T-fects: Near-core corrections and description in Doubled spaces

Valentí Vall Camell

Ludwig Maximilians Universität, Max Planck Institut für Physik
München

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with D. Lüst, S. Massai and E. Plauschinn

Recent advances in T/U-dualities and Generalized Geometries.
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Motivation and Summary

- Some CY_2 manifolds can be locally described as nontrivial T^2 meromorphic fibrations.
- Non-trivial fibrations on a compact base $\Rightarrow$ degeneration points.
- These manifolds admit non-geometric modifications (monodromies in the T-duality group). [Font,Garcia-Etxebarria,Lüst,Massai,Mayrhofer]
- T-fects:
  - Local descriptions of such degenerations.
  - (Flat-)T^2 fibration breaks down close to the degeneration.

Structure of the talk

1. T^2 fibrations and T-fects.
2. Special cases: NS5 $\rightarrow$ KK-monopole $\rightarrow$ Q-brane.
   Description, corrections and T-duality.
3. T-fects and corrections in Double spaces.

T^2 fibrations (semi-flat approximation)

Moduli space of T^2 with metric g and B-field B

$$\frac{O(2,2,\mathbb{R})}{O(2,\mathbb{R}) \times O(2,\mathbb{R})} / O(2,2,\mathbb{Z})$$

Parametrising the moduli space

The moduli space can be parametrised by 2 complex parameters:

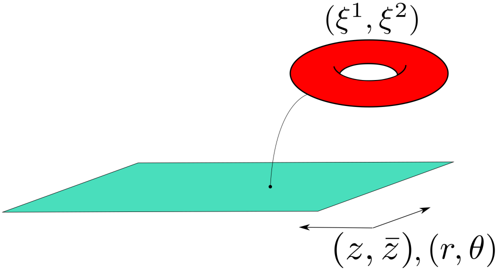
$$\text{Complex structure of the torus: } \tau = \frac{g_{12}}{g_{22}} + i\frac{\sqrt{\det g}}{g_{22}}$$

$$\text{Kähler structure of the torus: } \rho = B_{12} + i\sqrt{\det g}$$

In this language:

$$O(2,2) = SL(2)_\tau \times SL(2)_\rho \times \mathbb{Z}_2^{\tau\leftrightarrow\rho} \times \mathbb{Z}_2^{\tau\leftrightarrow-\bar{\rho}}$$

$SL(2,\mathbb{Z})$ acts on τ (and ρ) as Möbius transformations: $\tau' = \frac{a\tau+b}{c\tau+d}$.



T^2 fibrations

- We let the torus moduli vary along the base.
- Semi-flat ansatz: $[\tau = \tau(z, \bar{z}) = \tau_1 + i\tau_2 \mid \rho = \rho(z, \bar{z}) = \rho_1 + i\rho_2]$

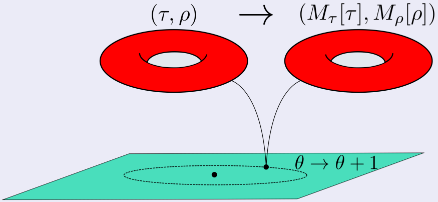
$$
\begin{aligned}
ds^2 &= e^{2\phi(z,\bar{z})}\rho_2\,\tau_2\,dz\,d\bar{z} + \frac{\rho_2}{\tau_2}|d\xi^2 + \tau\,d\xi^1|^2 \\
B &= \rho_1 d\xi^1 \wedge d\xi^2
\end{aligned}
$$

[Hellerman,McGreevy,Williams]

- If τ anf ρ are meromorphic functions of the base and $\nabla^2\phi = 0$
  $\Rightarrow$ Configurations are 1/2- (or 1/4-) BPS (solutions of SUGRA e.o.m.).

T-fects

Monodromies



Ansatz:

$$\tau(r,\theta) = e^{\theta\,\mathbf{m}_\tau}[\tau_0(r)]$$

with $e^{\mathbf{m}_\tau} = M_\tau \in SL(2,\mathbb{Z})_\tau$.

(same for ρ)

The function $\tau_0(r)$ is fixed (up to constants) by Cauchy-Riemann equations.

- Embedding the solution into 10D $\Rightarrow$ Transversal spaces of five-branes.
- Constructed for any monodromy in $SL(2,\mathbb{Z})_\tau \times SL(2,\mathbb{Z})_\rho$.
  Classified in terms of $SL(2,\mathbb{Z})$ conjugacy classes.
- $M_\tau \Leftrightarrow$ geometric (τ-fects). $M_\rho \Leftrightarrow$ non-geometric (ρ-fects).
- They have logarithmic divergences.
  To construct global models several T-fects are needed.
- Semi-flat approximation breaks down close to the degeneration.
  In some cases we know how to add corrections by hand.

