FRUSTRATION OF BEING ODD

Vanja Marić

PhD Students' Presentations at the RBI Theoretical Physics Department

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ThPhys @IRB

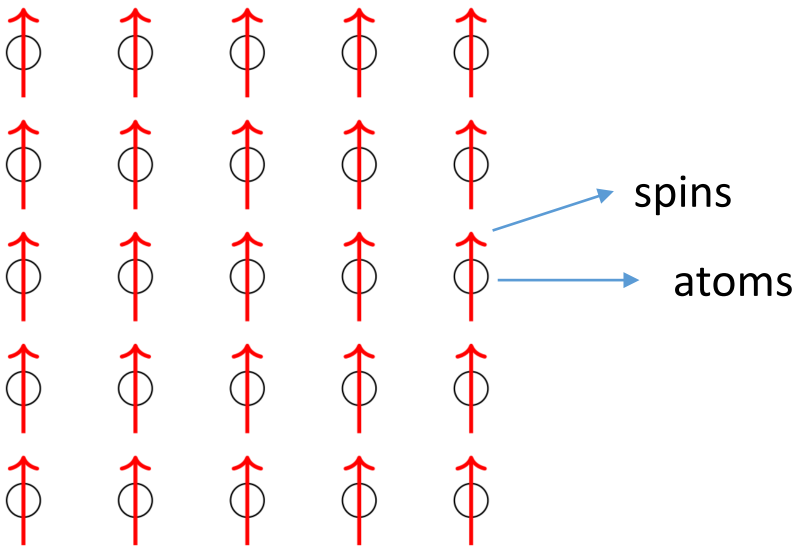
SISSA

Outline

- Magnetic systems, Landau theory
- Frustration
- Research problem: Frustration + Quantum Mechanics, exact results
- Examination of particular models
- Incompleteness of the Landau Theory (boundaries affect local order)
- Conclusions

FERROMAGNETS

- spins like to point in the SAME direction
- Order parameter: $\langle \vec{S}_j \rangle$



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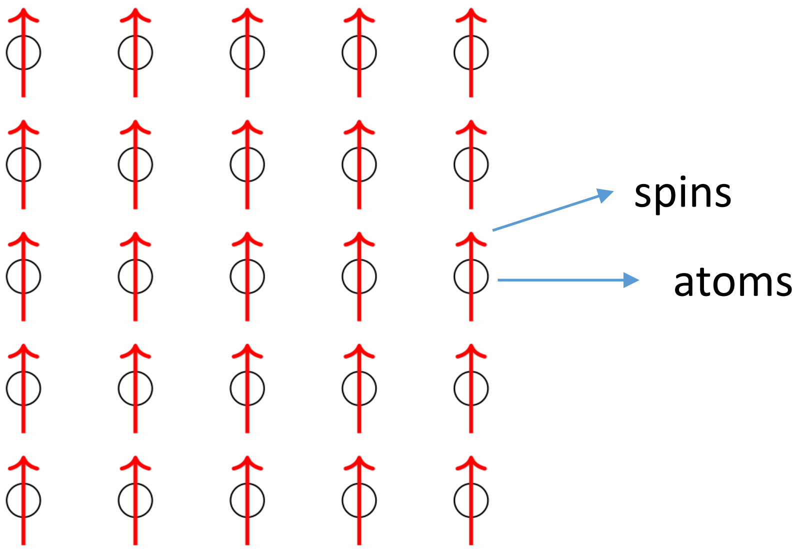
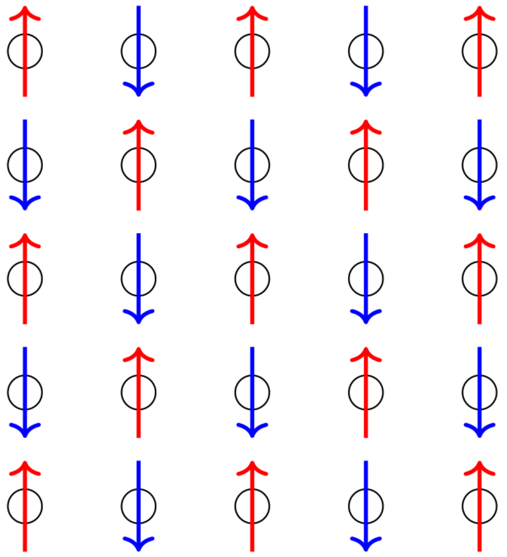


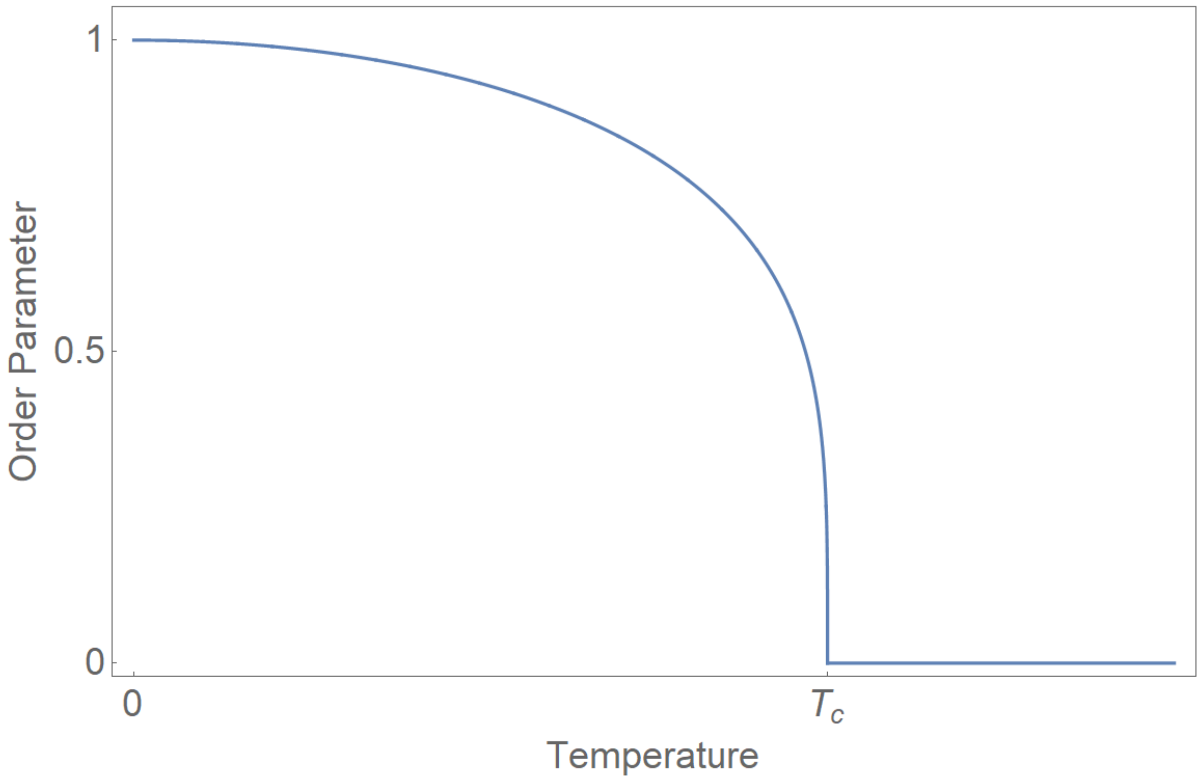
ANTIFERROMAGNETS

- spins like to point in the OPPOSITE directions
- Order parameter: $\langle (-1)^j \vec{S}_j \rangle$



