Non-Geometric Non-Abelian Supertubes

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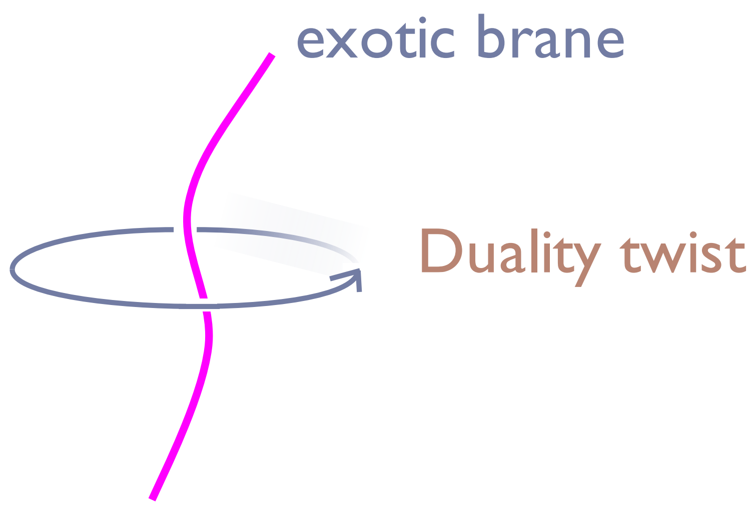
Recent Advances in T/U-dualities and Generalized Geometries

Introduction

Duality

Duality = symmetry in string theory

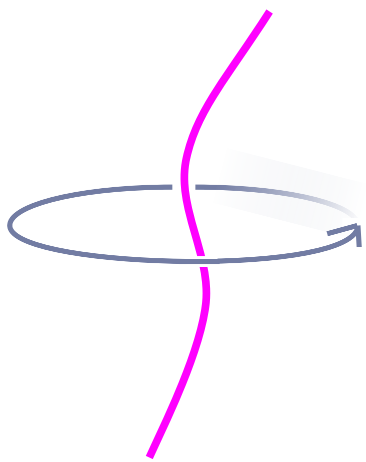
- DFT, EFT: make it manifest
- Twisting in compact directions → non-geom flux cpt'n
- Twisting in noncompact directions → exotic branes



Exotic branes

"Exotic brane"
(codim-2)

[Blau+O'Loughlin '97] [Hull '97]
[Elitzur+Giveon+Kutasov+Rabinovici '97]
[Obers+Pioline+Rabinovici '97]
[Obers+Pioline '98]
[de Boer+MS '10,'12]



Duality twist

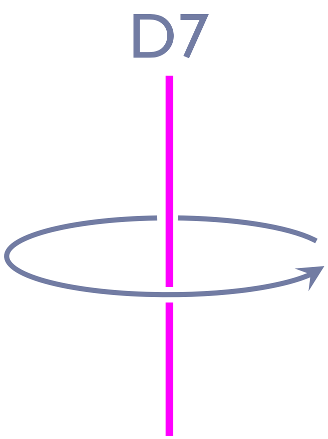
- Non-geometric
- Naturally lives in DFT/EFT
- Non-Abelian

Ex: (p,q) 7-branes in F-theory [Vafa '96]

- 10D IIB: $SL(2,\mathbb{Z})$ duality sym

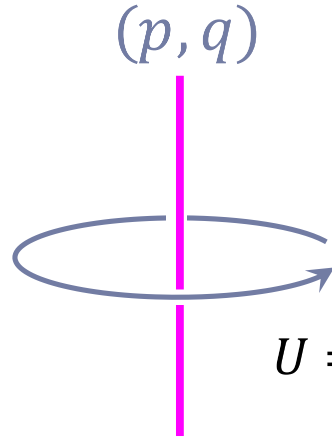
$$\tau = C^{(0)} + i\, e^{-\Phi}$$

$$U = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z}): \quad \tau \to \frac{a\tau + b}{c\tau + d}$$