Case I: The I_1 degeneration.

Monodromy: $\tau \to \tau + 1$

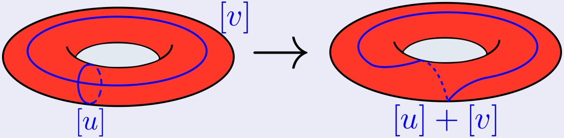
$\tau(z) = i \log(\mu/z)$

$$ds^2 = h(r)(dr^2 + r^2 d\theta + (d\xi^1)^2) + \frac{1}{h(r)}(d\xi^1 + \theta d\xi^2)^2$$

with $h(r) = \log(\mu/r)$, $\mu =$ constant.

Dehn twist

Monodromy for $\theta \to \theta + 1$:



$[u], [v]$ basis for $\pi_1(T^2)$

$$\begin{aligned} [u] &\to [u] \\ [v] &\to [v] + [u] \end{aligned}$$

Coordinate transformations:

$$\xi^1 \to \xi^1, \qquad \xi^2 \to \xi^2 - \xi^1$$

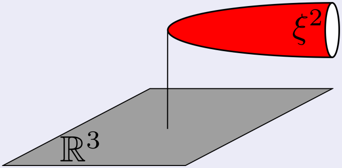
- Logarithmic behaviour.

The euclidian Taub-NUT space (KK-monopole)

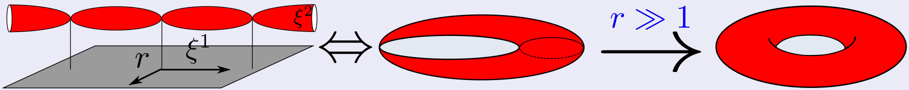
$$ds^2 = h(|\vec{x}|)|d\vec{x}|^2 + \frac{1}{h(|\vec{x}|)}(d\xi^2 + \omega)^2$$

with $|\vec{x}| \in \mathbb{R}^3$ and $\xi^2 \sim \xi^2 + 2\pi$.

- $h(|\vec{x}|) = 1 + \frac{1}{2|\vec{x}|}$.
- $d\omega = \star_3 dh$.



The KK-monopole on S^1 and the smearing limit.



$$h(r,\xi^1) = 1 + \sum_{n\in\mathbb{Z}} \frac{1}{2\sqrt{r^2 + (\xi^1 + 2\pi n)^2}} = \underbrace{\log(\mu/r)}_{\text{semiflat term}} + \sum_{k\neq 0} \underbrace{K_0(|k|r)}_{\sim e^{-|k|r}\to 0,\ r\gg 1} \quad e^{i\,k\,\xi^1}$$

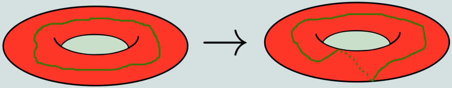
Strings trajectories in the semiflat limit.

Momentum(n^i)/winding(m^i) change under $\theta \to \theta + 1$:

$$\begin{aligned} (n^1, n^2) &\to (n^1 + n^2, n^2) \\ (m^1, m^2) &\to (m^1, m^2 - m^1) \end{aligned}$$

m^1 and n^2 **not conserved.**

Example: unwinding trajectory



- Isometry along ξ^1 is broken by the corrections to the semiflat limit.

Zero modes

- Shift along ξ^1. Corrections localise the shrinking cycle around ξ^1_0.
- **Dyonic collective coordinate** α: $B = \alpha d\Lambda$, $d\Lambda$ self-dual 2-form

$$\Lambda = \frac{1}{h(r)}(d\xi^2 + \theta d\xi^1) \qquad \text{[Sen]}$$

Case II: The (smeared) NS5 brane

Monodromy: $\rho \to \rho + 1$

$$\rho(z) = i \log(\mu/z)$$

- No twistings.
- Constant volume fiber.
- Non-trivial B-field, glued with a gauge transformation: $B \to B + \text{constant}$.

