Weyl Curvature Hypothesis

Ivica Smolić

University of Zagreb

Physics and Geometry Seminars

Nov 14$^{\text{th}}$ 2019

Overview

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- Context and motivation

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- **WCH** through $\lesssim 50$ papers

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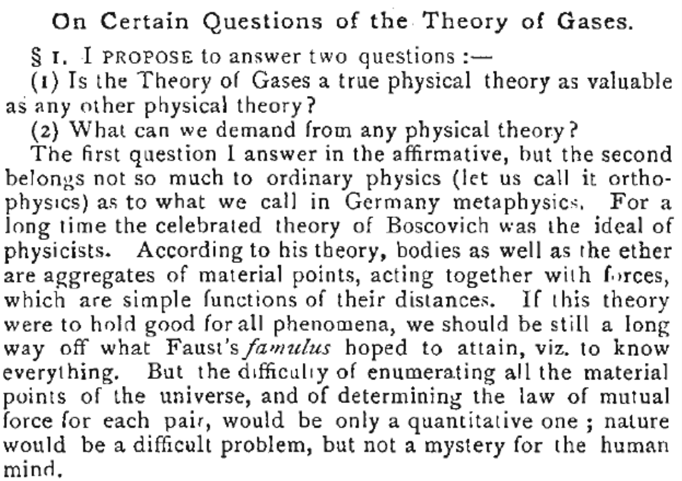
- Open questions

Problem of arrow of time

Ludwig Eduard Boltzmann

Boltzmann on arrow of time (1895)



On Certain Questions of the Theory of Gases.

§ 1. I PROPOSE to answer two questions :—

(1) Is the Theory of Gases a true physical theory as valuable as any other physical theory?

(2) What can we demand from any physical theory?

The first question I answer in the affirmative, but the second belongs not so much to ordinary physics (let us call it ortho-physics) as to what we call in Germany metaphysics. For a long time the celebrated theory of Boscovich was the ideal of physicists. According to his theory, bodies as well as the ether are aggregates of material points, acting together with forces, which are simple functions of their distances. If this theory were to hold good for all phenomena, we should be still a long way off what Faust's *famulus* hoped to attain, viz. to know everything. But the difficulty of enumerating all the material points of the universe, and of determining the law of mutual force for each pair, would be only a quantitative one; nature would be a difficult problem, but not a mystery for the human mind.

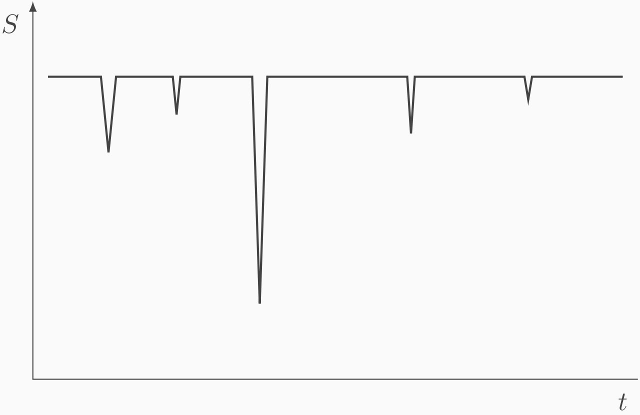
I will conclude this paper with an idea of my old assistant, Dr. Schuetz.

We assume that the whole universe is, and rests for ever, in thermal equilibrium. The probability that one (only one) part of the universe is in a certain state, is the smaller the further this state is from thermal equilibrium; but this probability is greater, the greater is the universe itself. If we assume the universe great enough, we can make the probability of one relatively small part being in any given state (however far from the state of thermal equilibrium), as great as we please. We can also make the probability great that, though the whole universe is in thermal equilibrium, our world is in its present state. It may be said that the world is so far from thermal equilibrium that we cannot imagine the improbability of such a state. But can we imagine, on the other side, how small a part of the whole universe this world is? Assuming the universe great enough, the probability that such a small part of it as our world should be in its present state, is no longer small.

If this assumption were correct, our world would return more and more to thermal equilibrium; but because the whole universe is so great, it might be probable that at some future time some other world might deviate as far from thermal equilibrium as our world does at present. Then the afore-mentioned H-curve would form a representation of what takes place in the universe. The summits of the curve would represent the worlds where visible motion and life exist.

LUDWIG BOLTZMANN.

Imperial University of Vienna.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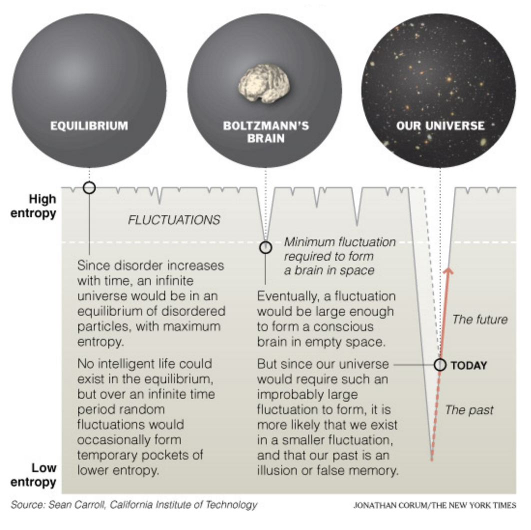
The New York Times, Jan 14 2008



Modern view, in a nutshell

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crucial question:

Why was S low in the vicinity of the Big Bang?

