(E)\nu_5\nu_6}$	$\rightarrow$	$\delta^{[\nu_1\cdots\nu_6]}_{[\mu_1\cdots\mu_6]}\mathbf{W}^{\mu_1\mu_2}_{\nu_1\nu_2}\mathbf{W}^{\mu_3\mu_4}_{\nu_3\nu_4}\mathbf{W}^{\mu_5\mu_6}_{\nu_5\nu_6}$	$32(2\mathbf{I}_1+\mathbf{I}_2)$
$-\frac{1}{\ell^2}\delta^{[\nu_1\cdots\nu_4]}_{[\mu_1\cdots\mu_4]}\mathbf{W}^{\mu_1\mu_2}_{(E)\nu_1\nu_2}\mathbf{W}^{\mu_3\mu_4}_{(E)\nu_3\nu_4}$	$\rightarrow$	$\delta^{[\nu_1\cdots\nu_5]}_{[\mu_1\cdots\mu_5]}\mathbf{W}^{\mu_1\mu_2}_{\nu_1\nu_2}\mathbf{W}^{\mu_3\mu_4}_{\nu_3\nu_4}\mathbf{S}^{\mu_5}_{\nu_5}+\mathbf{16}\mathbf{C}^{\mu\nu\lambda}\mathbf{C}_{\mu\nu\lambda}+\nabla^\mu\mathbf{J}_\mu$	$4\mathbf{I}_1-\mathbf{I}_2-\mathbf{I}_3$

$$\mathbf{J}_\mu=\mathbf{16}\mathbf{W}^{\kappa\lambda\nu}_{\mu}\mathbf{C}_{\kappa\lambda\nu}-\mathbf{2}\mathbf{W}^{\kappa\lambda}_{\nu\sigma}\nabla_\mu\mathbf{W}^{\nu\sigma}_{\kappa\lambda}$$



Lu-Pang-Pope CG in 6D

6D CG with an Einstein sector [Lu, Pang and Pope, 2013]

$$I_{CG} = \alpha_{CG} \int\limits_M d^6x \sqrt{-\hat{g}} \left(\frac{1}{4!} \delta^{[\nu_1 \cdots \nu_6]}_{[\mu_1 \cdots \mu_6]} W^{\mu_1\mu_2}_{\nu_1\nu_2} W^{\mu_3\mu_4}_{\nu_3\nu_4} W^{\mu_5\mu_6}_{\nu_5\nu_6} + \frac{1}{2} \delta^{[\nu_1 \cdots \nu_5]}_{[\mu_1 \cdots \mu_5]} W^{\mu_1\mu_2}_{\nu_1\nu_2} W^{\mu_3\mu_4}_{\nu_3\nu_4} S^{\mu_5}_{\nu_5} + 8 C^{\mu\nu\lambda} C_{\mu\nu\lambda} \right) + \alpha_{CG \partial M} d^5x \sqrt{-h} n^{\mu} \left(8 W_{\mu}^{\kappa\lambda\nu} C_{\kappa\lambda\nu} - W^{\kappa\lambda}_{\nu\sigma} \nabla_{\mu} W^{\nu\sigma}_{\kappa\lambda} \right).$$

- LPP action appears as type-B anomaly and one-loop divergences in 7D
- Variation of I_{CG} gives EOM in terms of Weyl, Cotton and Schouten tensors.
- Any Einstein-AdS spacetime is solution of LPP CG. [Anastasiou, Araya and RO, 2010.15146]

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$$\left.+8C^{\mu\nu\lambda}C_{\mu\nu\lambda}\right)+\alpha_{CG}\int_{\partial M}d^5x\sqrt{-h}n^{\mu}\left(8W^{\kappa\lambda\nu}_{\mu}C_{\kappa\lambda\nu}-W^{\kappa\lambda}_{\nu\sigma}\nabla_{\mu}W^{\nu\sigma}_{\kappa\lambda}\right).$$

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$$\left. + 8 C^{\mu\nu\lambda} C_{\mu\nu\lambda} \right) + \alpha_{CG} \partial_M d^5x \sqrt{-h} n^{\mu} \left(8 W^{\kappa\lambda\nu}_{\mu} C_{\kappa\lambda\nu} - W^{\kappa\lambda}_{\nu\sigma} \nabla_{\mu} W^{\nu\sigma}_{\kappa\lambda} \right).$$

