Branes, Wrapping Rules and Mixed-symmetry Potentials

Eric Bergshoeff

Groningen University

based on work with Fabio Riccioni

Recent Advances in T/U-dualities and Generalized Geometries

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The Problem

supergravity cannot accommodate T-duality !

The 'good' sector

- there is no problem with the supergravity fields that describe physical degrees of freedom. In each dimension these fields are related to each other via dimensional reduction

- for instance, maximal supergravity describes $128 + 128$ degrees of freedom in each dimension $D \leq 11$

The 'bad' sector

- D-dimensional maximal supergravity also contains high-rank form potentials that do not describe physical degrees of freedom and that are not controlled by the representation theory of the supersymmetry algebra. The problem is that they are not related to each other via dimensional reduction

- Key Example: (D-1)-form potentials are 'dual' to an integration constant. These high-rank potentials couple to domain walls.

Outline

Branes and p-form Potentials

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Wrapping Rules

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Branes and p-form Potentials

Wrapping Rules

‘Exotic Branes’ and Mixed-symmetry Potentials

Summary and Open Issues

Branes

Branes are extended objects with a number of worldvolume and transverse directions. They are an essential part of (non-perturbative) string theory

- The NS-NS 2-form B_2 suggests a half-supersymmetric string
- The 3-form C_3 of 11D sugra couples to a half-susy M2-brane

sugra potential $\leftrightarrow$ *half-supersymmetric brane*

Does it always work as simple as that?

Strings and T-duality

The T-duality group in D dimensions is SO(d,d; $\mathbb{Z}$) with $d = 10 - D$

The D-dimensional string couples to the NS-NS 2-form B_2 as well as 1-forms $B_{1,A}$ $(A = 1, \ldots, 2d)$ that transform as a vector under T-duality

To construct a gauge-invariant WZ term

$$\mathcal{L}_{\mathrm{WZ}}(D < 10) = B_2 + \eta^{AB} \mathcal{F}_{1,A} B_{1,B}$$

we need to introduce 'extra scalars' $b_{0,A}$ via $\mathcal{F}_{1,A} = d\, b_{0,A} + B_{1,A}$

Counting the Bosonic Worldvolume D.O.F.

$$D = 10 \quad : \qquad (10-2) = 8\,,$$

$$D < 10 \quad : \qquad (D-2) + 2(10-D) \neq 8\,!$$

Twice too many 'extra scalars' $b_{0,A}$ $\rightarrow$ 'doubled geometry'

Hull, Reid-Edwards (2006-2008)

Self-duality conditions on the extra scalars $b_{0,A}$ give correct counting

'Wess-Zumino term requirement'

the construction of a gauge-invariant WZ term may require, besides the embedding coordinates, the introduction of a number of extra worldvolume p-form potentials

worldvolume supersymmetry requires that these worldvolume fields fit into a multiplet with 16 supercharges

Does the 'WZ term requirement' always lead to the rule that

$$\text{potential} \;\Leftrightarrow\; \text{half-susy brane}\,?$$

Input from Supergravity

- The T-duality representations of all high-rank form potentials have been determined using three different techniques:

  - closure of the supersymmetry algebra

    de Roo, Hartong, Howe, Kerstan, Ortín, Riccioni + E.B. (2005-2010)

  - using the embedding tensor technique

    for a review, see de Wit, Nicolai, Samtleben (2008)

  - using the very extended Kac-Moody algebra E_{11}

    West (2001); Riccioni, West (2007); Nutma + E.B. (2007)

Question

given a $(p+1)$-form potential which (components of its) T-duality repres. couple to a half-supersymmetric brane ?

