Magnetization in the Frustrated XY chain

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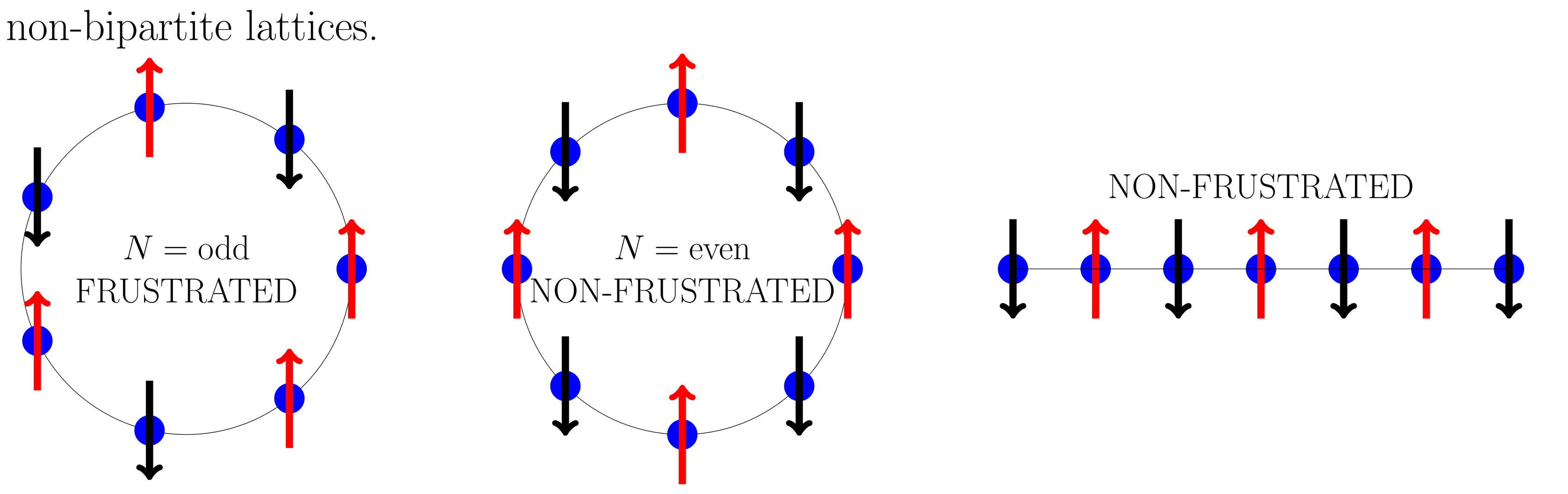
Abstract

We analyze the XY chain in a zero magnetic field. The interactions are antiferromagnetic, ferromagnetic or a combination of them. The periodic boundary conditions and an odd number of lattice sites are imposed to induce frustration. The model possesses peculiar symmetries, whose generators have a fermionic nature and anti-commute, resulting into (at least two-fold) exact degeneracy of every eigenstate, even in finite size systems. The degeneracy is between different parity sectors, thus allowing a non-zero ground state magnetization (and spontaneous symmetry breaking) even in a finite system. Based on the symmetries we develop a simple method of computing the magnetization exactly, which is otherwise a technical problem. We distinguish three different regions in the phase diagram. One is standard, with the magnetization in the thermodynamic limit having the same value as in the non-frustrated model. The second shows an algebraic decay of the magnetization with the system size, reaching zero in the thermodynamic limit. The third one shows an algebraic decay towards a non-zero value. The behavior of the magnetization in the latter two phases and even a distinction between them are novel phenomena, due to frustration. This model demonstrates that different boundary conditions may result in a different behavior of the magnetization in the thermodynamic limit.