$$U = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\tau \to \tau + 1$$

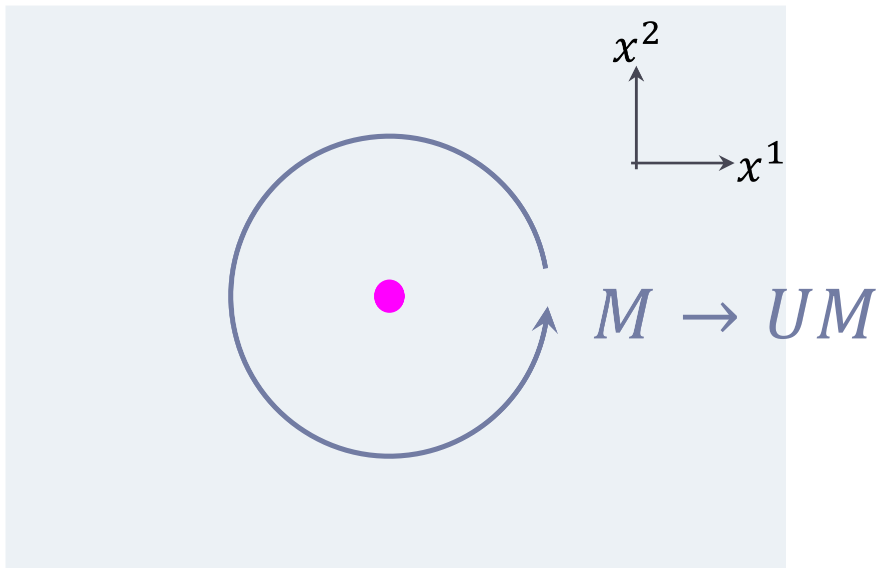


$$U = \begin{pmatrix} 1+pq & p^2 \\ -q^2 & 1-pq \end{pmatrix}$$

More examples

Can do the same using U-duality in lower D

E.g.: 3D

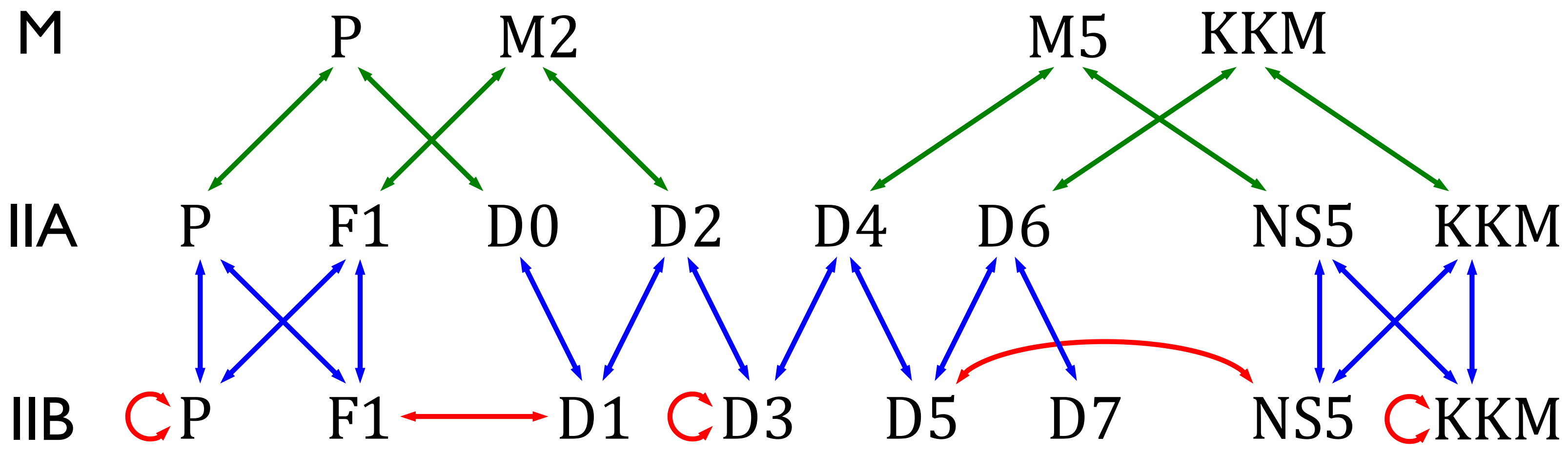


M: moduli

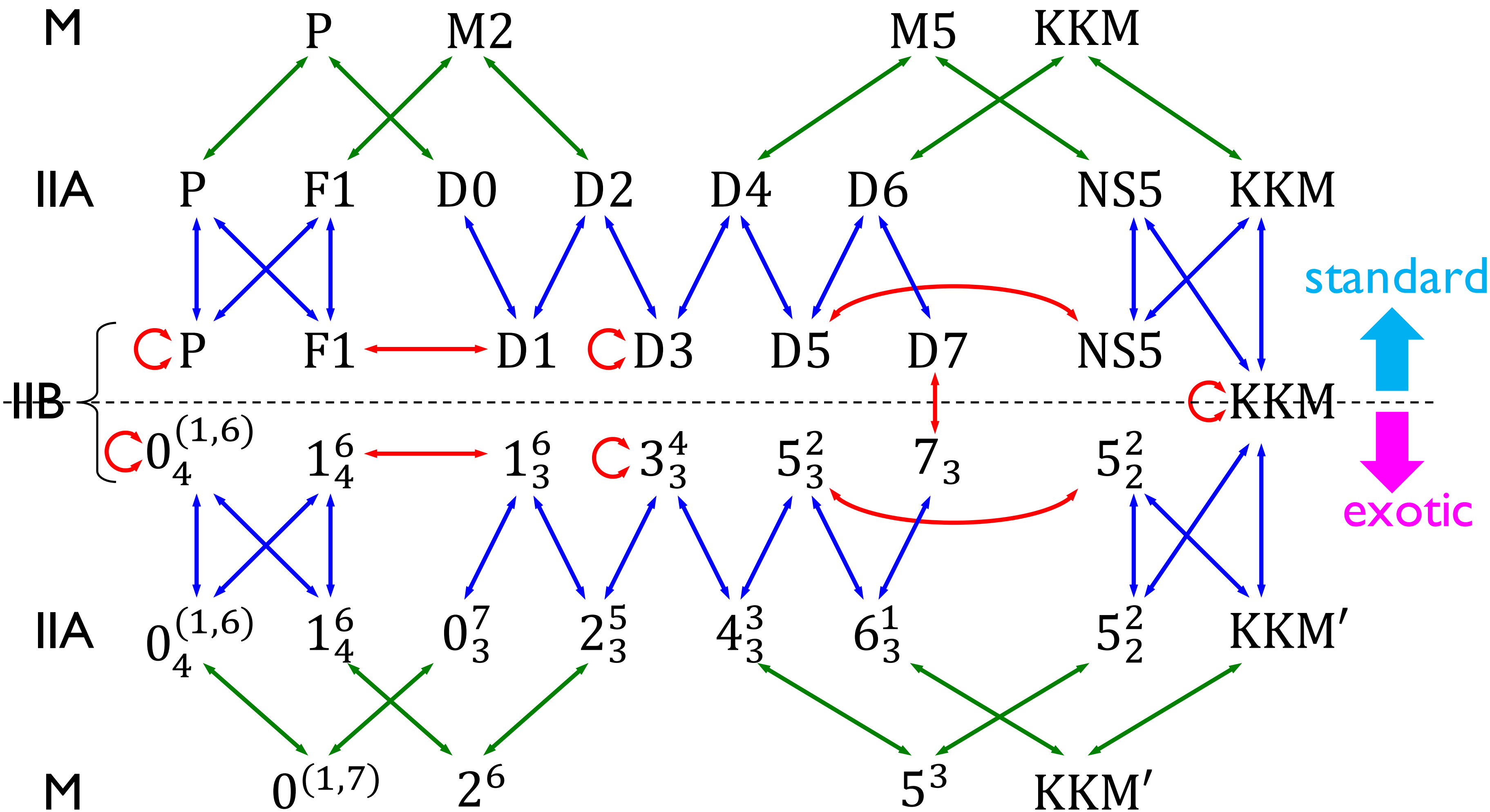
$U \in E_{8(8)}(\mathbb{Z})$

- Various possible twists
- Various kinds of brane

The duality web



The duality web



Example: 5^2_2-brane

Compactify 8,9 directions → $SL(2,\mathbb{Z})$ duality

$$\tau \equiv B_{89} + i\,\mathrm{vol}\left(T^2_{89}\right)$$

	1	2	3	4	5	6	7	8	9
NS5	·	·	O	O	O	O	O	~	~

$\tau \to \tau + 1$ $\quad U = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

T-duality along x^8

	1	2	3	4	5	6	7	8	9
KKM	·	·	O	O	O	O	O	⊙	~

T-duality along x^9

	1	2	3	4	5	6	7	8	9
5^2_2	·	·	O	O	O	O	O	⊙	⊙

$$\tau \to \frac{\tau}{-\tau + 1} \qquad U = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

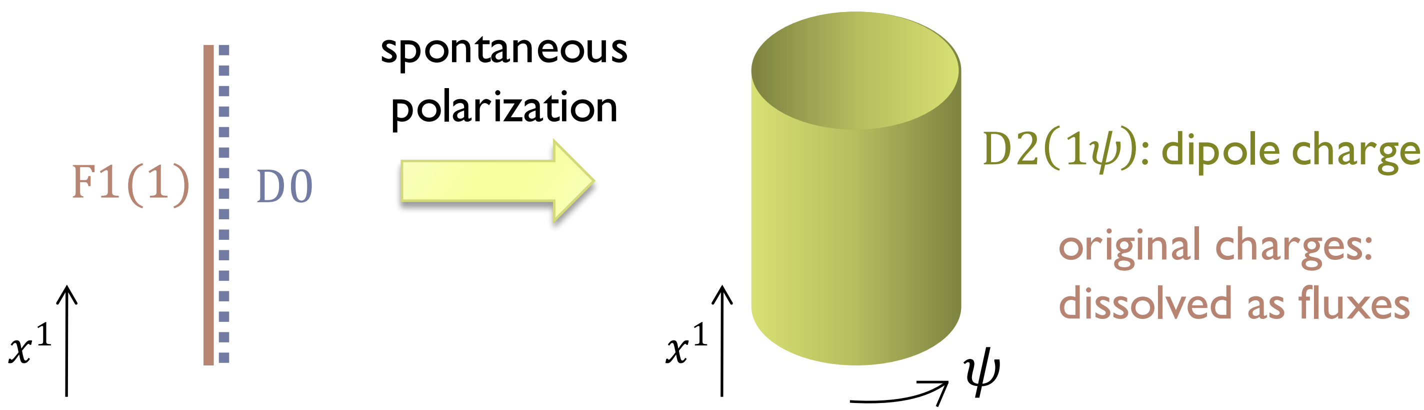
metric multivalued;
non-geometric

Relevance of codim-2 branes

- Codim-2 object problematic
  - Log divergences

$$V \sim \frac{1}{r^{d-2}} \xrightarrow{d=2} V \sim \log\left(\frac{\mu}{r}\right)$$

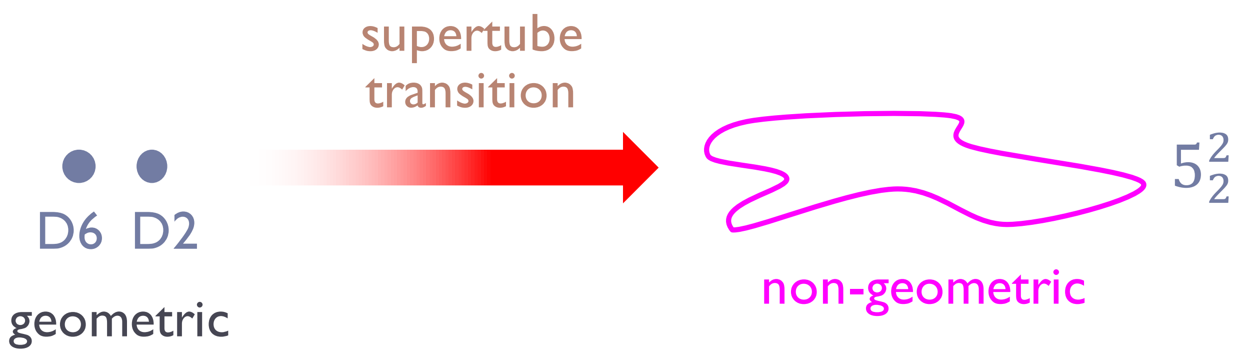
- Are they relevant? Why care?
- Supertube transition [Mateos+Townsend 2001]