NS5 brane solution

$$\begin{aligned} ds^2 &= h(|x^i|) dx^i dx_i \,, \quad x^i \in \mathbb{R}^4 \\ e^{2\phi} &= h(|x^i|),\ H = \star_4 dh(|x^i|), \end{aligned}$$

$$h(|x^i|) = 1 + \frac{1}{|x^i|^2}$$

Compactifying on a T^2 and smearing

$$\begin{aligned} h(r, \xi^1 \xi^2) &= 1 + \sum_{\vec{n} \in \mathbb{Z}^2} \frac{1}{r^2 + (\xi^1 + 2\pi n_1)^2 + (\xi^2 + 2\pi n_2)^2} \\ &= \underbrace{\log(\mu/r)}_{\text{semiflat term}} + \sum_{\vec{k} \in (\mathbb{Z}^2)^*} \underbrace{K_0(|k| r)}_{\sim e^{-|\vec{k}| r} \to 0,\ r \gg 1} \quad e^{i(k_1 \xi^1 + k_2 \xi^2)} \end{aligned}$$

Strings trajectories in the semiflat limit.

Momentum(n^i)/winding(m^i) change under $\theta \to \theta + 1$:

$$\begin{aligned} (n^1, n^2) &\to (n^1 + m^2, n^2 - m^1) \\ (m^1, m^2) &\to (m^1, m^2) \end{aligned}$$

n^i's are not conserved.

Zero modes

- Shift along ξ^1
- Shift along ξ^2

- Isometries along (ξ^1, ξ^2) are broken by the corrections to the semiflat limit.

T-duality in the semi-flat region.

The presence of isometries in the semi-flat region allows dualisation.

(Smeared) NS5 $\leftrightarrow$ I_1 degeneration (KK-monopole):

- $\rho(z) \leftrightarrow \tau(z)$. Or equivalently by gauging ξ^2-isometry (Buscher rules).
- $m^2 \leftrightarrow n^2$.
- Non-vanishing α in the KK-monopole is mapped to a shift along ξ^2.

T-duality of massive modes

Corrections to the smearing limit in the NS5 brane

$$\mathcal{C}_{k_1,k_2} \sim e^{-\lambda r} e^{i\left(\frac{k_1\xi^1}{R_1}+\frac{k_2\xi^2}{R_2}\right)}, \qquad \text{with } \lambda^2 = \left(\frac{k_1}{R_1}\right)^2 + \left(\frac{k_2}{R_2}\right)^2$$

$R_i \to$ Compactification radii. If $R_i \gg R_j$ (Direction j is smeared):

$$\mathcal{C}_{k_1,k_2} \sim \mathcal{C}_{k_i} = \mathcal{C}_{k_i,k_j=0} \sim e^{-k_i r} e^{i\frac{k_i\xi^i}{R_i}}$$

T-duality NS5→KK-monopole

- $\mathcal{C}_{k_1} \to \mathcal{C}_{k_1}$. Modes localising the shrinking cycle.
- $\mathcal{C}_{k_2} \to \tilde{\mathcal{C}}_{k_2} \sim e^{-k_2 r} e^{i\frac{k_2\tilde{\xi}_2}{R_2}}$. $\quad (\tilde{\xi} = \xi_L - \xi_R)$ [Gregory, Harvey, Moore]
  Checked (for big r) by world-sheet instantons techniques. [Harvey, Jensen]
- General case: $\mathcal{C}_{k_1,k_2} \to \mathcal{C}_{k_1,\tilde{k}_2} \sim e^{-\lambda r} e^{i\left(\frac{k_1\xi^1}{R_1}+\frac{k_2\tilde{\xi}_2}{R_2}\right)}$

- $\tilde{\xi}_2$ dual (winding) coordinate $\leftrightarrow$ Dyonic coordinate α.

Case III: T-fold

Apply to the NS5 a fiberwise $SL(2, \mathbb{Z})$ rotation: $\rho(z) \to -1/\rho(z)$

Monodromy:
$-\frac{1}{\rho} \to -\frac{1}{\rho} + 1$

$$\rho(z) = \frac{i}{\log(\mu/z)}$$

The Q-brane background(5^2_2 brane)

[de Boer,Sigemori; Haßler,Lüst]

$$\begin{aligned} ds^2 &= h(r)(dr^2 + r^2 d\theta) + \frac{h(r)}{h(r)^2 + \theta^2}\left[(d\xi^1)^2 + (d\xi^2)^2\right] \\ B &= -\frac{\theta}{h(r)^2 + \theta^2} d\xi^1 \wedge d\xi^2, \qquad \text{with } h(r) = \log(\mu/r) \end{aligned}$$

- Volume of the fiber: $V_{T^2}(\theta + 1) \neq V_{T^2}(\theta) \Rightarrow$ Non-geometric.