Frustration

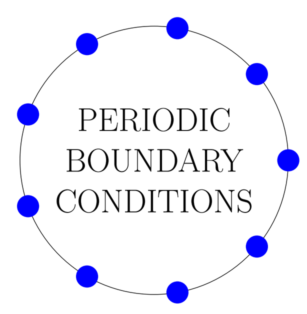
- Frustration is a competition of different interactions, tending to minimize the energy of the system, but unable to do it simultaneously. It is present in Ising antiferromagnets on non-bipartite lattices.



- Combined with the quantum effects it may result in a new behavior of the quantum-many body systems.

The XY chain

$$H = -\cos\phi \sum_{j=1}^{N} \sigma_j^y \sigma_{j+1}^y + \sin\phi \sum_{j=1}^{N} \sigma_j^x \sigma_{j+1}^x \,, \qquad N = \text{odd}\,,$$



The parity operators $P_x = \prod_{j=1}^{N} \sigma_j^x$, $P_y = \prod_{j=1}^{N} \sigma_j^y$, $P_z = \prod_{j=1}^{N} \sigma_j^z$ commute with the Hamiltonian

$$[H, P_\alpha] = 0\,, \quad \alpha = x, y, z$$

but anticommute among themselves

$$\{P_\alpha, P_\beta\} = 0\,, \quad \alpha \neq \beta.$$

These symmetries have immediate consequences:

- Exact (at least) two-fold degeneracy of every eigenstate, in particular the ground state.
- It is possible to break the parity symmetry in the ground state of a finite system.
- It is possible to have a non-zero magnetization in the ground state of a finite system.

Computing the Magnetization

- The magnetization $\langle \sigma_j^x \rangle$ in the ground state depends on the superposition coefficients. The order in the system is described by the maximal possible value.

A generic ground state is a superposition between different parity sectors

$$|\text{GS}\rangle = \alpha\,|\text{GS}+\rangle + \beta\,|\text{GS-}\rangle\ , \quad |\alpha|^2 + |\beta|^2 = 1\,, \quad P_z\,|\text{GS}\pm\rangle = \pm\,|\text{GS}\pm\rangle$$