Exotic supertubes

[de Boer+Shigemori '10,'12]

- Can be created out of ordinary branes



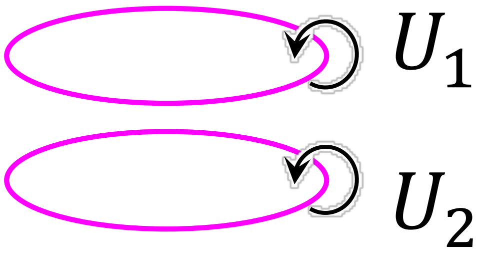
- More common than previously thought
- Relevance for black hole physics

→ Cf. Fuzzball proposal,
Microstate geometry program

The (ultimate) goal

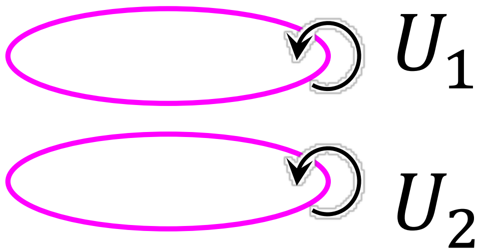
- Explicitly construct a config of codim-2 branes with black hole charges

  → It would represent a non-geometric BH microstate



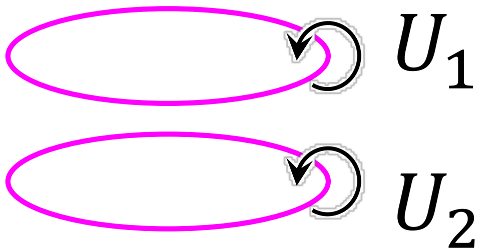
Previously…

[Park+MS 2015]



- Explicitly constructed configs of multiple tubes in sugra
- They are unbound
- Not a BH microstate ☹
- Related to $[U_1, U_2] = 0$?

This talk [Fernández-Melgarejo+Park+MS]



- Now $[U_1, U_2] \neq 0$
- Still unbound… ☹
  - Is the obstacle merely technical…?

“Harmonic solutions”

Susy solutions in 4D sugra

- Type IIA on $T^6 = T^2_{45} \times T^2_{67} \times T^2_{89}$

⇩

- $D = 4, \mathcal{N} = 2$ sugra with 2 vector multiplets (ignore hypers)

- The most general (timelike class) susy solution: completely specified by harmonic functions in $\mathbb{R}^3$:

$$H = \left(V, K^I, L_I, M\right), \qquad I = 1,2,3$$

$$\triangle H(\boldsymbol{x}) = 0 \qquad \boldsymbol{x} \in \mathbb{R}^3$$

[Behrndt-Lüst-Sabra '97]
[Bates-Denef '03]
[Gutowski-Reall '04]
[Bena-Warner '04]
[Meessen-Ortin '06]

"harmonic solutions"

10D IIA fields

$$ds^2_{10,\mathrm{str}} = -\frac{1}{\sqrt{V(Z-V\mu^2)}}(dt+\omega)^2 + \sqrt{V(Z-V\mu^2)}\,dx^i dx^i$$

$$+\sqrt{\frac{Z-V\mu^2}{V}}\left(Z_1^{-1}dx^2_{45} + Z_2^{-1}dx^2_{67} + Z_3^{-1}dx^2_{89}\right)$$

$$e^{2\Phi} = \frac{(Z-V\mu^2)^{3/2}}{V^{3/2}Z}\,, \qquad B_2 = \left(V^{-1}K^I - Z_I^{-1}\mu\right)J_I\,, \qquad \dots$$

$$Z = Z_1 Z_2 Z_3 \qquad J_1 \equiv dx^4 \wedge dx^5, \quad J_2 \equiv dx^6 \wedge dx^7, \quad J_3 \equiv dx^8 \wedge dx^9$$

$$Z_I = L_I + \frac{1}{2}C_{IJK}V^{-1}K^J K^K$$

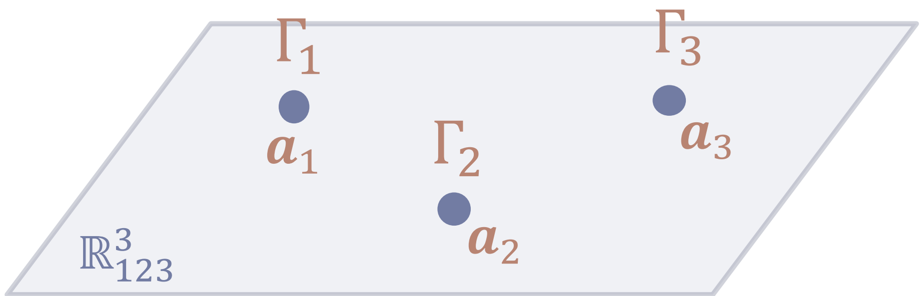
$$\mu = M + \frac{1}{2}V^{-1}K^I L_I + \frac{1}{6}C_{IJK}V^{-2}K^I K^J K^K$$

$$*_3\, d\omega = V dM - M dV + \frac{1}{2}\left(K^I dL_I - L_I dK^I\right)$$

General codim-3 solutions

$$H = (V, K^I, L_I, M), \qquad H(\boldsymbol{x}) = h + \sum_p \frac{\Gamma_p}{|\boldsymbol{x} - \boldsymbol{a}_p|}$$

— Describes multi-center config of branes in IIA on T^6_{456789}



$$V \leftrightarrow \mathrm{D6}(456789) \qquad \begin{array}{l} K^1 \leftrightarrow \mathrm{D4}(6789) \\ K^2 \leftrightarrow \mathrm{D4}(4589) \\ K^3 \leftrightarrow \mathrm{D4}(4567) \end{array} \qquad \begin{array}{l} L_1 \leftrightarrow \mathrm{D2}(45) \\ L_2 \leftrightarrow \mathrm{D2}(67) \\ L_3 \leftrightarrow \mathrm{D2}(89) \end{array} \qquad M \leftrightarrow \mathrm{D0}$$

Example: 4-charge BH

- Susy BH in 4D (4 supercharges)

N^0 D6(456789) $\quad V = 1 + \frac{N^0}{r} \quad K^I = 0$ No D4

N_1 D2(45)

N_2 D2(67) $\quad \Big\} \; L_I = 1 + \frac{N_I}{r} \quad M = 0$ No D0

N_3 D2(89)

- Single-center
- Macroscopic entropy: $S \sim \sqrt{N^0 N_1 N_2 N_3}$

Duality

- Duality group includes:

$$SL(2,\mathbb{Z})_1 \times SL(2,\mathbb{Z})_2 \times SL(2,\mathbb{Z})_3$$

T-duality in T^2_{89} & shift in B_{89}

- Transformation rule of harmonic func:

$$U = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SU(2,\mathbb{Z})$$

$$\Rightarrow \quad \begin{pmatrix} K^3 \\ V \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} K^3 \\ V \end{pmatrix}$$

And similarly for

$$\begin{pmatrix} -2M \\ L_3 \end{pmatrix}, \quad \begin{pmatrix} -L_1 \\ K^2 \end{pmatrix}, \quad \begin{pmatrix} -L_2 \\ K^1 \end{pmatrix}$$

Torus moduli

- Complexified Kähler modulus for T^2_{89}:

$$\tau^3 = B_{89} + i\,\mathrm{vol}(T^2_{89})$$
$$= \left(\frac{K^3}{V} - \frac{\mu}{Z_3}\right) + \frac{i\sqrt{V(Z - V\mu^2)}}{Z_3 V}$$