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however, we are forgetting about the

gravitational degrees of freedom!

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  Proceedings of the First Marcel Grossmann Meeting on GR

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- R. Penrose: *Singularities and time–asymmetry* (1979)

  General Relativity, an Einstein centenary survey

Why Weyl?

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Ricci $\leftrightarrow$ matter via Einstein

$$R_{ab} = 8\pi \left(T_{ab} - \frac{1}{D-2} T g_{ab} \right)$$

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$$R_{ab} = 8\pi \left(T_{ab} - \frac{1}{D-2} T g_{ab} \right)$$

"subtract Ricci from Riemann $\rightarrow$ Weyl"

$$C_{abcd} = R_{abcd} + \frac{2}{D-2} \left(g_{a[c} R_{d]b} - g_{b[c} R_{d]a} \right) - \\ - \frac{2}{(D-1)(D-2)} R \, g_{a[c} g_{d]b}$$

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conformal transformation

$$\tilde{g}_{ab} = \Omega^2 \, g_{ab}$$

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conformal co/invariance of Weyl tensor

$$\widetilde{C}^a{}_{bcd} = C^a{}_{bcd} \qquad \text{but} \qquad \widetilde{C}_{abcd} = \Omega^2 \, C_{abcd}$$

Weyl and Petrov

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Type	multiplicity	$\Psi_{ABCD} =$
I	$\{1,1,1,1\}$	$\alpha_{(A}\,\beta_B\,\gamma_C\,\delta_{D)}$
II	$\{2,1,1\}$	$\alpha_{(A}\,\alpha_B\,\gamma_C\,\delta_{D)}$
D	$\{2,2\}$	$\alpha_{(A}\,\alpha_B\,\beta_C\,\beta_{D)}$
III	$\{3,1\}$	$\alpha_{(A}\,\alpha_B\,\alpha_C\,\beta_{D)}$
N	$\{4\}$	$\alpha_A\,\alpha_B\,\alpha_C\,\alpha_D$
O	–	0

Weyl and Petrov

$$\begin{tikzcd}
I \arrow[d] \arrow[dr] & & \\
II \arrow[r] \arrow[d] \arrow[dr] & D \arrow[d] \arrow[dr] & \\
III \arrow[r] & N \arrow[r] & O
\end{tikzcd}$$

"It seems that in some way the Weyl tensor gives a measure of the entropy in the space–time geometry. The initial curvature singularity would then be one with large Ricci tensor and vanishing Weyl tensor (zero entropy in the geometry); the final curvature singularity would have Weyl tensor much larger than Ricci tensor (large entropy in the geometry).

These ideas are indeed somewhat imprecise, but it would seem worth while to pursue them further."

R. Penrose: *Space–Time Singularities* (1977)

Solidification of the idea

Early cosmological considerations

Early cosmological considerations

- Goode and Wainwright, CQG **2** (1985) 99–115

  In some cosmological models

$$\lim_{T\to 0^+} R_{ab}R^{ab} = \infty \,, \quad \lim_{T\to 0^+} C_{abcd}C^{abcd} = \infty$$

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  $\approx$ sing. which can be removed by rescaling the metric

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- Newman, Proc. R. Soc. Lond. A **443** (1993)
  ...proof for $p = \rho/3$

- Goode, CQG **8** (1991)
  Goode, Coley and Wainwright, CQG **9** (1992)

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- Barrow and Hervik, CQG **19** (2002)

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Roger Penrose: By way of clarification I should say that I never meant the Weyl tensor to be a measure of gravitational entropy. I merely wanted it to be zero at the big bang.

TR: Really? Well, if that's true I apologize.

Roger Penrose: You're not the only person to have that misunderstanding.

Weyl $\rightarrow$ Bel–Robinson

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- Pelavas and Coley, Int. J. Theor. Phys. **42** (2006) 1301

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$$T_{abcd} = \Psi_{ABCD}\overline{\Psi}_{A'B'C'D'}$$

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$$W = T_{abcd}u^a u^b u^c u^d \geq 0$$

More systematics

via Clifton, Ellis and Tavakol, CQG **30** (2013) 125009

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E4 $S_{\rm grav} \;\to\; S_{\rm BH}$

E5 $\delta S_{\rm grav} \geq 0$ as structures form in Universe

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  CQG **31** (2014) 075021; CQG **32** (2015) 165012

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- Maki and Morita, Springer Proc. Math. Stat. **60** (2014) 311
  ...Weyl in Jacobson's approach
- Acquaviva, Kofroň and Scholtz, CQG **35** (2018)
  ...dissipative terms (?)

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- Weyl^2/Ricci^2 vs Bel–Robinson vs ...?
- bottom $\rightarrow$ up derivation of gravitational entropy?
- how local is S_{grav}?

Thank you for the attention!