Phases and phase transitions

- Order parameter vanishes above the critical temperature



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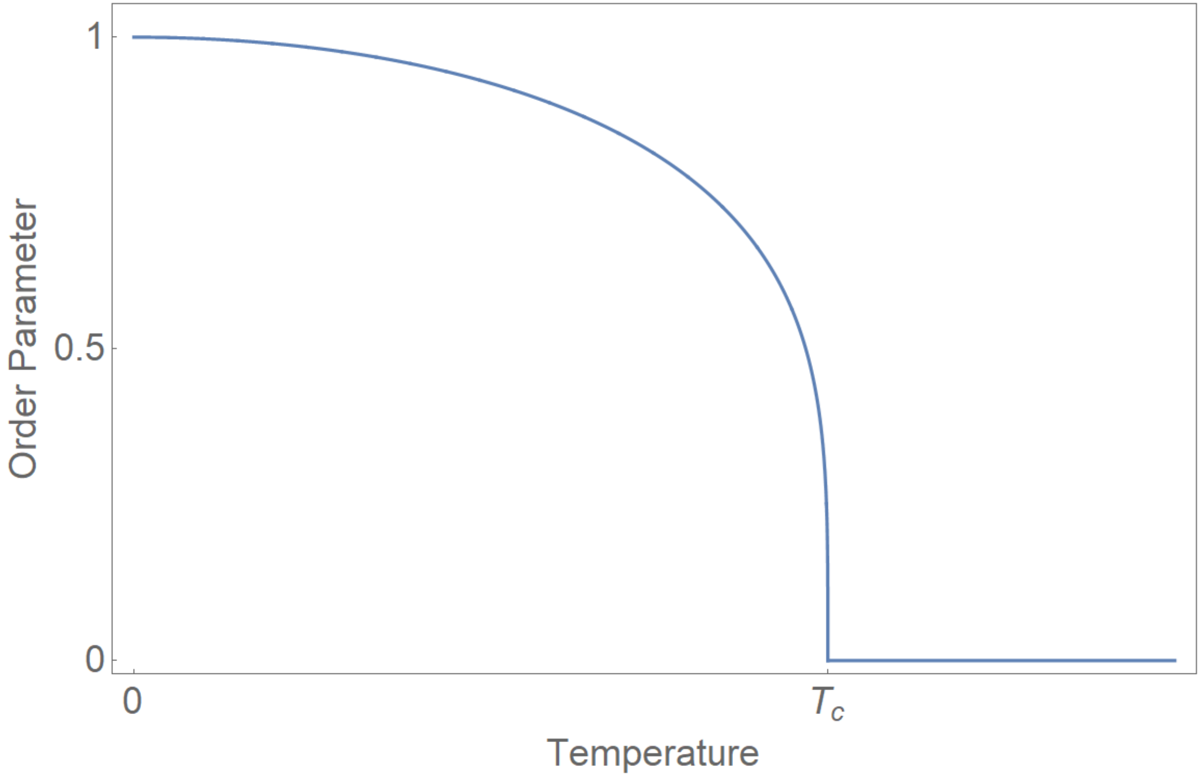

Phase transitions:

- Classical: Changing the temperature.
- Quantum: Changing a parameter of the Hamiltonian at zero temperature.

Landau Theory

- Phase transitions – Spontaneous Symmetry Breaking
    - Exceptions: BKT transition, ...

- Order parameter – Zero in one phase, non-zero in the other.

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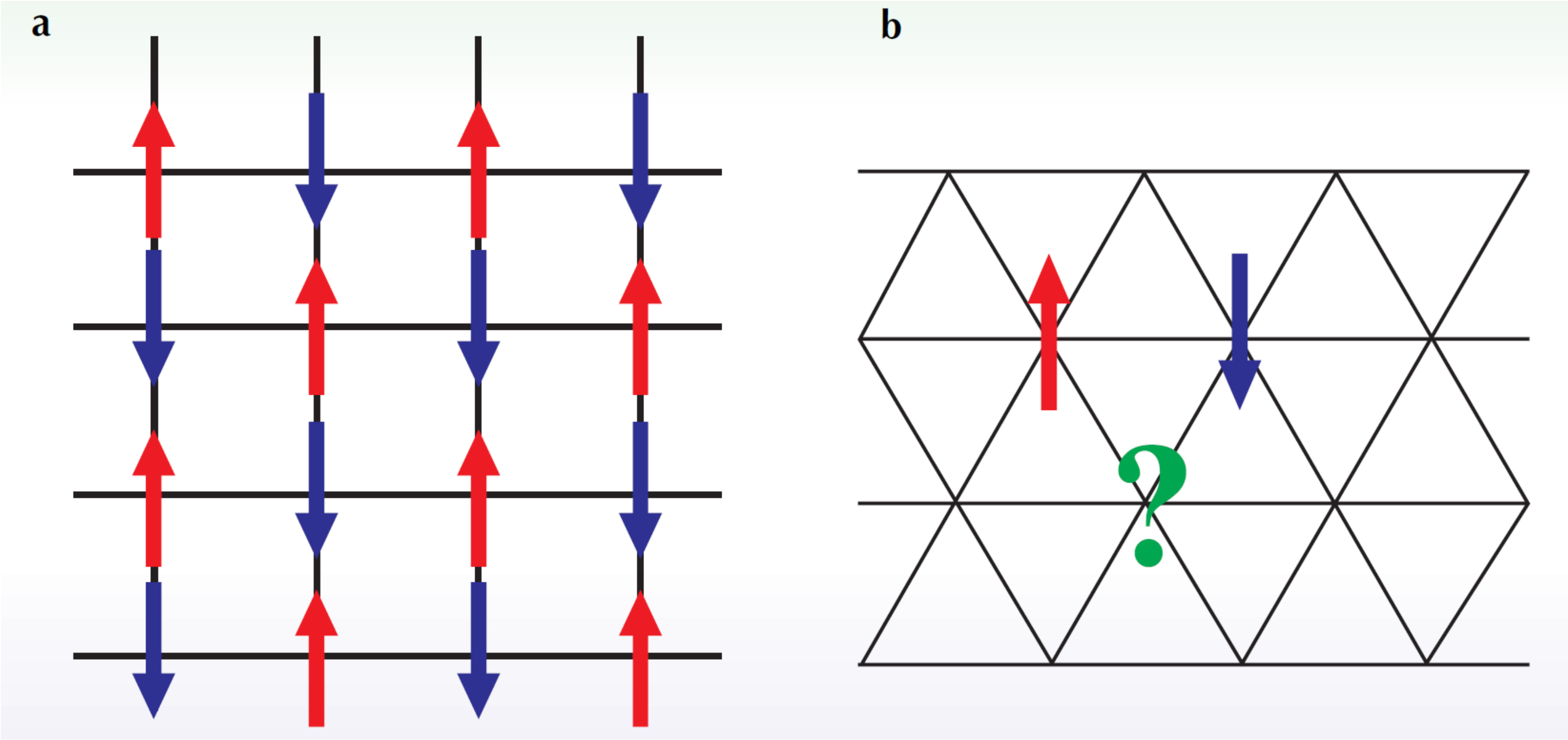
  - In antiferromagnets critical behavior depends on the shape of the lattice (related to Frustration)

- Local order does not depend on the boundaries.