A scaling symmetry

All potentials transform as a representation of the T-duality group O(d,d) and scale under a scaling symmetry

The scaling weight α determines the dependence of the brane tension T on the string coupling constant g_s via

$$\mathrm{T} \sim (g_s)^{\alpha}$$

This scaling weight is invariant under dimensional reduction

A universal pattern arises

α	potentials	branes
$\alpha = 0$	$B_{1,A}\,,\,B_2$	fundamental
$\alpha = -1$	$C_{2n+1,a}\,,\,C_{2n,\dot{a}}$	Dirichlet
$\alpha = -2$	$D_{D-4},\,D_{D-3,A},\,D_{D-2,A_1A_2},\,D_{D-1,A_1\cdots A_3},\,D_{D,A_1\cdots A_4}$	solitonic
$\vdots$	$\vdots$	$\vdots$

A $(a, \dot{a})$ are vector (spinor)-indices of T-duality

$\alpha = -3$: S-dual of D7-brane

$\alpha = -4$: S-dual of D9-brane

Branes with $\alpha < -4$ have no ten-dimensional brane origin!

Outcome Wess-Zumino Term Requirement

Riccioni + E.B. (2010)

There is a simple group-theoretical characterization of which (components of the) T-duality representation couple to a half-supersymmetric brane

- the (group-theoretical) details can be found in our papers
- Comparing branes in different dimensions an interesting patterns arises ...

Outline

'Wrapping Rules'

the wrapping rules of 'standard geometry'

$$\text{any brane} \quad \begin{cases} \text{wrapped} & \rightarrow \quad \text{undoubled} \\ \text{unwrapped} & \rightarrow \quad \text{undoubled} \end{cases}$$

only works for D-branes !

Counting D-branes

Dp-brane	IIA/IIB	9	8	7	6	5	4	3
0	1/0	1	2	4	8	16	32	64
1	0/1	1	2	4	8	**16**	**32**	64
2	1/0	1	2	4	8	**16**	32	64
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
8	1/0	1						
9	0/1							

spinors $(\mathrm{Dp})_\alpha\,,\qquad \alpha = 1 \cdots 2^{9-D}$

Fundamental Branes

the wrapping rules of fundamental branes are given by

$$\mathrm{T}_{\mathsf{F}} \sim 1 \ : \quad \begin{cases} \text{wrapped} & \rightarrow \quad \text{doubled} \\ \text{unwrapped} & \rightarrow \quad \text{undoubled} \end{cases}$$

the extra input comes from pp-waves

Two points of view:

'new objects' (pp-waves) or 'doubled geometry'

Counting Fundamental Branes

Fp-brane	IIA/IIB	9	8	7	6	5	4	3
0		2	4	6	8	**10**	**12**	14
1	1/1	1	1	1	1	**1**	1	1

$(\mathrm{F0})_A$ and F1

$A = 1, \ldots, 2(10 - D)$

Solitonic Branes with $T \geq 3$

the wrapping rules of solitonic branes are given by

$$\mathrm{T_S} \sim (g_s)^{-2} \; : \quad \begin{cases} \text{wrapped} & \rightarrow \quad \text{undoubled} \\ \text{unwrapped} & \rightarrow \quad \text{doubled} \end{cases}$$

For instance, in 9D we have two solitonic 5-branes coming from an un-wrapped NS5-brane and a KK monopole

$$\text{10D KK monopole}: \quad \begin{cases} 5+1 \text{ worldvolume directions} \\ 1 \text{ isometry direction} \\ 3 \text{ transverse directions} \end{cases}$$

Counting Solitonic Branes with $T \geq 3$

Sp-brane	IIA/IIB	9	8	7	6	5	4	3
0						**1**	**12**	
1					1	**10**		
2				1	8			
3			1	6				
4		1	4					
5	1/1	2						

S(D-5)-brane and S(D-4)-brane$_A$

Solitonic Branes with $T \leq 2$

Sp-brane	IIA/IIB	9	8	7	6	5	4	3
0						1	12	84
1					1	10	60	280
2				1	8	40	160	560
3			1	6	24	80	240	
4		1	4	12	32	80		
5	1/1	2	4	8	16			

The red numbers follow from imposing the Wess-Zumino term requirement

A Numerical Coincidence?