$R_8 = R_9 = l_s$

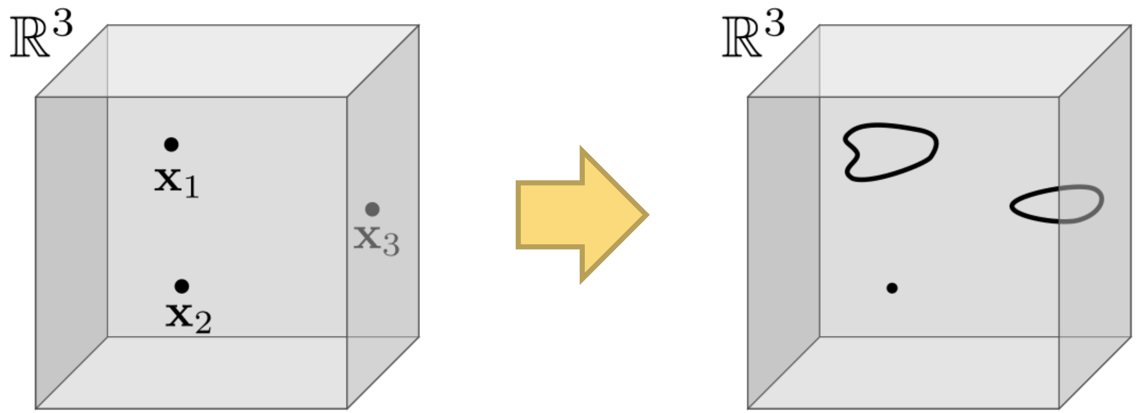
We likewise have τ^1, τ^2 for T^2_{45}, T^2_{67}

Transforms under $SL(2,\mathbb{Z})_3$ as:

$$\tau^3 \to \frac{a\tau^3 + b}{c\tau^3 + d}, \qquad U = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SU(2,\mathbb{Z})$$

Codim-2 solutions [Park+MS 2015]

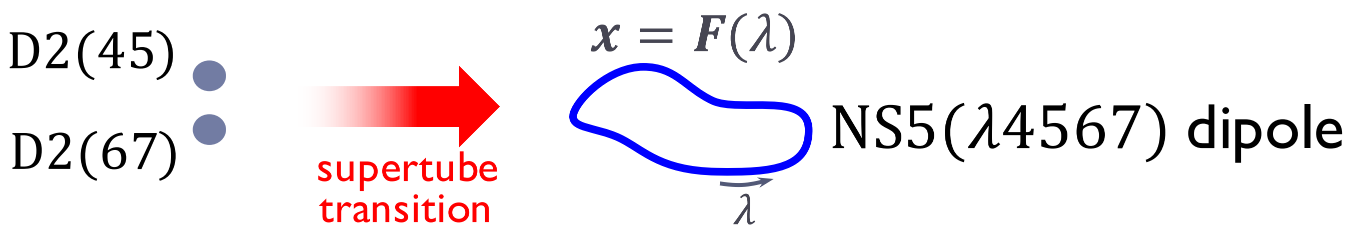
- Harmonic functions can have codim-2 singularities too



- There is a physical reason to consider them: supertube transition.

D2+D2→NS5 (1)

NS5 along general curve in $\mathbb{R}^3$:



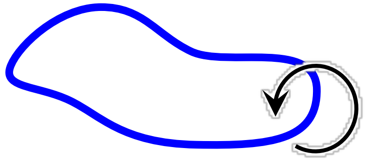
$$V = 1, \quad K^1 = 0, \quad K^2 = 0, \quad K^3 = \gamma$$
$$L_1 = f_2, \quad L_2 = f_1, \quad L_3 = 1, \quad M = -\frac{\gamma}{2}$$

$$f_1 = 1 + \frac{Q_1}{L}\int_0^L \frac{d\lambda}{|\boldsymbol{x}-\boldsymbol{F}(\lambda)|}, \qquad f_2 = 1 + \frac{Q_1}{L}\int_0^L \frac{|\dot{\boldsymbol{F}}(\lambda)|^2 d\lambda}{|\boldsymbol{x}-\boldsymbol{F}(\lambda)|}$$

$$d\gamma = *_3\, d\alpha, \qquad \alpha_i = \frac{Q_1}{L}\int_0^L \frac{\dot{F}_i(\lambda) d\lambda}{|\boldsymbol{x}-\boldsymbol{F}(\lambda)|}$$

D2+D2→NS5 (2)

$$V = 1, \quad K^1 = 0, \quad K^2 = 0, \quad K^3 = \gamma$$
$$L_1 = f_2, \quad L_2 = f_1, \quad L_3 = 1, \quad M = -\frac{\gamma}{2}$$



$$\begin{pmatrix} K^3 \\ V \end{pmatrix} \to \begin{pmatrix} K^3 + 1 \\ V \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} K^3 \\ V \end{pmatrix}$$

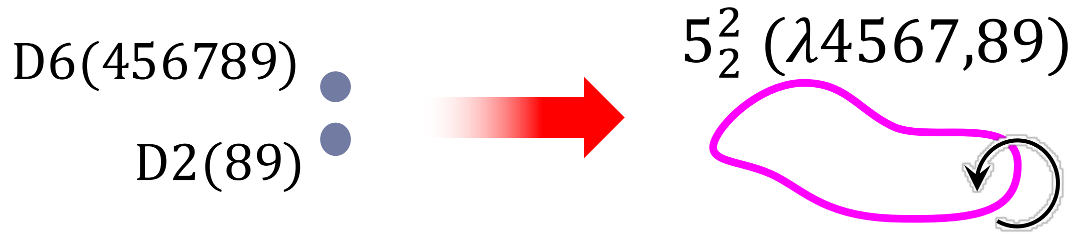
$$\tau^3 \to \tau^3 + 1$$

$SL(2,\mathbb{Z})$ monodromy $U = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

: correct monodromy for NS5

D6+D2→ 5^2_2

$$V = f_2, \quad K^1 = \gamma, \quad K^2 = \gamma, \quad K^3 = 0$$
$$L_1 = 1, \quad L_2 = 1, \quad L_3 = f_1, \quad M = 0$$



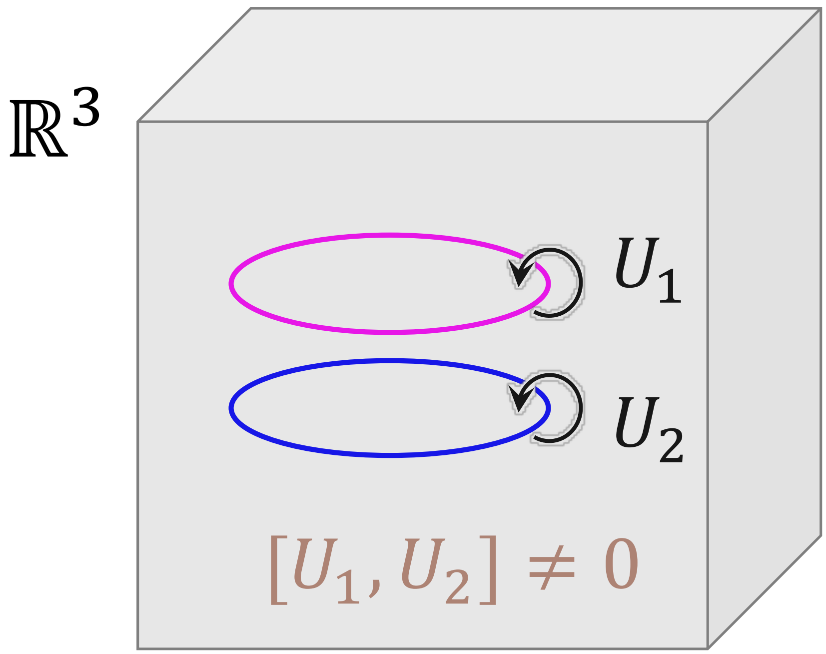
$$\begin{pmatrix} -L_1 \\ K^2 \end{pmatrix} \to \begin{pmatrix} -L_1 \\ L_1 + K^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -L_1 \\ K^2 \end{pmatrix} \qquad \tau^3 \to \frac{\tau^3}{-\tau^3 + 1}$$