Frustration

- interactions in conflict



Picture taken from [Moessner,Ramirez 2006]

Interest

<u>Applications</u>

Materials with new properties

- Quantum technologies

<u>Fundamental Side</u>

- Phases of matter
- Spin ices, spin liquids
- Ability of such condensed matter systems to mimic different systems (e.g. Artificial light)

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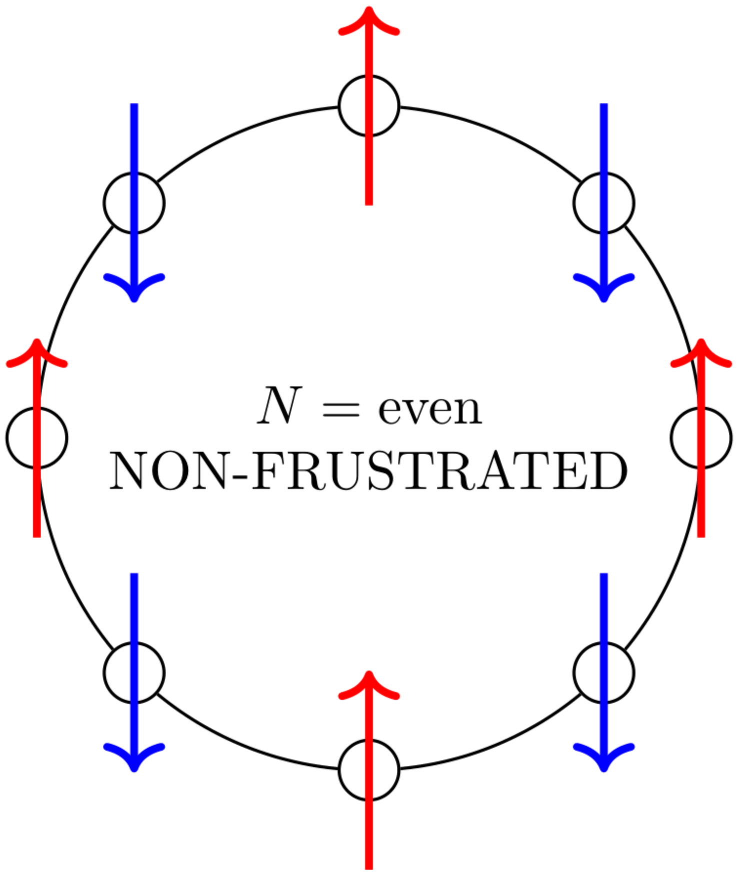
Frustration + Quantum Mechanics, Exact results?

1D: Frustration through Boundary Conditions

$$H = \sum_j S_j S_{j+1}$$



1D: Frustration through Boundary Conditions

- periodic boundary conditions and odd system size

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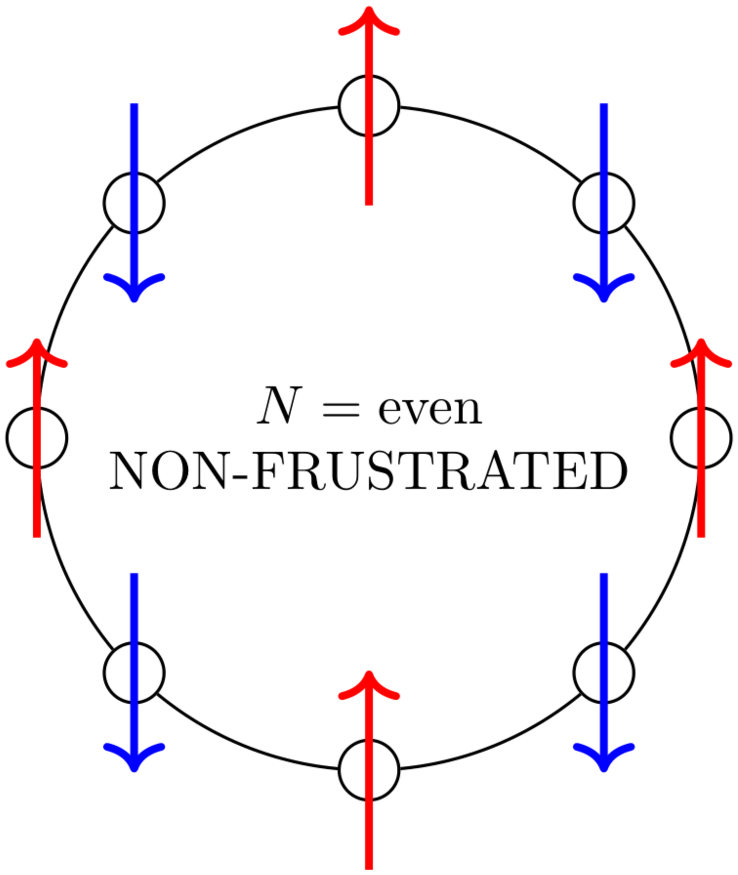

