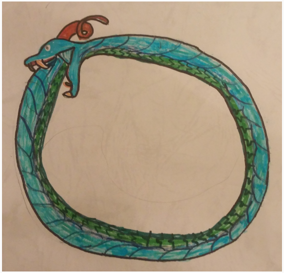
Double Field Theory as Stringy Gravity

Foundation



Implication

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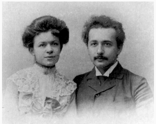
8th June 2017, Recent Advances in T/U-dualities and Generalized Geometries @ Rudjer Bošković Institute, Zagreb

Prologue

Ever since Einstein formulated General Relativity (GR), the Riemannian metric, $g_{\mu\nu}$, has been privileged to be the only geometric and hence gravitational field.



- All other fields are meant to be 'extra' matters.
- The coupling of GR to matters, *e.g.* to the Standard Model, are then 'minimally' determined through the explicitly appearing metric and covariant derivatives in Lagrangians, which ensure both diffeomorphisms and local Lorentz symmetry.

- *Symmetry dictates interaction.* C. N. Yang

- On the other hand, string theory suggests us to put a two-form gauge potential, $B_{\mu\nu}$, and a scalar dilaton, ϕ, on an equal footing along with the metric.

  Forming the massless sector of closed strings, they are ubiquitous in all string theories,

$$\int \mathrm{d}^D x \sqrt{-g} e^{-2\phi} \left(R_g + 4\partial_\mu \phi \partial^\mu \phi - \tfrac{1}{12} H_{\lambda\mu\nu} H^{\lambda\mu\nu} \right) \qquad \text{where} \qquad H = \mathrm{d}B\,,$$

  and can be transformed to each other by T-duality. Buscher

This talk summarizes my works in collaborations with Imtak Jeon, Kanghoon Lee, Soo-Jong Rey, Yuho Sakatani, Charles Melby-Thompson, Rene Meyér, and Kang-Sin Choi:

1105.6294, 1206.3478,1210.5078, 1304.5946, 1307.8377, 1505.01301, 1506.05277, 1507.07545, 1508.01121, 1606.09307, 1609.04265. *c.f.* last page for detailed list.

- Essentially I will argue that Double Field Theory, initiated by Siegel, Hull, Zwiebach, has by now evolved to a 'gravitational' theory, *i.e.* **Stringy Gravity**, potentially alternative to GR, which postulates the entire closed string massless sector to be geometric and thus gravitational, dictated by Symmetry Principle:
  - **O(D, D) T-duality**
  - **a pair of local Lorentz Symmetries, Spin$(1, D-1)_L \times$ Spin$(D-1, 1)_R$**
  - **Diffeomorphisms (on doubled-yet-gauged spacetime)**
  - **'Coordinate Gauge Symmetry'**

– Talk Content –

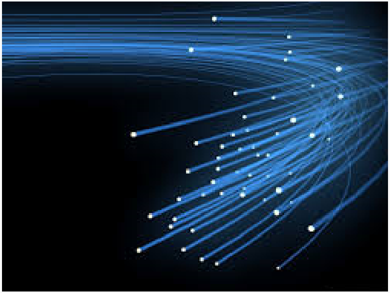
Part I. Geometric Foundation

Stringy Gravity is formulated on 'doubled-yet-gauged' spacetime.

Part II. Gravitational Implication

Stringy Gravity modifies GR at "short" distance and may solve the Dark Matter/Energy problems.

Geometric Foundation



Doubled-yet-Gauged

- *Notation*

Index	Representation	Metric (raising/lowering indices)
$A, B, \cdots$	$\mathbf{O}(D,D)$ & Diffeomorphism vector	$\mathcal{J}_{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$p, q, \cdots$	$\mathbf{Spin}(1, D{-}1)_L$ vector	$\eta_{pq} = \mathrm{diag}(-++\cdots+)$
$\alpha, \beta, \cdots$	$\mathbf{Spin}(1, D{-}1)_L$ spinor	$C_{\alpha\beta}, \qquad (\gamma^p)^T = C\gamma^p C^{-1}$
$\bar{p}, \bar{q}, \cdots$	$\mathbf{Spin}(D{-}1, 1)_R$ vector	$\bar{\eta}_{\bar{p}\bar{q}} = \mathrm{diag}(+--\cdots-)$
$\bar{\alpha}, \bar{\beta}, \cdots$	$\mathbf{Spin}(D{-}1, 1)_R$ spinor	$\bar{C}_{\bar{\alpha}\bar{\beta}}, \qquad (\bar{\gamma}^{\bar{p}})^T = \bar{C}\bar{\gamma}^{\bar{p}}\bar{C}^{-1}$

– Here D denotes the dimenison of the physical spacetime. In this talk, $D \equiv 4$ or 10.