Strings trajectories in the semiflat limit.

Momentum(n^i)/winding(m^i):

$$\begin{aligned}(n^1, n^2) &\rightarrow (n^1, n^2)\\ (m^1, m^2) &\rightarrow (m^1 + n^2, m^2 - n^1)\end{aligned}$$

m^i**'s are not conserved**

Zero modes

Under T-duality:

<u>NS5</u>		<u>Q-brane</u>
Shifts	$\longrightarrow$	Gauge transformations

$\Rightarrow$ Two dyonic coordinates.

T-duality of massive modes

See that under the duality $\rho \leftrightarrow -1/\rho$: $\vec{m} \leftrightarrow \vec{n}$.

$$\mathcal{C}_{k_1,k_2} \rightarrow \tilde{\mathcal{C}}_{k_1,k_2} \sim e^{-\lambda r} e^{i\left(\frac{k_1\tilde{\xi}_1}{R_1} + \frac{k_2\tilde{\xi}_2}{R_2}\right)}$$

Consistent with the argumentations before.

- Dual (winding) coordinates $\leftrightarrow$ dyonic coordinates.

T-fects in Double spaces

Embedding $SL(2)$ monodromies into $SO(2,2)$

$\underline{A_\rho : \rho \to \rho + 1}$

$$\left(\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right),$$

$\underline{A_\tau : \tau \to \tau + 1}$

$$\left(\begin{array}{cc|cc} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{array}\right),$$

$\underline{B_\rho : -1/\rho \to -1/\rho + 1}$

$$\left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{array}\right)$$

Factorised dualities in $O(2,2)$

Element $e_i \Leftrightarrow$ Buscher procedure along direction i.

$\underline{e_2 : \tau \leftrightarrow \rho}$

$$\left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right),$$

$\underline{e_1 : \tau \leftrightarrow -1/\rho}$

$$\left(\begin{array}{cc|cc} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right),$$

$\underline{e = e_2 e_1 : \rho \leftrightarrow -1/\rho}$

$$\left(\begin{array}{cc|cc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right)$$

See that: $A_\tau = e_2^T A_\rho \, e_2, \quad B_\rho = e_1^T A_\tau \, e_1 = e^T A_\rho e \, \ldots$

- We describe T-fects as T^4 fibrations by noticing $SO(2,2) \subset SL(4)$. (Following the formalism used in [Hull,Reid-Edwards]).
- Extra toroidal coordinates, $(\tilde{\xi}_1, \tilde{\xi}_2) \Leftrightarrow$ winding coordinates.
- For all τ- and ρ- fects, the T^4 factorises into $T^2 \times T^2$.

Ricci-flat $T^2 \times T^2$ semi-flat fibration

$$ds^2 = e^{2\chi}\tau_2\tilde{\tau}_2 dz d\bar{z} + \frac{1}{\tau_2}|d\xi^2 + \tau d\xi^1|^2 + \frac{1}{\tilde{\tau}_2}|d\tilde{\xi}_2 + \tilde{\tau} d\tilde{\xi}_1|^2$$

with $\tau(z)$, $\tilde{\tau(z)}$ meromorphic functions of the base. $\nabla^2\chi = 0$.

- $O(2,2)$ structure $\Rightarrow \tilde{\tau} = -1/\tau$.
- Describes τ-fects. Metrics for ρ-fects are obtained by $\xi^i \leftrightarrow \tilde{\xi}_i$.
- $I_1 \times I_1$ degeneration? Corrections to the semi-flat limit not known.
- Global issues need further investigation. Naively, too many degenerations are obtained.

Next, this fibrations are studied within Double Field Theory.

General DFT facts

- Coordinate content: $X^M = (x^i, \tilde{x}_i)$. Momentum dual to $\tilde{x}_i$ is winding.
- We combine g and B into an $O(d,d)$ tensor called "generalised metric"

$$\mathcal{H} = \begin{pmatrix} g - Bg^{-1}B & Bg^{-1} \\ -g^{-1}B & g^{-1} \end{pmatrix}$$

  For a T^2, $\mathcal{H}$ coincides with the metric obtained by embedding $SL(2)_{\tau,\rho} \subset SO(2,2) \subset SL(4)$.
- DFT background-independent action: $S = \int dX^M e^{-2\Phi} \mathcal{R}(\mathcal{H})$
  $\mathcal{R}(\mathcal{H})$ "generalised Ricci scalar".
- Closure of the algebra $\Rightarrow$ strong constraint: $\partial_M \partial^M ... = 0$
- Strong constraint $\Leftrightarrow$ the fields depend only on half of the coordinates.
  Independence on $\tilde{x}_i \Leftrightarrow$ SUGRA. Independence on $x^i \Leftrightarrow$ isometry.
- DFT generalised duality: to all DFT configuration one can apply transformations of the type $x^i \leftrightarrow \tilde{x}_i$ and obtain another DFT solution.