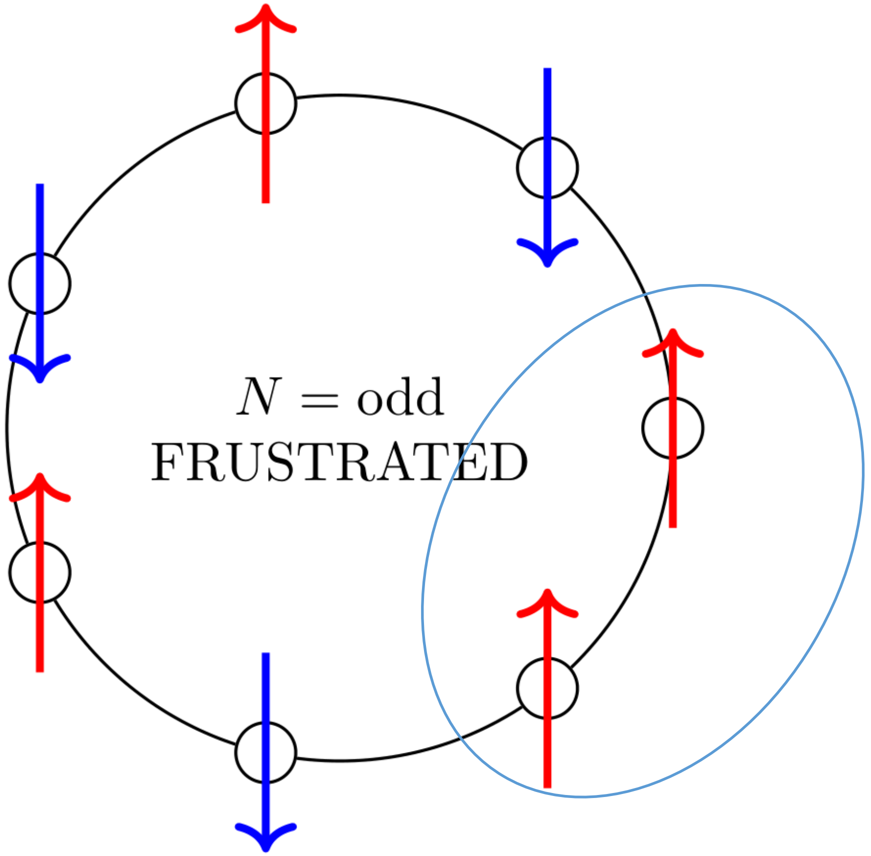

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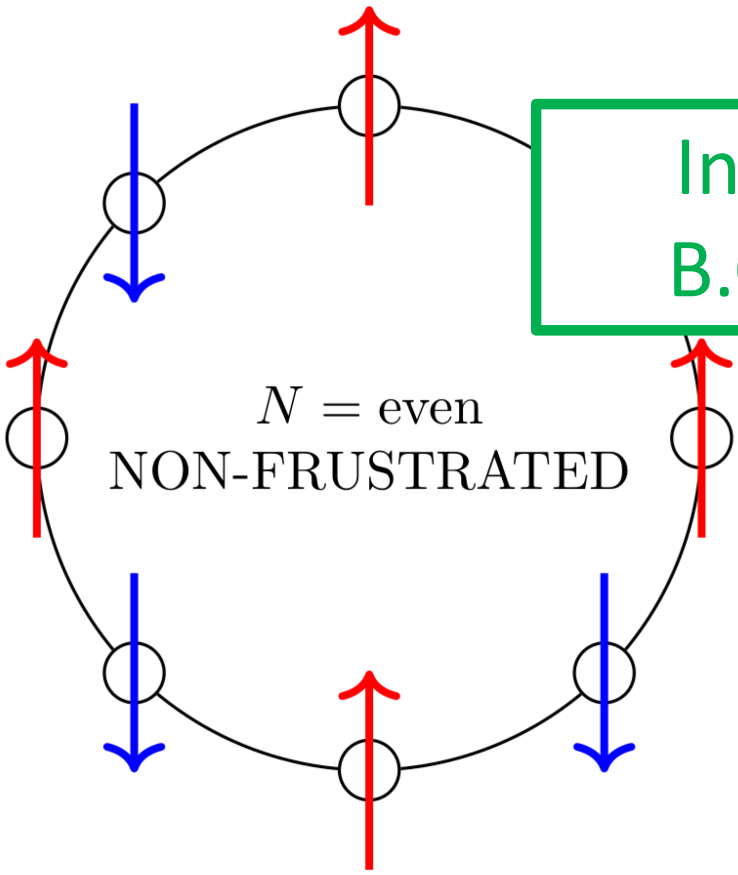
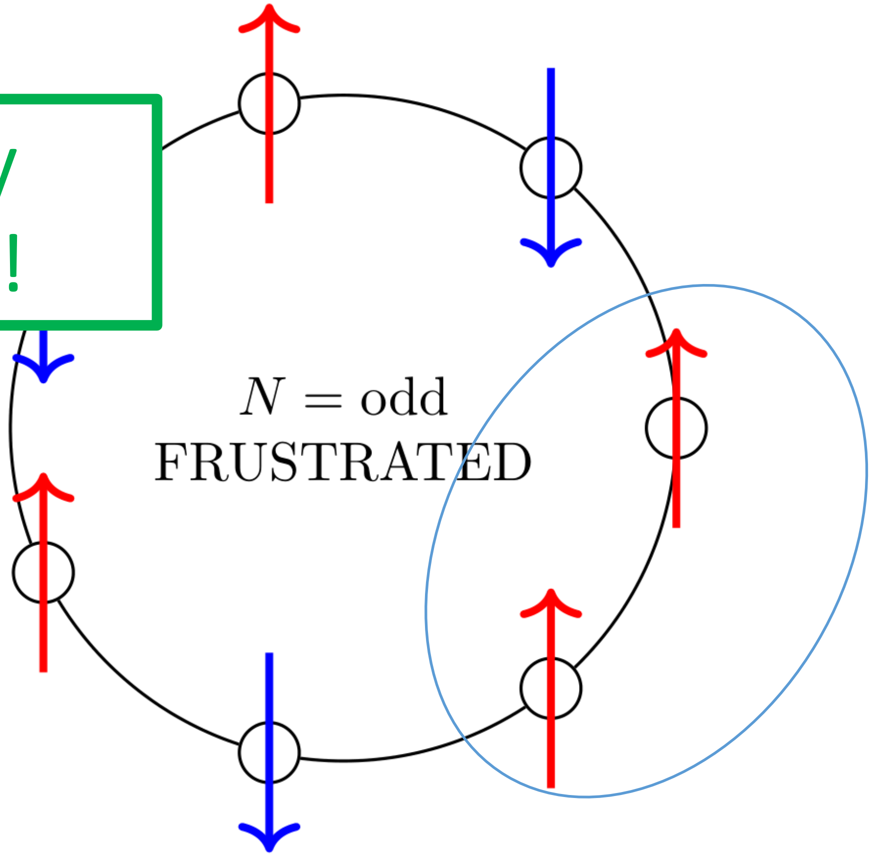


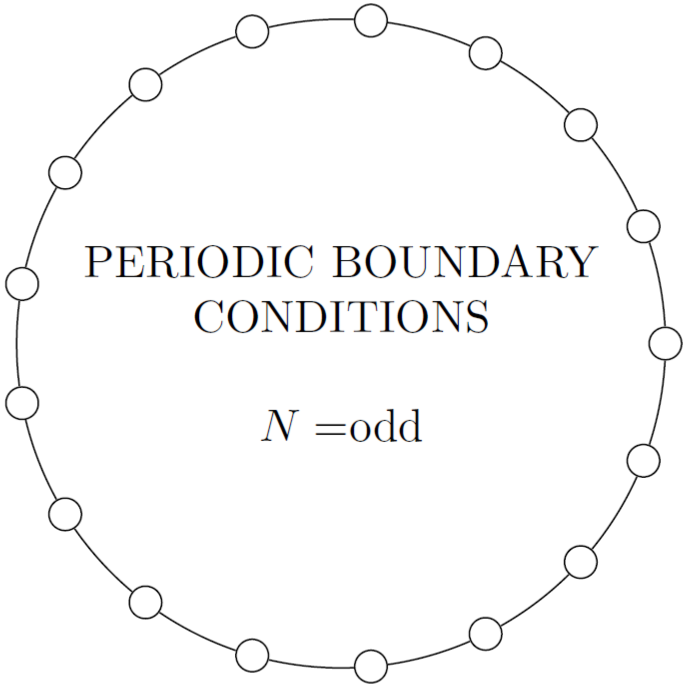
In Landau theory
B.C. not relevant!