– Each symmetry rotates its own indices *exclusively*: spinors are $\mathbf{O}(D,D)$ singlet!

– The constant $\mathbf{O}(D,D)$ metric, $\mathcal{J}_{AB}$, naturally decomposes the doubled coordinates into two parts,

$$x^A = (\tilde{x}_\mu, x^\nu)\,, \qquad \partial_A = (\tilde{\partial}^\mu, \partial_\nu)\,,$$

where μ, ν are D-dimensional curved indices.

- *Doubled-yet-gauged spacetime*

Stringy Gravity adopts a *doubled-yet-gauged coordinate system*: the doubled coordinates are 'gauged' by an equivalence relation,

$$x^A \sim x^A + \Delta^A(x)\,,$$

such that each equivalence class, or gauge orbit in $\mathbb{R}^{D+D}$, represents a single physical point in $\mathbb{R}^D$.

In the above, Δ^A is an arbitrary derivative-index-valued $\mathbf{O}(D,D)$ vector. This means that its superscript index must be identifiable as that of derivative, $\partial^A = \mathcal{J}^{AB}\partial_B$.

For example, with arbitrary functions, Φ_1, Φ_2 belonging to the theory, $\Delta^A = \Phi_1 \partial^A \Phi_2$.

The equivalence relation can be realized by requiring that *all* the fields/functions in Stringy Gravity should be invariant under the coordinate gauge symmetry shift,

$$\Phi(x+\Delta) = \Phi(x) \quad \Longleftrightarrow \quad \Delta^A \partial_A = 0\,.$$

This invariance is equivalent to the 'section condition' in DFT, $\partial_A \partial^A = 2\,\partial_\mu \tilde{\partial}^\mu = 0$, which can be generically solved, up to $\mathbf{O}(D,D)$ rotations, by letting $\tilde{\partial}^\mu \equiv 0$, and hence

$$(\tilde{x}_\mu\,,\, x^\nu) \;\sim\; (\tilde{x}_\mu + \Phi_1 \partial_\mu \Phi_2\,,\, x^\nu) \quad : \quad \tilde{x}_\mu \text{ coordinates are gauged}\,.$$

$\mathbf{O}(D,D)$ transformations then rotate the gauged directions (and the section).

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- *Diffeomorphisms*

Diffeomorphism covariance in doubled-yet-gauged spacetime are given by

$$\delta x^{A}=\xi^{A}\,,\qquad\qquad \delta\partial_{A}=-\partial_{A}\xi^{B}\partial_{B}=(\partial^{B}\xi_{A}-\partial_{A}\xi^{B})\partial_{B}\,,$$

and for a covariant tensor (or tensor density with weight ω),

$$\delta T_{A_1\cdots A_n}=-\omega\partial_{B}\xi^{B}T_{A_1\cdots A_n}+\textstyle\sum_{i=1}^{n}(\partial_{B}\xi_{A_i}-\partial_{A_i}\xi_{B})T_{A_1\cdots A_{i-1}}{}^{B}{}_{A_{i+1}\cdots A_n}\,.$$

This is the passive counterpart of the 'generalized Lie derivative', $\hat{\mathcal{L}}_{\xi}$ Siegel 1993.

– Unlike $\mathbf{O}(D,D)$ rotations, diffeomorphisms leave the gauged directions of the doubled coordinates, *e.g.* $\{\tilde{x}_{\mu}\}$, invariant, and preserve the section, $\{x^{\nu}\}$.

\# The usual infinitesimal one-form, $\mathrm{d}x^{A}$, is not diffeomorphic covariant,

$$\delta(\mathrm{d}x^{A})=\mathrm{d}x^{B}\partial_{B}\xi^{A}\neq(\partial_{B}\xi^{A}-\partial^{A}\xi_{B})\mathrm{d}x^{B}\,,$$

which I will recall later to define the 'proper length'.