: correct monodromy for 5^2_2

Plan of attack

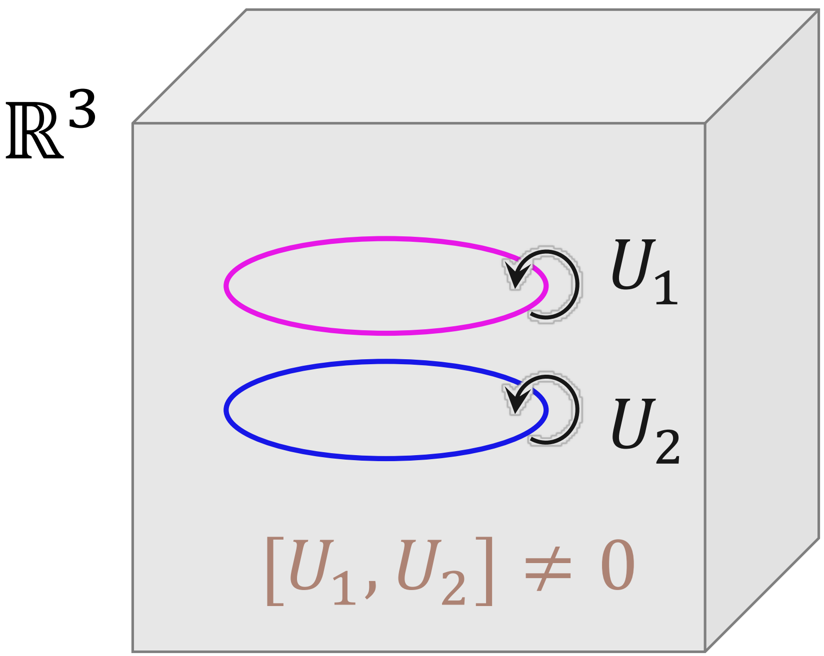
The goal

- Construct a config of supertubes with non-commuting (non-Abelian) monodromies:



The issue

- Getting full solution in 3D is technically hard



Need a pair of 3D harmonic funcs s.t.

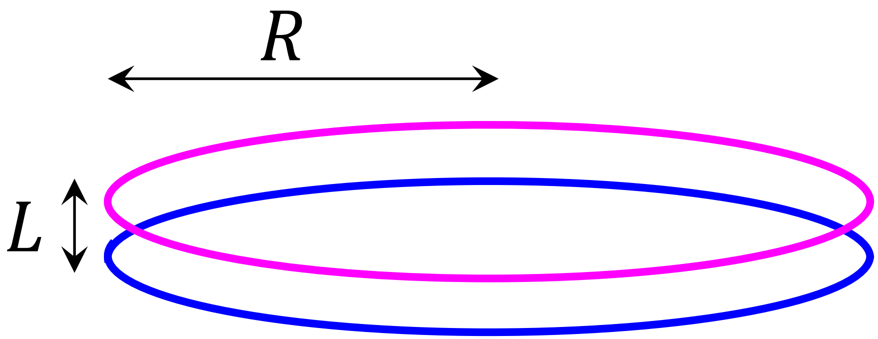
$$\begin{cases} \begin{pmatrix} F \\ G \end{pmatrix} \to \begin{pmatrix} F+G \\ G \end{pmatrix} & \text{around tube 1} \\ \begin{pmatrix} F \\ G \end{pmatrix} \to \begin{pmatrix} F \\ -F+G \end{pmatrix} & \text{around tube 2} \end{cases}$$

⇓

difficult… ☹

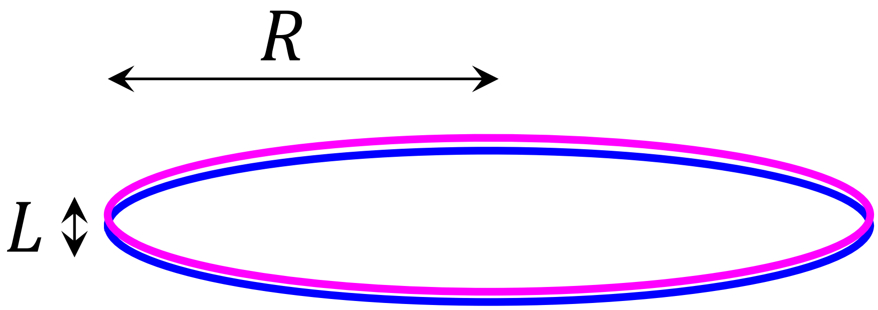
e.g. $U_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, $U_2 = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$