Frustration + Quantum Mechanics

Quantum XY chain – xAFM, yFM

$$H = \sum_{j=1}^{N} \sigma_j^x \sigma_{j+1}^x - \lambda \sum_{j=1}^{N} \sigma_j^y \sigma_{j+1}^y \ , \ \lambda \in (0,1)$$



Order Parameter – (staggered) magnetization

$$\langle \sigma_j^x \rangle_{\mathrm{GS}} = ?$$

$$Z_2 \text{ symmetry: } \sigma_j^x \to -\sigma_j^x$$

- Combination of analytical and numerical methods

How to diagonalize the model? $H = \sum_{j=1}^{N} \sigma_j^x \sigma_{j+1}^x - \lambda \sum_{j=1}^{N} \sigma_j^y \sigma_{j+1}^y \ , \ \lambda \in (0,1)$

Mapping spins to fermions – Jordan-Wigner Transformation, Fourier transform, Bogoliubov rotation

Z_2 symmetry: $\sigma_j^x \to -\sigma_j^x$ $\quad [H, \Pi^z] = 0 \quad \Pi^z = \prod_{j=1}^{N} \sigma_j^z$

$$H = \tfrac{1+\Pi^z}{2} H^+ \tfrac{1+\Pi^z}{2} + \tfrac{1-\Pi_z}{2} H^- \tfrac{1-\Pi_z}{2}$$

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$$H^{\pm} = \sum_{q \in \Gamma^{\pm}} \varepsilon(q) \left(\hat{a}_q^{\dagger} \hat{a}_q - \tfrac{1}{2} \right)$$

$$\begin{aligned} \epsilon(q) &= \left| e^{i2q} - \lambda \right|, \ q \neq 0, \pi \ , \\ \epsilon(0) &= -\epsilon(\pi) = 1 - \lambda \ , \end{aligned}$$

$$\Gamma^- = \{2\pi k/N : k = 1..., N\}$$
$$\Gamma^+ = \{2\pi(k + \tfrac{1}{2})/N : k = 1, ..., N\}$$

Construction of the ground states...

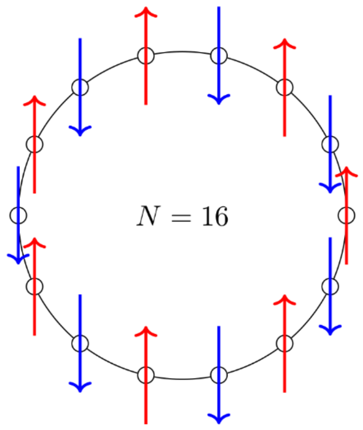
New method of computing the magnetization

Results

$$H = \sum_{j=1}^{N} \sigma_j^x \sigma_{j+1}^x - \lambda \sum_{j=1}^{N} \sigma_j^y \sigma_{j+1}^y \ , \ \lambda \in (0,1)$$

Without frustration (System size *N*=Even):

- Gapped
- Magnetization antiferromagnetic



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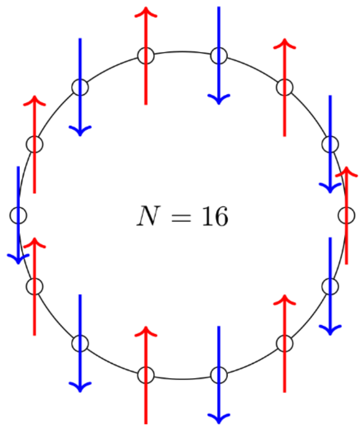

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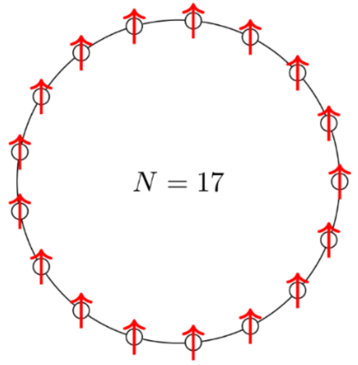

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Mesoscopic Ferromagnetic Order

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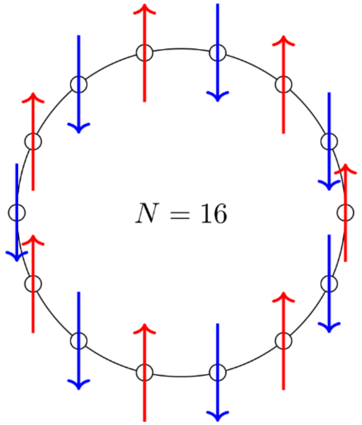

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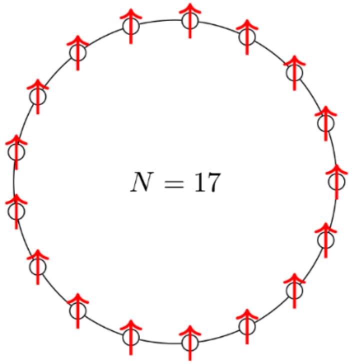