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- *Fundamental fields : building blocks of Stringy Gravity*

The geometric and hence gravitational fields in Stringy Gravity consist of a pair of vielbeins and a dilaton which are $\mathbf{O}(D,D)$ covariant:

$$V_{Ap}\,,\qquad \bar{V}_{A\bar{p}}\,,\qquad d\,,$$

They represent the massless sector of closed strings (*i.e.* NS-NS sector, *c.f.* R-R sector, $\mathcal{C}^{\alpha}{}_{\bar{\alpha}}$).

– The pair of vielbeins satisfy four defining properties,

$$V_{Ap}V^{A}{}_{q}=\eta_{pq}\,,\qquad \bar{V}_{A\bar{p}}\bar{V}^{A}{}_{\bar{q}}=\bar{\eta}_{\bar{p}\bar{q}}\qquad V_{Ap}\bar{V}^{A}{}_{\bar{q}}=0\,,\qquad V_{Ap}V_{B}{}^{p}+\bar{V}_{A\bar{p}}\bar{V}_{B}{}^{\bar{p}}=\mathcal{J}_{AB}\,,$$

such that they are the "square-roots" of projectors,

$$P_{A}{}^{B}=V_{Ap}V^{Bp}\,,\qquad \bar{P}_{A}{}^{B}=\bar{V}_{A\bar{p}}\bar{V}^{B\bar{p}}$$

satisfying

$$P^{2}=P\,,\qquad \bar{P}^{2}=\bar{P}\,,\qquad P\bar{P}=0\,,\qquad P+\bar{P}=\mathbf{1}\,.$$

$\Rightarrow$ The difference of the two projectors sets the DFT-metric, $\mathcal{H}_{AB}=P_{AB}-\bar{P}_{AB}$.

– The dilaton gives rise to the $\mathbf{O}(D,D)$ invariant integral measure with weight one,

$$e^{-2d}\,.$$

Naturally the cosmological constant term in Stringy Gravity should be $e^{-2d}\Lambda_{\mathrm{SG}}$.

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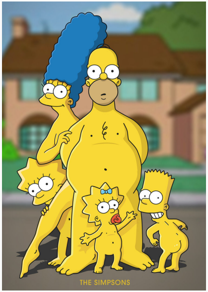
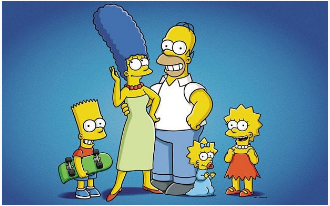
Now, let me talk about the **diffeomorphism covariant derivatives** that turn out to involve two stages.

Semi-covariant derivatives which are not by themselves automatically covariant.

They are naked and vulnerable, being potentially anomalous under diffeomorphisms.



Only after being properly dressed up, through appropriate contractions with the projectors (vielbeins), they become **completely covariant.**



- *Semi-covariant derivative :* Jeon-Lee-JHP 2010, 2011

$$\nabla_C T_{A_1 A_2 \cdots A_n} := \partial_C T_{A_1 A_2 \cdots A_n} - \omega_T \Gamma^B{}_{BC} T_{A_1 A_2 \cdots A_n} + \sum_{i=1}^{n} \Gamma_{CA_i}{}^B T_{A_1 \cdots A_{i-1} B A_{i+1} \cdots A_n} \,.$$

The stringy version of the Christoffel connection has been uniquely determined,

$$\Gamma_{CAB}=2\big(P\partial_C P\bar{P}\big)_{[AB]}+2\Big(\bar{P}_{[A}{}^D\bar{P}_{B]}{}^E-P_{[A}{}^D P_{B]}{}^E\Big)\partial_D P_{EC}-\tfrac{4}{D-1}\Big(\bar{P}_{C[A}\bar{P}_{B]}{}^D+P_{C[A}P_{B]}{}^D\Big)\Big(\partial_D d+(P\partial^E P\bar{P})_{[ED]}\Big)$$

by demanding the compatibility, $\nabla_A P_{BC} = \nabla_A \bar{P}_{BC} = \nabla_A d = 0$, and some extra "torsionless" conditions.