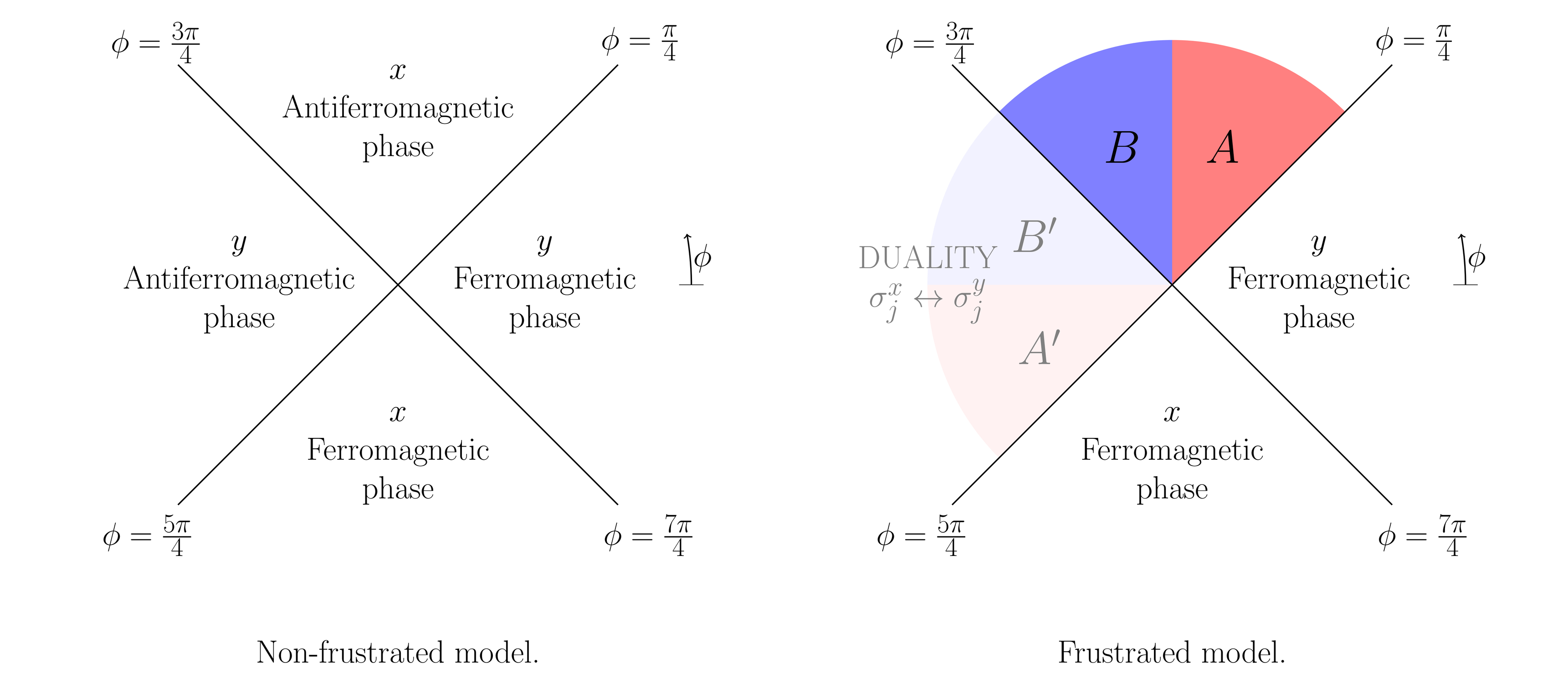
and the magnetization is

$$\langle \text{GS}|\, \sigma_j^x\, |\text{GS}\rangle = \alpha^* \beta\, \langle \text{GS}+|\, \sigma_j^x\, |\text{GS}-\rangle + \text{c.c.}$$

It can be computed in our model from the exact solution and the relation

$$\langle \text{GS}+|\, \sigma_j^x\, |\text{GS}-\rangle = \langle \text{GS}+|\, \sigma_j^x P_x\, |\text{GS}+\rangle = \langle \text{GS}+|\, \prod_{l\neq j}^{N} \sigma_l^x\, |\text{GS}+\rangle\ .$$

Quantum Phases



Phase A - Mesoscopic Magnetization

- Two-fold degenerate translationally invariant ground state.
- The magnetization is not staggered and goes to zero in the thermodynamic limit as $\langle \sigma_j^x \rangle = \frac{1}{N}(1 - \cot^2\phi)^{\frac{1}{4}}$.