Colliding limit



- Take the limit $L \ll R$

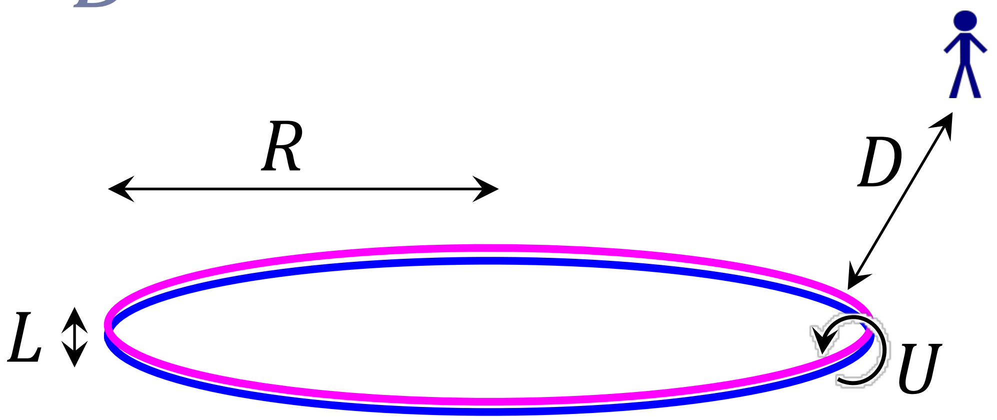
Colliding limit



- Take the limit $L \ll R$

Far region

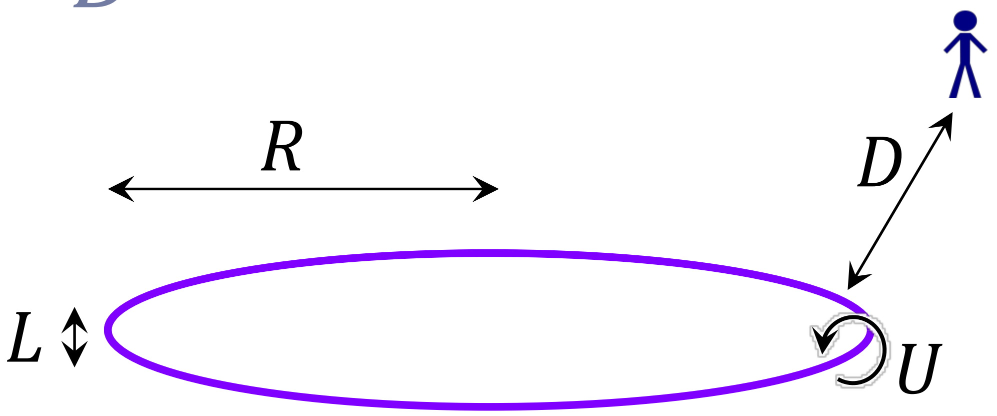
$L \ll R \sim D$



- Looks like one ring in 3D with $U = U_2 U_1$

Far region

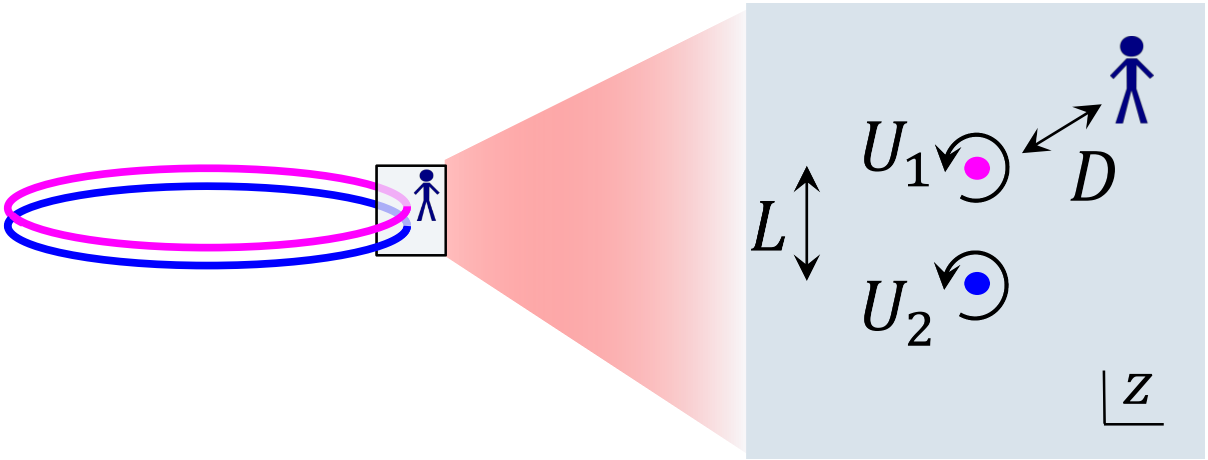
$L \ll R \sim D$



- Looks like one ring in 3D with $U = U_2 U_1$

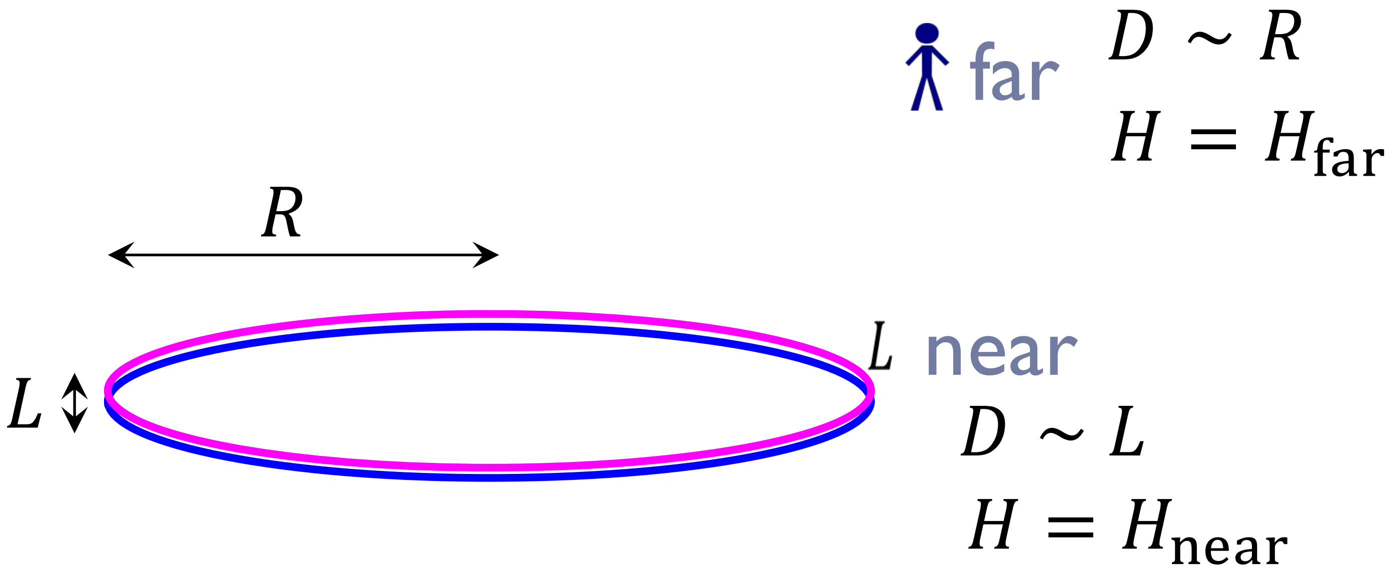
Near region

$D \sim L \ll R$



- Two infinite straight branes in 2D

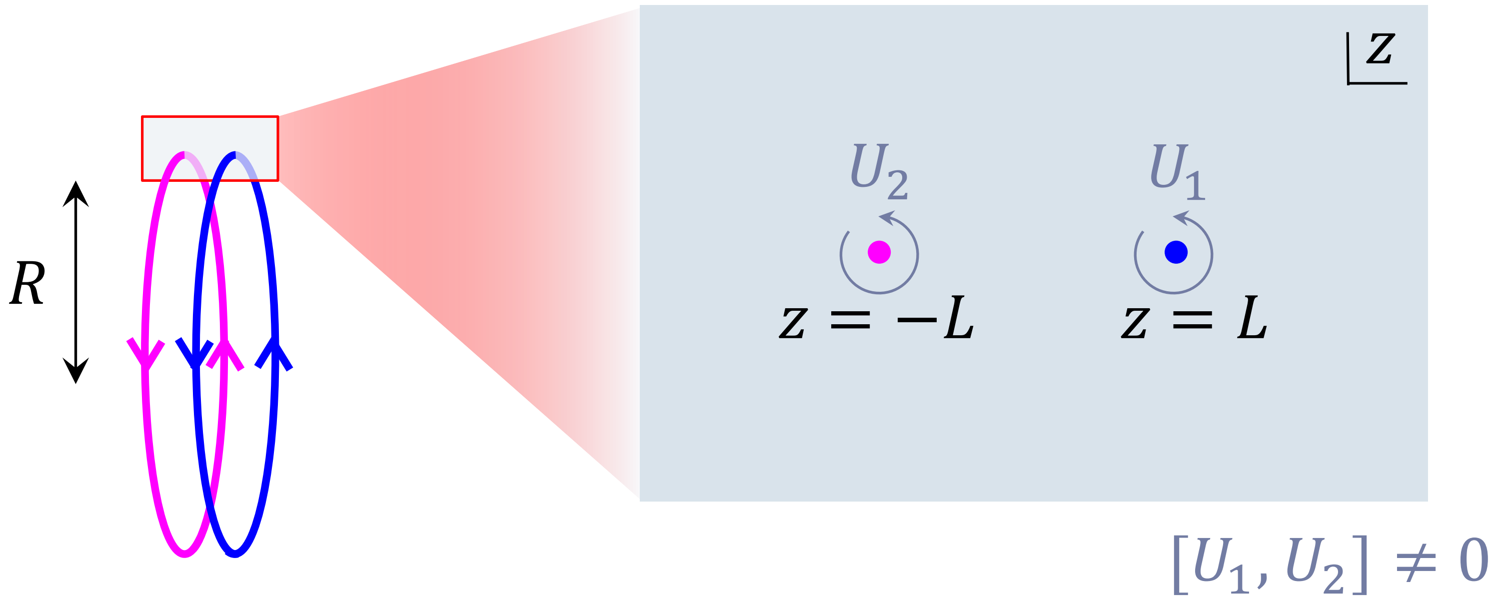
Strategy



- Construct solutions in two regimes
- Match two solutions in intermediate regime

Near region

Non-Abelian supertubes in 2D



- Assume that there are two different types of circular supertubes, like NS5 and 5_2^2
- Zooming in, can use 2D approximation

Config with $\tau^3(z)$

Focus on configs with $\tau^1 = \tau^2 = i,\ \tau^3 = \tau^3(z)$.