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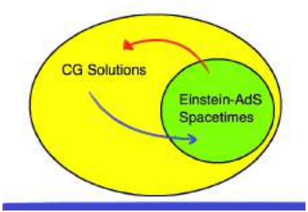
And back...(Einstein gravity from CG in 6D)

LPP CG action decomposed into Einstein and non-Einstein parts:

$$I_{CG} = -4!\alpha_{CG}\int\limits_{M} d^6x\sqrt{-g}\left[P_6\left(W_{(E)}\right)+Q\left(W_{(E)},H\right)\right]$$
$$-\alpha_{CG}\int\limits_{\partial M} d^5x\sqrt{-h}n^{\mu}\left(W^{\kappa\lambda}_{(E)\nu\sigma}\nabla_{\mu}W^{\nu\sigma}_{(E)\kappa\lambda}\right).$$



Back to Einstein gravity (with an extra term)

Einstein condition, and $\alpha_{\mathrm{E}} = -\frac{\ell^4}{384\pi G}$:

$$I_{CG}\left[E\right] = \frac{1}{16\pi G}\int\limits_{M} d^6x\sqrt{-g}\left(R + \frac{20}{\ell^2} - \frac{\ell^4}{72}(Euler)_6\right) + \frac{\ell^4}{384\pi G}\int\limits_{\partial M} d^5x\sqrt{-h}n^{\mu}\left(W_{(E)\nu\sigma}^{\kappa\lambda}\nabla_{\mu}W_{(E)\kappa\lambda}^{\nu\sigma}\right),$$

Performing asymptotic expansions:

$$\Delta I = \frac{\ell^8}{192\pi G}\int\limits_{\partial M} d^5x\sqrt{-h}W^{ijkl}(h)W_{ijkl}(h) + \ldots$$

CG action for Einstein spaces = Renormalized Einstein AdS action

$$I_{\mathrm{CG}}\left[E\right] = I_{\mathrm{HR}}$$

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Curvature-squared terms and conical defects

Riemann squared term

$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(Rie^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,Rie^2 + 8\pi\,(1-\alpha)\int_\Sigma d^2y\sqrt{\gamma}\left(R_{ABAB} - \mathcal{K}^{(A)}_{mn}\mathcal{K}^{mn}_{(A)}\right) + \dots$$

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$$\int_{M^{(\alpha)}} d^4x\sqrt{g}\left(Ric^{(\alpha)}\right)^2 = \int_M d^4x\sqrt{g}\,Ric^2 + 4\pi\,(1-\alpha)\int_\Sigma d^2y\sqrt{\gamma}\left(R_{AA} - \frac{1}{2}\mathcal{K}^{(A)}\mathcal{K}_{(A)}\right) + \dots$$

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[Fursaev-Patrushev-Solodukhin, 2013]

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Conformal Gravity and Conical Defects

4D Conformal Gravity

$$I_{CG}=\frac{\ell^2}{64\pi G}\int\limits_M d^4x\,\sqrt{-g}\,W_{\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta}-\frac{\pi\ell^2}{2G}\chi\left[M\right]$$

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$$I_{CG}=\frac{\ell^2}{64\pi G}\int\limits_M d^4x\sqrt{g}\left(Rie^2-2Ric^2+\frac{1}{3}R^2\right)-\frac{\pi\ell^2}{2G}\chi\left[M\right]$$

In a manifold with a conical singularity

$$\int\limits_{M^{(\alpha)}} d^4x\sqrt{g}\left|W^{(\alpha)}\right|^2=\int\limits_M d^4x\sqrt{g}\left|W\right|^2+8\pi\left(1-\alpha\right)\int\limits_\Sigma d^2y\sqrt{\gamma}K_\Sigma+O\left(\left(1-\alpha\right)^2\right)$$

Conformal Gravity and Conical Defects

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Conformal Gravity and Conical Defects

4D Conformal Gravity

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Conformal Gravity and Conical Defects