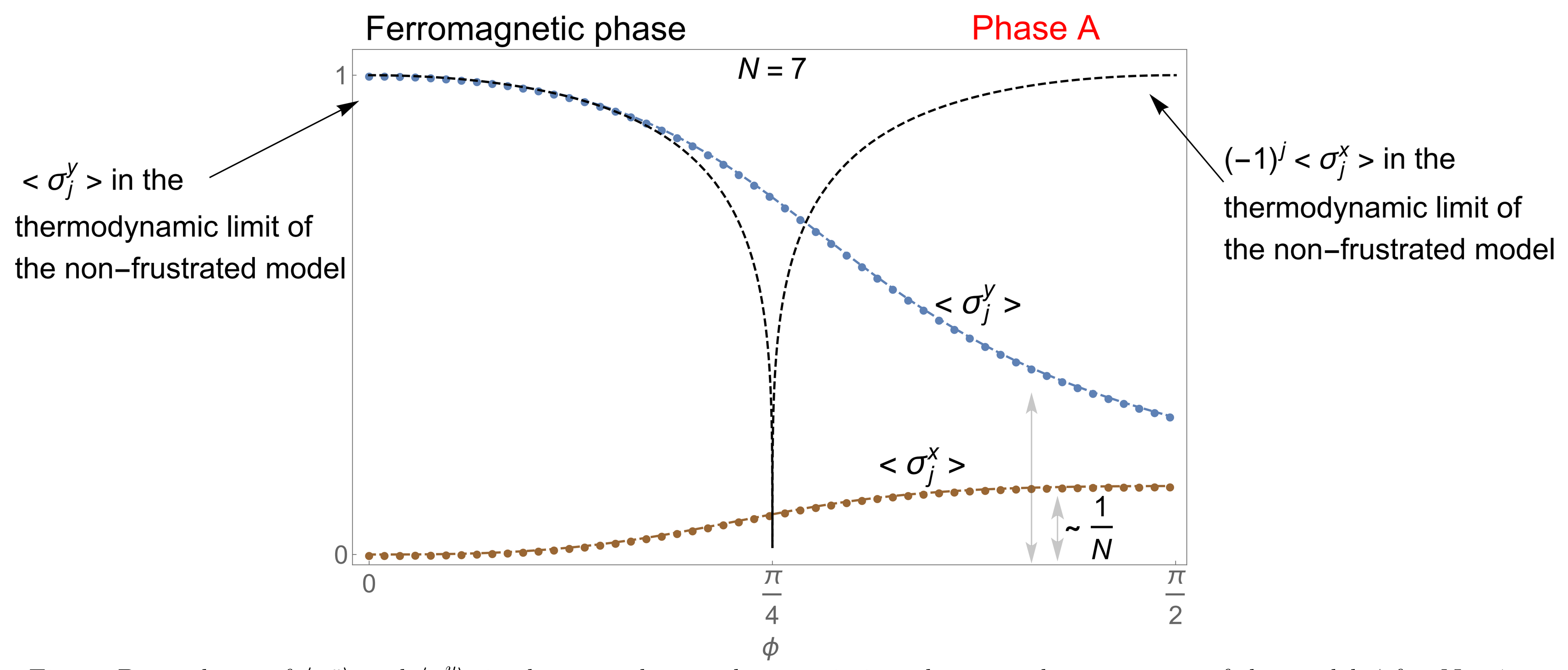


Fig. 1: Dependence of $\langle \sigma_j^x \rangle$ and $\langle \sigma_j^y \rangle$, in the ground state that maximizes them, on the parameter of the model ϕ for $N = 7$.

Phase B - Breaking of Translational Invariance

- Four-fold degenerate ground state, $|\text{GS}\rangle = \alpha_1\,|\text{GS}_1+\rangle + \alpha_2\,|\text{GS}_2+\rangle + \beta_1\,|\text{GS}_1-\rangle + \beta_2\,|\text{GS}_2-\rangle$.
- It is possible to break the translational invariance. The maximum of the magnetization over all lattice sites does not go necessarily to zero in the thermodynamic limit.