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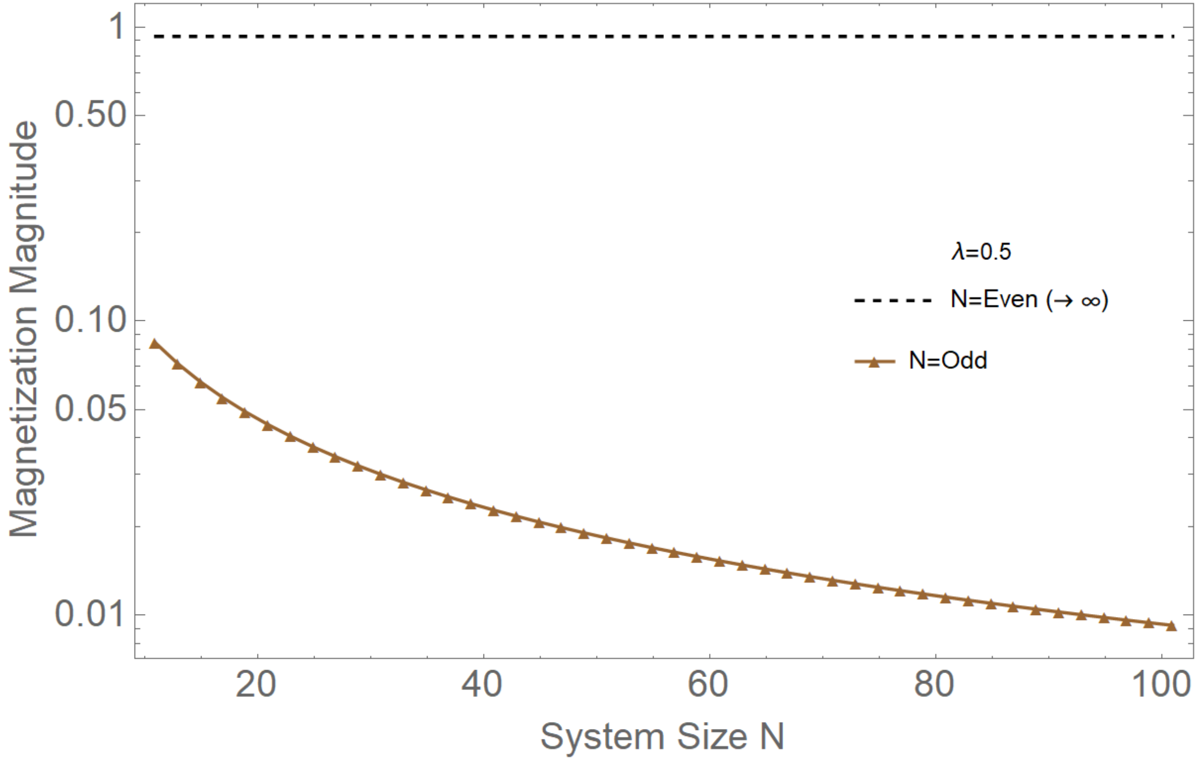

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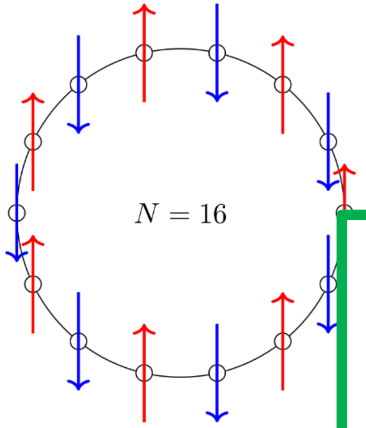

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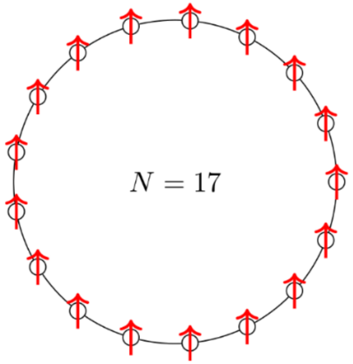

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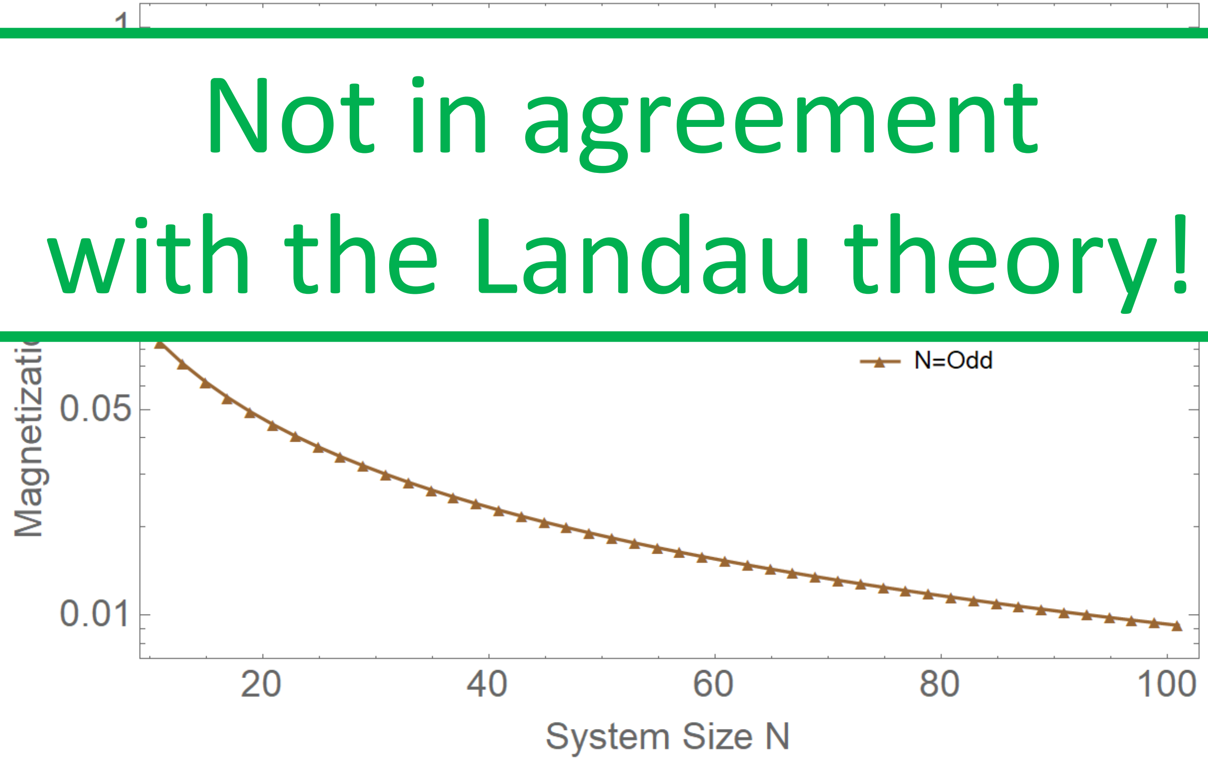
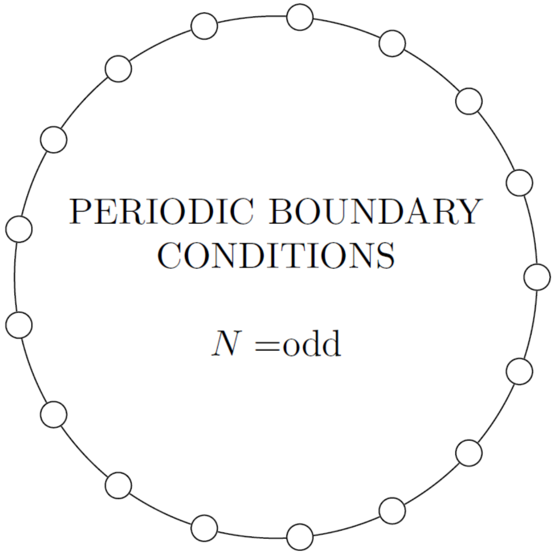


Not in agreement with the Landau theory!