General solution:

$$V = \frac{1}{2}(g + \bar{g}) \qquad K^1 = K^2 = \frac{i}{2}(g - \bar{g}) \qquad K^3 = \frac{i}{2}(f - \bar{f})$$

$$L_1 = L_2 = \frac{1}{2}(f + \bar{f}) \qquad L_3 = \frac{1}{2}(g + \bar{g}) \qquad M = -\frac{i}{4}(f - \bar{f})$$

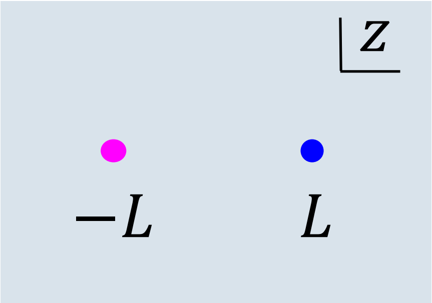
$f(z), g(z)$: holomorphic

$$\tau^3(z) = \frac{if(z)}{g(z)}$$

The question:

Find a pair of holomorphic functions

$$f(z),\ g(z)$$



such that the quantity

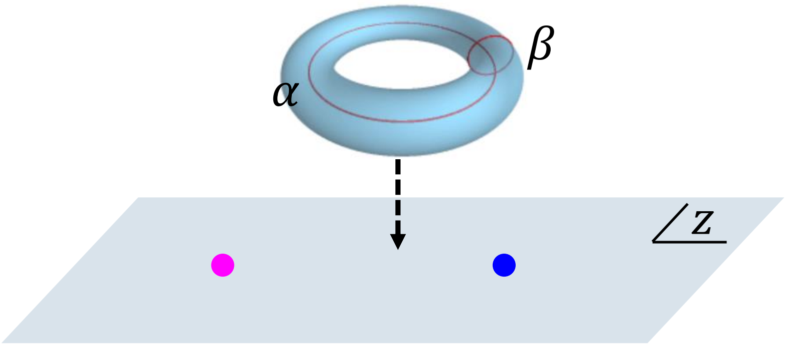
$$\tau^3(z) = \frac{if(z)}{g(z)}$$

has non-trivial $SL(2,\mathbb{Z})$ monodromy around some singular points on the z plane, and

$$\mathrm{Im}\,\tau^3(z) \geq 0$$

The answer:

Seiberg-Witten:
Consider torus fibration over z-plane!



$$\begin{pmatrix} if(z) \\ g(z) \end{pmatrix} = h(z) \begin{pmatrix} \partial_z a_D \\ \partial_z a \end{pmatrix}$$

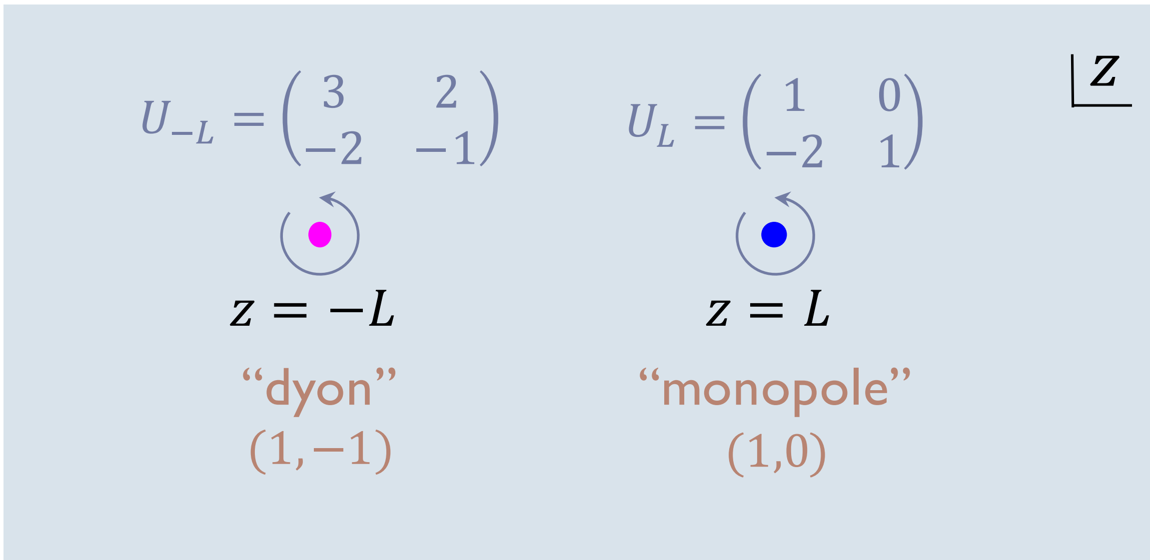
set $h(z) = 1$

$$a_D = \int_\alpha \lambda, \qquad a = \int_\beta \lambda,$$

λ: holomorphic 1-form
(SW 1-form)

A config of non-Abelian supertubes

- Use SW's very original solution



$$a_D(z) = i \int_L^z \sqrt{\frac{z-x}{L^2-x^2}} \qquad a(z) = \int_{-L}^L \sqrt{\frac{z-x}{L^2-x^2}}$$

Behavior near singularities

- $z \sim L$ (monopole)

$$V = L_3 \sim -\log|z-L| \qquad K^1 = K^2 \sim \arg(z-L)$$

$$L_1 = L_2 \sim \text{const.} \qquad K^3 = -2M \sim 0$$

$$\Rightarrow \mathrm{D6}(456789) + \mathrm{D2}(89) \rightarrow 5_2^2(34567{,}89)$$

- $z \sim -L$ (dyon)

$$V = L_3 \sim -\arg(z+L) \qquad K^1 = K^2 \sim -\log|z+L|$$

$$L_1 = L_2 \sim -\log|z+L| \qquad K^3 = -2M \sim \arg(z+L)$$

$$\Rightarrow \begin{array}{lll} \mathrm{D4}(6789) + \mathrm{D4}(4589) & \rightarrow & 5_2^2(34567{,}89) \\ \mathrm{D2}(45) + \mathrm{D2}(67) & \rightarrow & \mathrm{NS5}\ (34567) \end{array}$$

We do have non-commutative pair of supertubes

Behavior for $|z| \gg L$

- $z \sim L$ (monopole)

$$V = L_3 \sim \frac{1}{\sqrt{z}} \qquad K^1 = K^2 \sim \frac{i}{\sqrt{z}}$$

$$L_1 = L_2 \sim \frac{1}{\sqrt{z}} \log z \qquad K^3 = -2M \sim \frac{i}{\sqrt{z}} \log z$$

→ To be matched with the far-region behavior

Define

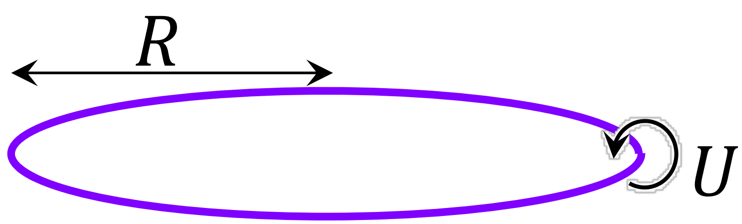
$$V \sim \frac{1}{\sqrt{z}} \equiv G_{\text{match}} \qquad K^3 \sim \frac{i}{\sqrt{z}} \log z \equiv F_{\text{match}}$$

Far region

Setup

- One circular ring in 3D with monodromy

$$U = U_{-L} U_L = \begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}$$



- Need to find a pair of harmonic funcs with this monodromy $\begin{pmatrix} F \\ G \end{pmatrix} \to \begin{pmatrix} -F + 2G \\ -G \end{pmatrix}$