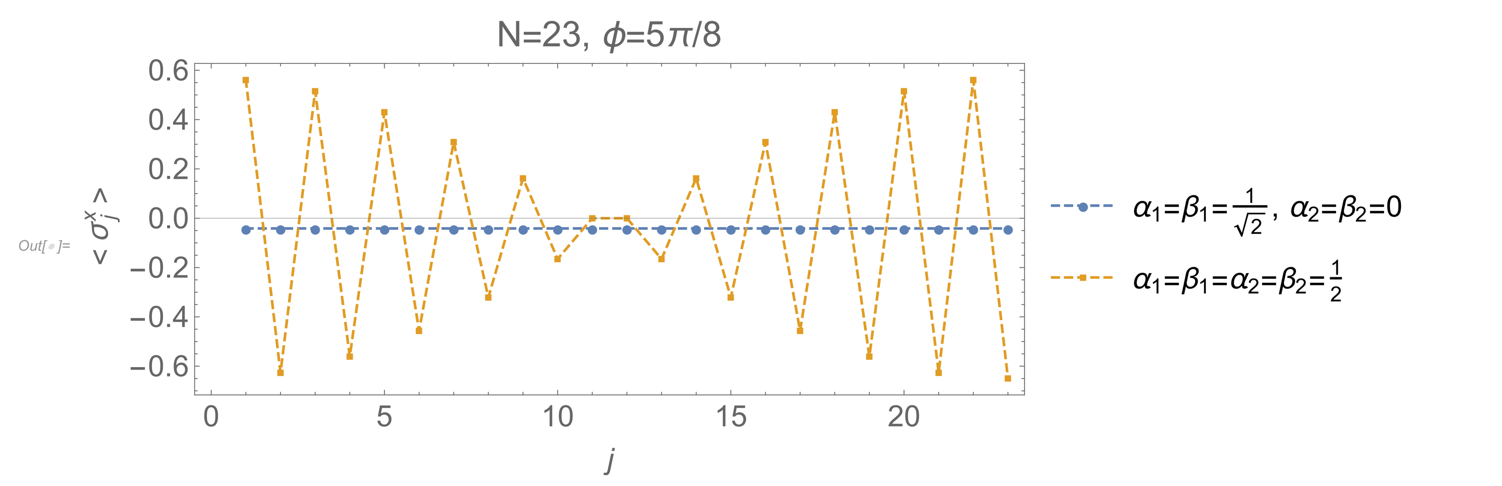


Fig. 2: Dependence of the magnetization on the lattice site for $N = 23$, $\phi = 5\pi/8$ and two different choices of superposition coefficients. For one choice the translational invariance is broken.

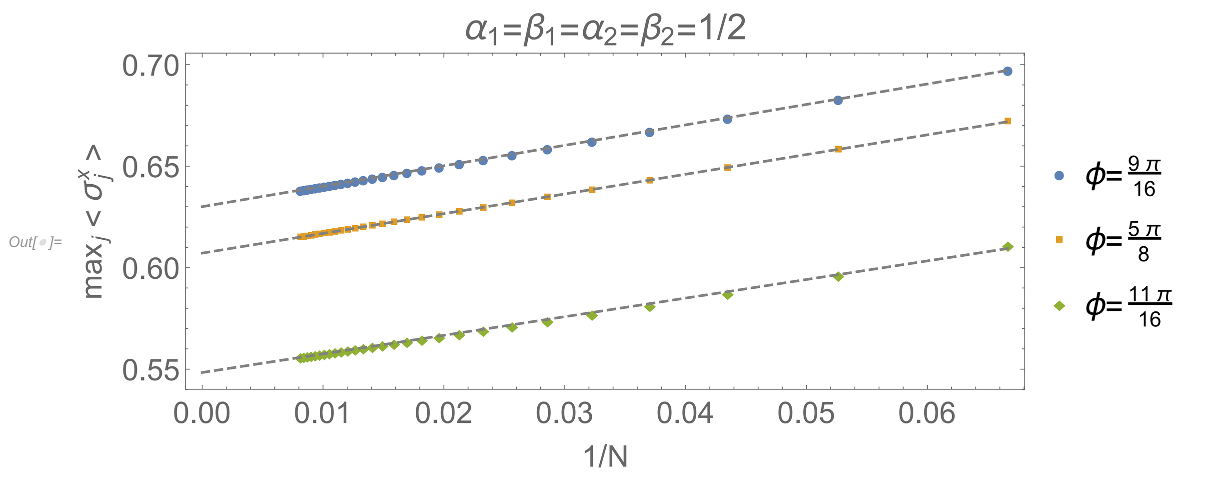


Fig. 3: Dependence of the maximum of the magnetization over all lattice sites on the system size. The linear fits (dashed lines) intersect the y-axis in the non-zero value, indicating a non-zero value in the thermodynamic limit.

Conclusion

- Different boundary conditions may result in a different behavior of the magnetization in the thermodynamic limit.
- Three different phases characterized by the magnetization.
- Connection between frustration and supersymmetries?

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