Mesoscopic Ferromagnetic Order

XY chain – xAFM, yAFM



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Magnetization $\langle \sigma_j^x \rangle_{\mathrm{GS}} = ?$

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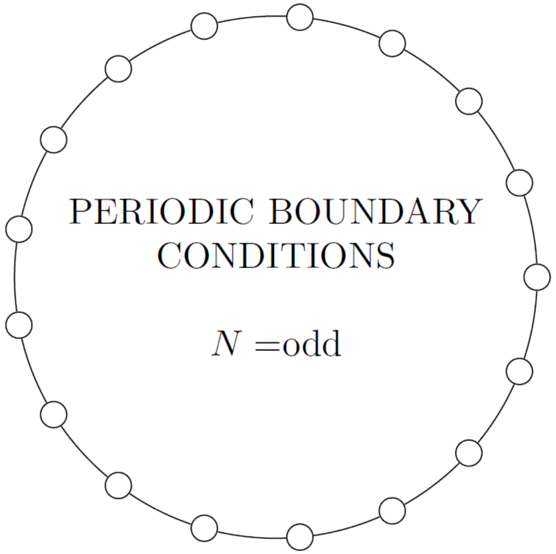

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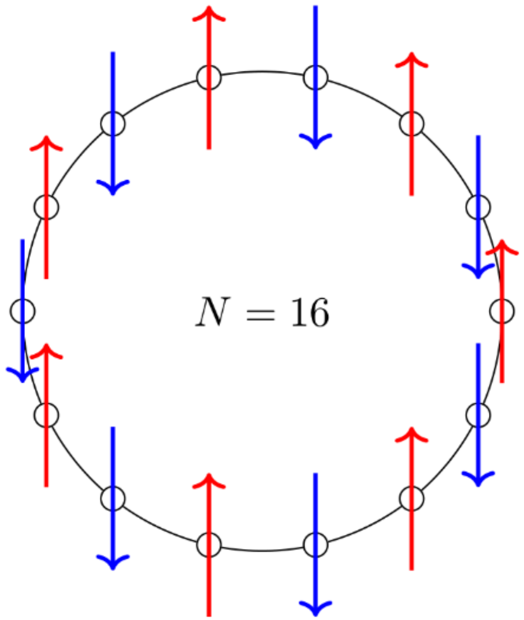
Without frustration:

- Gapped
- Standard Antiferromagnetic order
- Two-fold ground state degeneracy



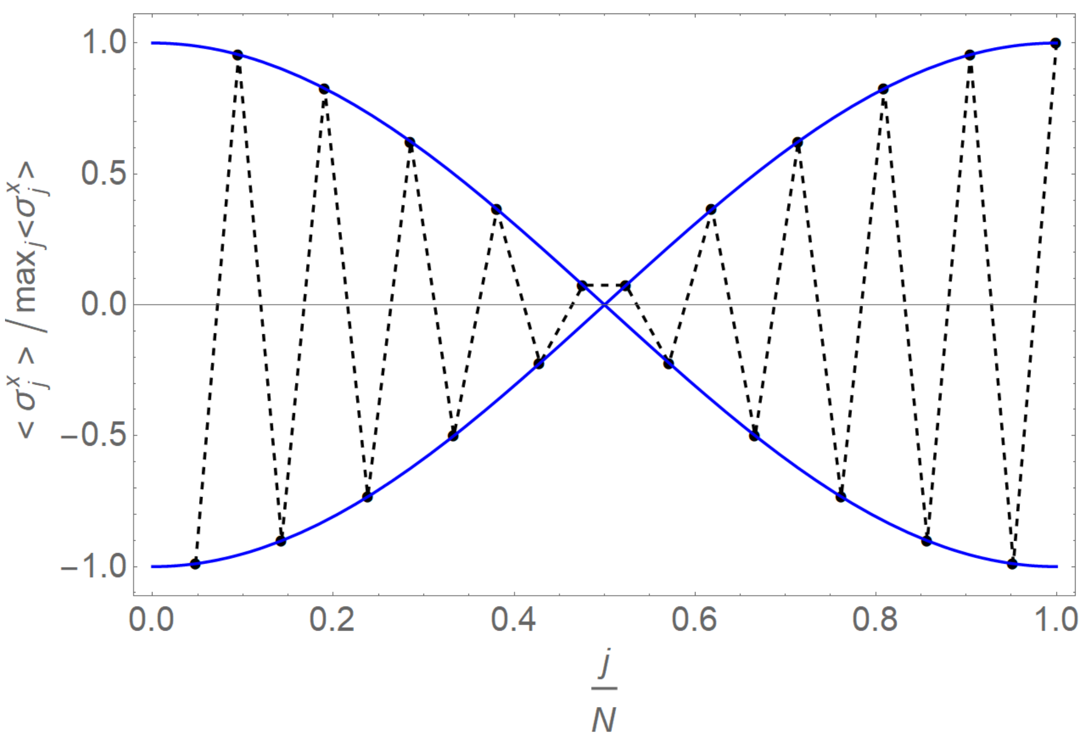
$\langle \sigma_j^x \rangle \simeq (-1)^j (1-\lambda^2)^{1/4}$

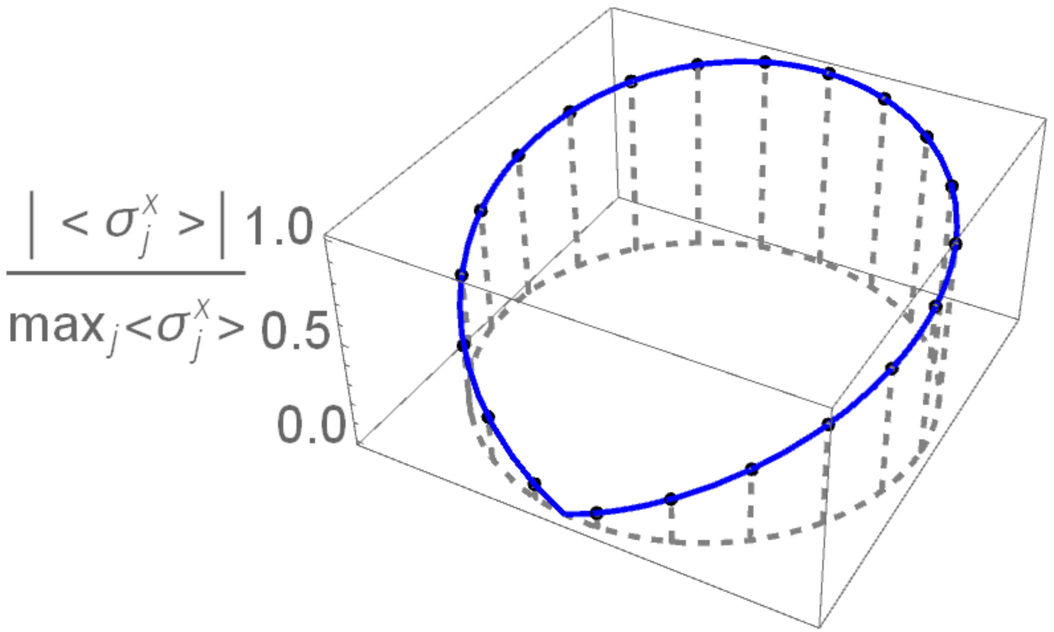
With Frustration: Breaking of Translational Symmetry