T-fects in DFT

T-fects as DFT solutions.

$$"ds^2_{DFT}" = \mathcal{H}_{MN} d\xi^M d\xi^N = e^{2\phi}\tau_2 dz d\bar{z} + \frac{1}{\tau_2}|d\xi^2 + \tau d\xi^1|^2 + \frac{1}{\tilde{\tau}_2}|d\tilde{\xi}_2 + \tilde{\tau} d\tilde{\xi}_1|^2$$

with $\tau(z) = -1/\tilde{\tau}(z)$ meromorphic functions of the base and $\nabla^2\phi = 0$.

- Same fiber as the T^4 fibration, different base pre-factor.
- $\mathcal{H}$ is not a metric. Interpreted as such it is not Ricci-flat.
- It is a solution of DFT equations of motion. (It is "generalised-Ricci-flat")
- The coordinates on the base can be formally doubled and then $\mathcal{H}$ is an $O(d,d)$ matrix.

Corrections and generalised duality

- Consider corrections to the smearing limit of the NS5, uplift it to DFT and apply generalised duality.
- The result is a solution of the DFT e.o.m. that reproduce the discussed corrections.

B-field for the localised NS5 $\left[h = \log(\mu/r) + \sum_{k_1,k_2} \mathcal{C}_{k_1,k_2}(r, \xi^1, \xi^2)\right]$:

- Gauge choice: $B = \theta d\xi^1 \wedge d\xi^2 + \Pi_1 d\theta \wedge d\xi^1 + \Pi_2 d\theta \wedge d\xi^2$
- $\Pi_i = \Pi_i(r, \xi^1, \xi^2)$ and $dB = \star_4 dh$. $\Pi_i \to 0$ for $r \gg 1$

The localised KK-monopole

$$\begin{aligned} ds^2 &= \tilde{h}\left[dr^2 + r^2 d\theta^2 + (d\xi^1)^2\right] + \frac{1}{\tilde{h}}\left[d\xi^2 + \theta\, d\xi^1 + \tilde{\Pi}_2\, d\theta\right]^2, \\ B &= \tilde{\Pi}_1\, d\theta \wedge d\xi^1, \qquad \text{with } \tilde{h} = h(r, \xi^1, \tilde{\xi}_2),\ \tilde{\Pi}_i = \Pi_i(r, \xi^1, \tilde{\xi}_2) \end{aligned}$$

The localised Q-brane

$$\begin{aligned} ds^2 &= \tilde{h}\left[dr^2 + r^2 d\theta^2\right] + \frac{\tilde{h}}{\tilde{h}^2 + \theta^2}\left[(d\xi^1 + \tilde{\Pi}_1 d\theta)^2 + (d\xi^2 + \tilde{\Pi}_2 d\theta)^2\right], \\ B &= \frac{-\theta}{\tilde{h}^2 + \theta^2}\left(d\xi^1 + \tilde{\Pi}_1 d\theta\right) \wedge \left(d\xi^2 + \tilde{\Pi}_2 d\theta\right), \end{aligned}$$

with $\tilde{h} = h(r, \tilde{\xi}_1, \tilde{\xi}_2)$, $\tilde{\Pi}_i = \Pi_i(r, \tilde{\xi}_1, \tilde{\xi}_2)$

DFT equations of motion explicitly checked.

Conclusions

- We describe T^2 fibrations and T-fects. Local descriptions can be constructed for any monodromy in $SL(2,\mathbb{Z})_\tau \times SL(2,\mathbb{Z})_\rho$
- We describe corrections to the duality chain:
  NS5 brane $\rightarrow$ KK-monopole $\rightarrow$ Q-brane.
- Winding modes are crucial to understand near-core physics.

  Winding non-conservation $\Leftrightarrow$ dual-coordinates dependence $\Leftrightarrow$ Dyonic modes
- Winding physics is not catured by supergravity.

  Dual-coordinate dependent corrections can only be seen by string-like objects.
- We construct new DFT solutions that capture these corrections
- Open questions:
  - How is this discussion generalised to any monodromy (outside geometric orbit)?
  - What happens with the strong constraint in the general case?