Conformal Invariant in codimension-2 (Graham-Witten anomaly)

$$K_{\Sigma} = W^{mn}_{mn} - P^{(A)}_{mn} P^{mn}_{(A)} \,,$$

Traceless part of the extrinsic curvature

$$P^{(A)}_{mn} = \mathcal{K}^{(A)}_{mn} - \frac{1}{2} \mathcal{K}^{(A)} \gamma_{mn}$$

Conformal Gravity Action

$$I^{(\alpha)}_{CG} = I_{CG} + \frac{(1-\alpha)}{4G} L_{\Sigma} + O\left((1-\alpha)^2\right)$$

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Renormalized Area

Minimal surface/Einstein ambient space

$$\mathcal{A}_{\text{ren}} = L_{\Sigma_{\text{min}}}[E]$$

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$$A_{ren}\left[\Sigma\right] = \frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left[W^{mn}_{(\text{E})mn} - P^{(A)}_{mn}P^{mn}_{(A)}\right] - 2\pi\ell^2\chi\left[\Sigma\right]$$

Ren. Area [Alexakis-Mazzeo, 2010] / Ren. HEE [Anastasiou-Araya-RO, 2018]

$$A_{\text{ren}}\left[\Sigma\right] = \frac{\ell^2}{4}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\,\delta^{pq}_{mn}\left(\mathcal{R}^{mn}_{pq} + \frac{1}{\ell^2}\delta^{mn}_{pq}\right) - 2\pi\ell^2\chi\left[\Sigma\right]$$

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Functional defined on compact and orientable 2D surfaces immersed in $\mathbb{R}^3$

In terms of the (spatial) mean curvature

$$W[\Sigma]=\int_{\Sigma} d^2y\sqrt{\gamma}H^2$$

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For pure/global AdS$_4$ as ambient space, constant-time slice

$$W = 0 \qquad\qquad \mathcal{K}^{(t)} = 0$$

Gauss-Codazzi relations

$$\mathcal{A}_{\text{ren}}[\Sigma] = -\frac{\ell^2}{2}\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left(R^{ij}_{ij} - \mathcal{R} + 2H^2\right) - 2\pi\ell^2\chi[\Sigma]$$

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For a compact surface

$$\int\limits_{\Sigma_{\text{comp}}} d^2y\sqrt{\hat{\gamma}}\hat{\mathcal{R}} = 4\pi\chi[\Sigma_{\text{comp}}]$$

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Bonus Track: Reduced Hawking Mass

Arbitrary Σ, Einstein ambient space

$$L_{\Sigma}\left[E\right]=\frac{\ell^{2}}{4}I_{H}\left[\Sigma\right]-2\pi\ell^{2}\chi\left[\Sigma\right]$$

Reduced Hawking Mass I_H [Fischetti and Wiseman, 2016]

$$I_{H}\left[\Sigma\right]=2\int\limits_{\Sigma}d^{2}y\sqrt{\gamma}\left[\mathcal{R}+\frac{2}{\ell^{2}}-\frac{1}{2}\left(\mathcal{K}^{(A)}\right)^{2}\right]$$

Generalizes Renormalized Area Functional

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Bonus Track: Reduced Hawking Mass

Arbitrary Σ, Einstein ambient space

$$L_{\Sigma}\left[E\right] = \frac{\ell^2}{4} I_H\left[\Sigma\right] - 2\pi\ell^2\chi\left[\Sigma\right]$$

Reduced Hawking Mass I_H [Fischetti and Wiseman, 2016]

$$I_H\left[\Sigma\right] = 2\int\limits_{\Sigma} d^2y\sqrt{\gamma}\left[\mathcal{R} + \frac{2}{\ell^2} - \frac{1}{2}\left(\mathcal{K}^{(A)}\right)^2\right]$$

Generalizes Renormalized Area Functional

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Outlook

Conformal Invariance in the Bulk $\Longrightarrow$ Conformal Invariance in Codimension-2

Renormalization in the Bulk $\Longrightarrow$ Renormalization in Codimension-2

Conformal Invariance $\Longrightarrow$ Renormalization (???)

Renormalized Volume $\Longrightarrow$ Renormalized Area (in conically singular manifolds)

Conformal Gravity in higher even dimensions $D \geq 8$? (with N.Boulanger)

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