Gapless spectrum, Four-fold ground state degeneracy

$$\frac{\langle \sigma_j^x \rangle}{\max_{j'} \langle \sigma_{j'}^x \rangle} = (-1)^j \cos\left(\pi \frac{j}{N}\right)$$

Magnetization non-zero





Two results combined

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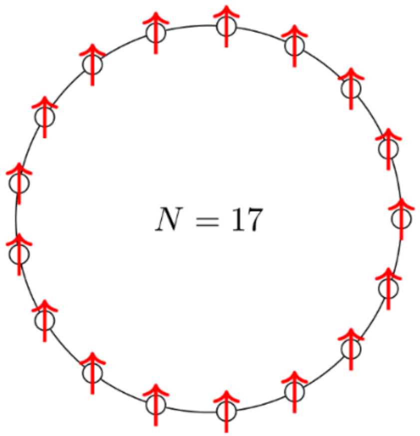
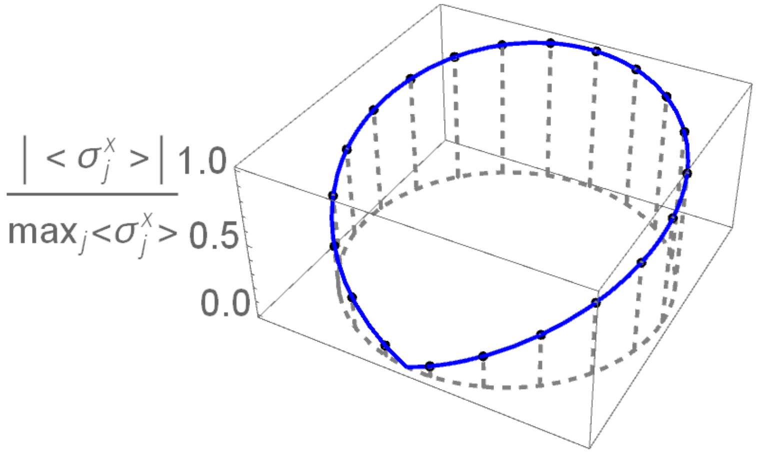


GS DEGENERACY=2



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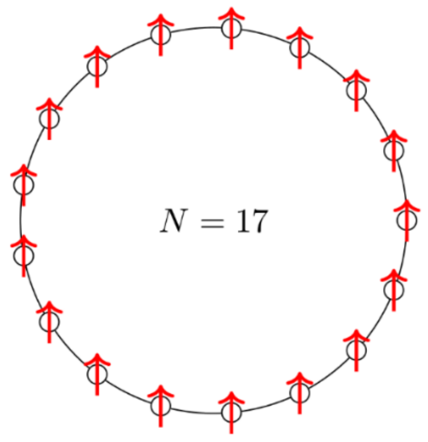
NEW QUANTUM PHASE TRANSITION!

$$\lambda = 0$$



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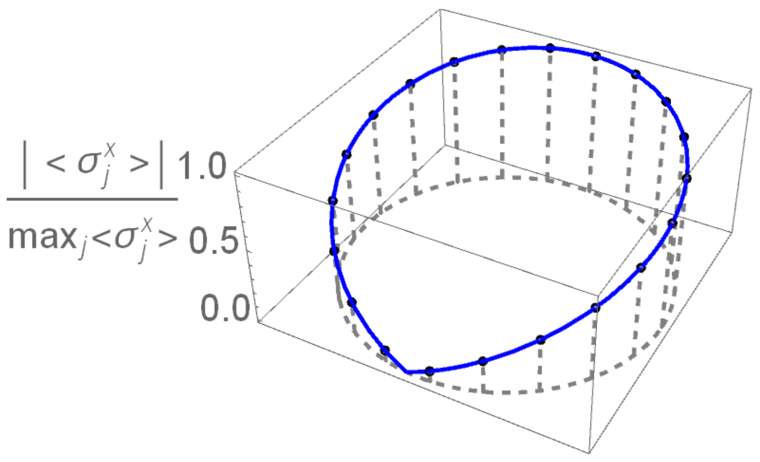

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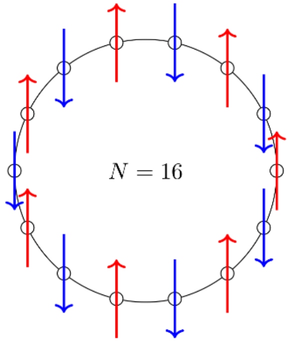
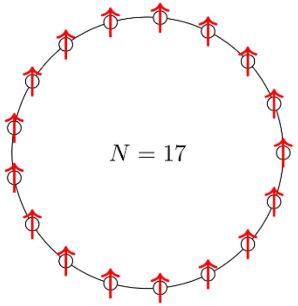
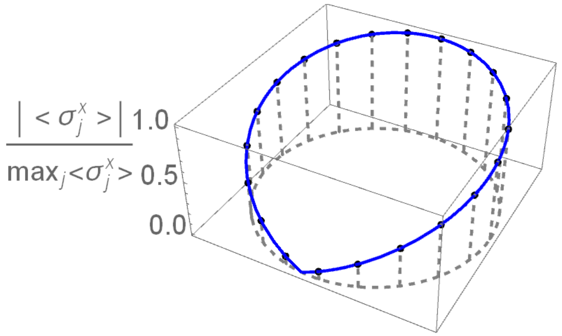
Conclusions







- Frustration with Quantum Mechanics leads to new types of order
  - Mesoscopic Ferromagnetic order, Breaking of translational symmetry, new Quantum Phase Transition
- Different behavior of systems of even and odd size, however large they are.
- Boundary conditions influence local order, however large system is.
- Incompleteness of the Landau theory.

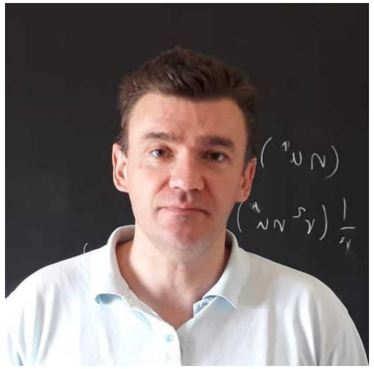
Reference: V. Marić, S. M. Giampaolo, D. Kuić, and F. Franchini. “The Frustration of being Odd: How Boundary Conditions can destroy Local Order”. arXiv:1908.10876, 2019.

Reference: V. Marić, S. M. Giampaolo, D. Kuić, and F. Franchini. In preparation.

People involved in the project

Fabio Franchini

Salvatore Marco Giampaolo



Domagoj Kuić

IRB ThPhys @IRB

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- Thank you for your attention!