Sp-brane	IIA/IIB	9	8	7	6	5	4	3
0						1	12	84
1					1	10	60	280
2				1	8	40	**160**	**560**
3			1	6	24	80	**240**	
4		1	4	12	32	80		
5	1/1	2	4	8	16			

Precisely the same numbers are reproduced by the solitonic wrapping rule !

Question

what is the 10D origin of the solitonic branes with $T \leq 2$?

Note: extra input is needed to fill up the T-duality representations!

standard supergravity is not sufficient!

Outline

T-duality

- At the level of (linearized) supergravity T-duality can be recovered by assuming that these theories can be extended with a set of mixed-symmetry potentials with an underlying E_{11}-symmetry

- To recover T-duality at the level of branes we assume that these mixed symmetry potentials are in one to one correspondence with extended objects called 'exotic branes'. They have worldvolume, transverse and special isometry directions

see, e.g., Obers, Pioline (1999); Lozano-Tellechea, Ortín (2001)

see also work by de Boer and Shigemori (2010, 2012) → 'T-folds'

A 7D Example

$\alpha = -2$	$D_3,\ D_{4,A},\ D_{5,[AB]},\ D_{6,[ABC]},\ D_{7,[ABCD]}$	$D_{6+n,n}\ (n = 0, 1, 2, 3)$

The 7D solitonic domain wall 6-forms $D_{6,[ABC]}$ $(A = 1, \ldots, 6)$ transform as 20 under $SO(3,3)$. These 6-forms are dual to (constant) fluxes

10D origin	mixed-symmetry	flux (a=1,2,3)
NS5 (5_2)	D_6	H_{abc} (1)
KK5 (5_2^1)	$D_{7,1}$	${f^a}_{bc}$ (9)
5_2^2	$D_{8,2}$	${Q^{ab}}_c$ (9)
5_2^3	$D_{9,3}$	R^{abc} (1)

see also Haßler, Lüst (2013); Kimura, Sasaki (2013); Chatzistavrakidis, Gautason, Moutsopoulos, Zagermann (2013)

Extending the Buscher Rules

Lombardo, Riccioni, Risoli (2016)

$$\alpha = -2 : \qquad 0 \overset{T_x}{\longleftrightarrow} x, x \qquad\qquad x \overset{T_x}{\longleftrightarrow} x$$

$$D_6 \leftrightarrow D_{6x,x}\,, \qquad\qquad D_{5x} \rightarrow D_{5x}$$

compactification 10 to 9: $D_6 \rightarrow D_6$ plus D_{5x}

T-duality in x: $D_6 \rightarrow D_{6x,x}$: doubled and $D_{5x} \rightarrow D_{5x}$: undoubled

Universal T-duality Rules

Lombardo, Riccioni, Risoli (2016)

$$\alpha = -n \;:\; \underbrace{x, x, ..., x}_{p} \xleftrightarrow{\mathrm{T}_x} \underbrace{x, x, ..., x}_{n-p} \qquad p = 0, 1, \cdots, [n/2]$$

$$n = 2 : \qquad 0 \xleftrightarrow{\mathrm{T}_x} x, x \qquad x \xleftrightarrow{\mathrm{T}_x} x \qquad p = 0, 1$$

	potential	IIA	IIB
$\alpha = -3$	$E_{D-2,\dot{a}}$ $E_{D-1,A\dot{a}}$ $E_{D,A_1A_2\dot{a}}$	$E_{8+n,2m+1,n}$	$E_{8+n,2m,n}$

$$n = 3 : \qquad 0 \xleftrightarrow{\mathrm{T}_x} x, x, x \qquad x \xleftrightarrow{\mathrm{T}_x} x, x \qquad p = 0, 1$$

S-dual of D7-brane satisfies double-double wrapping rule

What about Branes without a 10D Brane Origin?