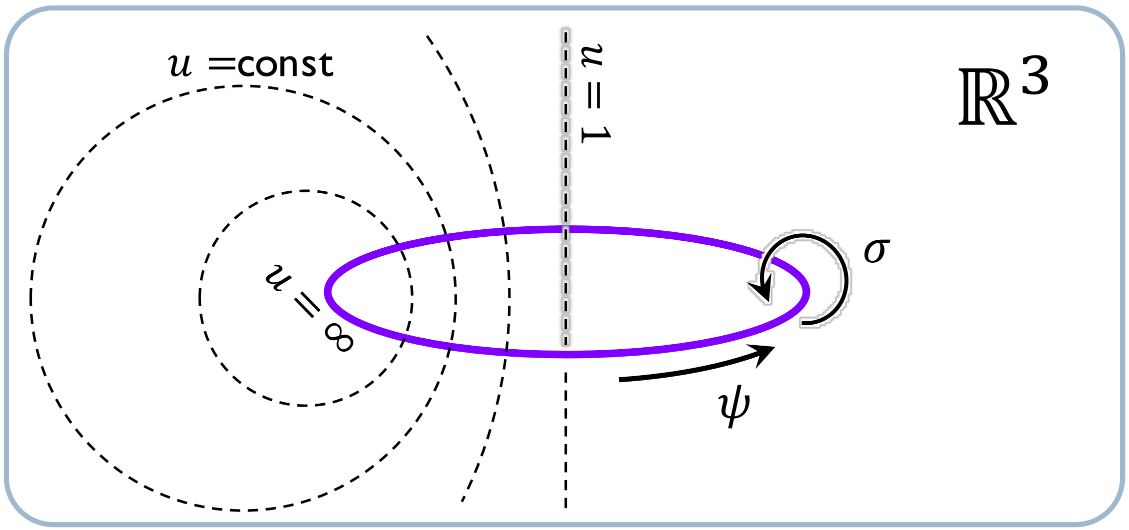
- Near the ring, they must behave like the large-$|z|$ limit of 2D harmonic funcs we found

Toroidal coordinates

$$x^1 = \frac{\sqrt{u^2-1}}{u-\cos\sigma} R\cos\psi, \qquad x^2 = \frac{\sqrt{u^2-1}}{u-\cos\sigma} R\sin\psi, \qquad x^3 = \frac{\sin\sigma}{u-\cos\sigma} R.$$

$$dx_{123}^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 = \frac{R^2}{(u-\cos\sigma)^2}\left[\frac{du^2}{u^2-1} + (u^2-1)d\psi^2 + d\sigma^2\right]$$

$$1 \le u < \infty, \qquad 0 \le \psi < 2\pi, \qquad 0 \le \sigma < 2\pi.$$



- Near-ring limit:
  $u \to \infty$
- $\mathbb{R}^3$ infinity:
  $u \to 1, \sigma = 0$

Ansatz

$$H = \sqrt{u - \cos\sigma}\, e^{im\sigma} f(u)$$

⇩ Laplace eq separates

$$\triangle H \propto (1 - 4m^2) f + 8uf' + 4(u^2 - 1) f'' = 0$$

⇩ regular soln at 3D infinity ($u = 1$):

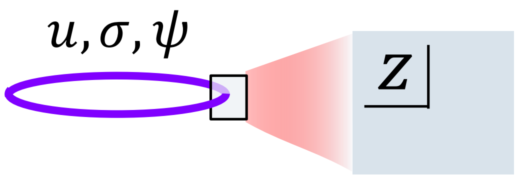
$$H \propto P_{m-1/2}(u)$$ (Legendre func/polynom)

- How do we determine m?

Matching

- Near the ring ($u \to \infty$), toroidal coordinates give

$$u^{-1} e^{i\sigma} \leftrightarrow z$$



- Matching with near-region solution means

$$G_{\text{match}} \sim 1/\sqrt{z} \leftrightarrow \sqrt{u}\, e^{-i\sigma/2}$$

$$\Rightarrow \quad m = -1/2$$

$$\Rightarrow \quad H = \sqrt{u - \cos\sigma}\, e^{i\sigma/2} \equiv G$$

- This one has monodromy

$$G \to -G$$

Cf. We need

$$\begin{pmatrix} F \\ G \end{pmatrix} \to \begin{pmatrix} -F + 2G \\ -G \end{pmatrix}$$

Ansatz 2

$$H = \sqrt{u - \cos\sigma}\, e^{im\sigma}(f(u) + i\sigma g(u))$$

Again, Laplace eq separates

$$4(u^2-1)f''(u) + 8uf'(u) + (1-4m^2)f(u) = 8mg(u)$$

$$4(u^2-1)g''(u) + 8ug'(u) + (1-4m^2)g(u) = 0.$$

f: Legendre. g: known once f is known

Take $m = -1/2$

$$H = \frac{1}{\pi}\sqrt{u - \cos\sigma}\, e^{\frac{i\sigma}{2}}(i\log(u+1) + \sigma) \equiv F$$

- Near-ring ($u \to \infty$) behavior:

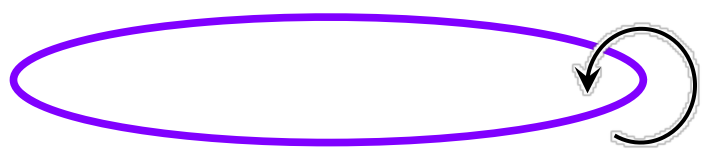
$$H \sim i\sqrt{u}\, e^{\frac{i\sigma}{2}} \log\left(ue^{-i\sigma}\right) \leftrightarrow \frac{i}{\sqrt{z}}\log z = F_{\text{match}} \checkmark$$

Monodromy

$$\begin{pmatrix} F \\ G \end{pmatrix} = \sqrt{u - \cos\sigma}\, e^{i\sigma/2} \begin{pmatrix} \frac{1}{\pi}\left[i \log(u+1) + \sigma\right] \\ 1 \end{pmatrix}$$

$$\rightarrow -\sqrt{u - \cos\sigma}\, e^{i\sigma/2} \begin{pmatrix} \frac{1}{\pi}\left[i \log(u+1) + \sigma\right] + 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -F + 2G \\ -G \end{pmatrix} = \underbrace{\begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}}_{U} \begin{pmatrix} F \\ G \end{pmatrix} \quad \checkmark$$



$\sigma \rightarrow \sigma + 2\pi$

Discussion

Summary

- We found expressions in far & near regions
- They match in the intermediate region

$$\begin{pmatrix} F \\ G \end{pmatrix} = \begin{cases} \sqrt{u - \cos\sigma}\, e^{i\sigma/2} \begin{pmatrix} \frac{1}{\pi}\left[i \log(u+1) + \sigma\right] \\ 1 \end{pmatrix} & \text{far} \\[2em] \dfrac{1}{\sqrt{z}} \begin{pmatrix} i \log z \\ 1 \end{pmatrix} & \text{near} \end{cases}$$

- Matching can be made better order by order

Some physical analysis

- Condition on metric signature

⇨ Far-region solution with 1 tube breaks down for

$$u \geq \log\frac{8R}{L} - 1$$

(recall that tube is at $u = \infty$)

Very close to tube, description in terms of 2 tubes must take over

Similar to O7 decomposing into (p,q) 7-branes

Some physical issues…

- The behavior at 3D infinity:

$$V \sim \frac{1}{r}, \qquad K^I \sim M \sim 0, \qquad L_1 = L_2 \sim \frac{\log(R/L)}{r}, \qquad L_3 \sim \frac{1}{r}$$

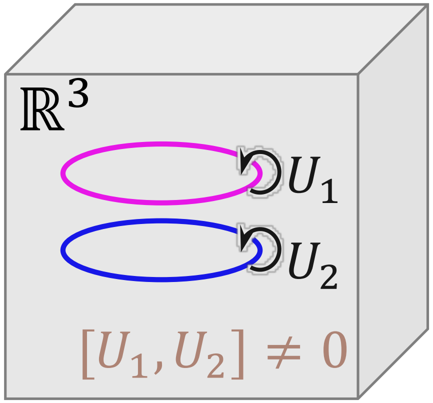
  - Has D6,D2 charges (those of 4D BH)
  - Asymptotic moduli = 0 → $AdS_2 \times S^2$
  - No condition on L, R → unbound ☹

- Angular momentum $J = 0$
  - What supports supertubes?
  - Good for microstates? cf. Sen

Conclusions

Conclusions

- Explicitly constructed a config of non-geometric non-Abelian supertubes



- Unbound ☹
  - Not a BH microstate
  - Way out? $h(z)$?
- Codim-2 harmonic solutions: unexplored
  - Split attractor flow, marginal stability, wall crossing, QQM, …
  - Codim-1
  - DFT/EFT?

Thanks!