Riccioni + E.B. , in preparation

	potential	IIA	IIB
$\alpha = -4$	$F_{D-1,A_1\ldots A_{d-3}}$ $\quad F_{D,A,B_1\ldots B_{d-3}}$	$F_{9+n,3+m,m,n}$	

The $F_{9,3}$ family of branes satisfies the double-double wrapping rule times multiplicity $\binom{d}{3}$ where d is the number of compact directions

$D \backslash p$	0	1	2	3	4	5	F	$\binom{d}{3}$
7						1	$F_{6xyz,xyz}$	1
6					2	2	$F_{6xyz,xyz}$, $F_{5xyzw,xyz}$	4
5				4	8		–	10
4			8	24			–	20

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	potential	IIA	IIB
$\alpha = -4$	$F_{D-1,A_1 \dots A_{d-3}} \quad F_{D,A,B_1 \dots B_{d-3}}$	$F_{9+n,3+m,m,n}$	

The $F_{9,3}$ family of branes satisfies the double-double wrapping rule times multiplicity $\binom{d}{3}$ where d is the number of compact directions.

$D \backslash p$	0	1	2	3	4	5
7						1
6					8	8
5				40	80	
4			160	480		

Outline

Summary

- In this talk I reviewed the classification of the potentials and branes of maximal supergravity and showed how this suggests the introduction of mixed-symmetry potentials and exotic branes

- The whole brane classification can be re-constructed by simple T-duality and wrapping rules

Compare with DFT

Where does B_6 fits into DFT ?

In SUGRA one can dualize B_2 into B_6 without dualizing the metric tensor $g_{\mu\nu}$ but in DFT B_2 is part of the generalized metric $\mathcal{H}_{MN}$!

Exotic Dualization

Boulanger, Sundell, West (2015)

$$S[b] = -\frac{1}{12}\int d^D x\, H^{abc} H_{abc} = -\frac{1}{4}\int d^D x\, \left(\partial^a b^{bc}\, \partial_a b_{bc} - 2\, \partial_a b^{ab}\, \partial^c b_{cb}\right)$$

$$S[Q, D] = \int d^D x\big(-\tfrac{1}{4}\, Q^{a|bc} Q_{a|bc} + \tfrac{1}{2}\, Q_{a|}{}^{ab} Q^{c|}{}_{cb} - \tfrac{1}{2}\, D^{ab|cd}\, \partial_a Q_{b|cd}\big)$$

$$\partial_{[a} Q_{b]|cd} = 0 \quad \Rightarrow \quad Q_{a|bc} = \partial_a b_{bc}$$

We now have a mixed-symmetry potential $D^{ab|cd} \overset{D=10}{\sim} D_{8,2}$!

Linearized DFT

Use formulation with generalized fluxes $\mathcal{F}_{ABC}$

Aldazabal, Baron, Marques, Nunez (2011); Geissbuhler (2011)

Grana, Marques (2011); Geissbulher, Marques, Nunez, Penas (2013)

Duality leads to 4-form potential D^{ABCD}

Hohm, Penas, Riccioni + E.B. (2016)

$$
\begin{aligned}
D^{\mu_1\cdots\mu_4} &\quad\rightarrow\quad B_6 \\
D^{\mu_1\cdots\mu_3}{}_{\mu_4} &\quad\rightarrow\quad h_{7,1} \\
D^{\mu_1\mu_2}{}_{\mu_3\mu_4} &\quad\rightarrow\quad D_{8,2} \\
D^{\mu_1}{}_{\mu_2\cdots\mu_4} &\quad\rightarrow\quad D_{9,3} \\
D_{\mu_1\cdots\mu_4} &\quad\rightarrow\quad D_{10,4}
\end{aligned}
$$

Can we define brane effective actions in DFT?

Cp. to talk by David Berman

Chatzistavrakidis, Gautason, Moutsopoulos, Zagermann (2014); Ortín, Riccioni + E.B., work in progress

Take Home Message

Can we understand the role of mixed-symmetry potentials better ?

See, e.g., Bunster, Henneaux (2013)

Thanks for your Attention !