- *Semi-covariant Riemann-like curvature :*

$$S_{ABCD} = S_{[AB][CD]} = S_{CDAB} := \tfrac{1}{2}\left(R_{ABCD} + R_{CDAB} - \Gamma^E{}_{AB}\Gamma_{ECD}\right) , \qquad S_{[ABC]D} = 0 \,,$$

where R_{ABCD} denotes the ordinary "field strength" of the stringy Christoffel connection,

$$R_{CDAB}=\partial_A\Gamma_{BCD}-\partial_B\Gamma_{ACD}+\Gamma_{AC}{}^E\Gamma_{BED}-\Gamma_{BC}{}^E\Gamma_{AED} \quad \Leftarrow \quad \mathrm{d}\Gamma + \Gamma\wedge\Gamma \,.$$

By construction, it transforms infinitesimally as 'total derivative',

$$\delta S_{ABCD} = \nabla_{[A}\delta\Gamma_{B]CD} + \nabla_{[C}\delta\Gamma_{D]AB} \,.$$

- *Semi-covariant 'Master' derivative :*

$$\mathcal{D}_A := \partial_A + \Gamma_A + \Phi_A + \bar{\Phi}_A = \nabla_A + \Phi_A + \bar{\Phi}_A \,.$$

The twofold spin connections are determined in terms of the above stringy Christoffel connection by requiring the compatibility with the vielbeins,

$$\mathcal{D}_A V_{Bp} = \nabla_A V_{Bp} + \Phi_{Ap}{}^q V_{Bq} = 0 \,, \qquad \mathcal{D}_A \bar{V}_{B\bar{p}} = \nabla_A \bar{V}_{B\bar{p}} + \bar{\Phi}_{A\bar{p}}{}^{\bar{q}} \bar{V}_{B\bar{q}} = 0 \,.$$

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- *Completely covariant derivatives & curvatures*
  - Tensors,

$$P_C{}^D \bar{P}_{A_1}{}^{B_1} \cdots \bar{P}_{A_n}{}^{B_n} \nabla_D T_{B_1 \cdots B_n} \quad \Longrightarrow \quad \mathcal{D}_p T_{\bar{q}_1 \bar{q}_2 \cdots \bar{q}_n} \,,$$
$$\bar{P}_C{}^D P_{A_1}{}^{B_1} \cdots P_{A_n}{}^{B_n} \nabla_D T_{B_1 \cdots B_n} \quad \Longrightarrow \quad \mathcal{D}_{\bar{p}} T_{q_1 q_2 \cdots q_n} \,,$$

$$\mathcal{D}^p T_{p \bar{q}_1 \bar{q}_2 \cdots \bar{q}_n} \,, \qquad \mathcal{D}^{\bar{p}} T_{\bar{p} q_1 q_2 \cdots q_n} \,; \qquad \mathcal{D}_p \mathcal{D}^p T_{\bar{q}_1 \bar{q}_2 \cdots \bar{q}_n} \,, \qquad \mathcal{D}_{\bar{p}} \mathcal{D}^{\bar{p}} T_{q_1 q_2 \cdots q_n} \,.$$

  - Fermions, ρ^α, $\rho'^{\bar{\alpha}}$, $\psi^\alpha_{\bar{p}}$, $\psi'^{\bar{\alpha}}_p$,

$$\gamma^p \mathcal{D}_p \rho \,, \quad \bar{\gamma}^{\bar{p}} \mathcal{D}_{\bar{p}} \rho' \,, \quad \mathcal{D}_{\bar{p}} \rho \,, \quad \mathcal{D}_p \rho' \,, \quad \gamma^p \mathcal{D}_p \psi_{\bar{q}} \,, \quad \bar{\gamma}^{\bar{p}} \mathcal{D}_{\bar{p}} \psi'_q \,, \quad \mathcal{D}_{\bar{p}} \psi^{\bar{p}} \,, \quad \mathcal{D}_p \psi'^p \,.$$

  - RR sector, $\mathcal{C}^\alpha{}_{\bar{\alpha}}$ *$\mathbf{O}(D,D)$ covariant extension of H-twisted cohomology*

$$\mathcal{D}_\pm \mathcal{C} := \gamma^p \mathcal{D}_p \mathcal{C} \pm \gamma^{(D+1)} \mathcal{D}_{\bar{p}} \mathcal{C} \bar{\gamma}^{\bar{p}} \,, \quad (\mathcal{D}_\pm)^2 = 0 \quad \Longrightarrow \quad \mathcal{F} := \mathcal{D}_+ \mathcal{C} \quad (\text{RR flux}) \,.$$

  - Curvatures,

$$P_A{}^C \bar{P}_B{}^D S_{CD} \quad (\text{Ricci-like}) \,, \qquad (P^{AC} P^{BD} - \bar{P}^{AC} \bar{P}^{BD}) S_{ABCD} \quad (\text{scalar}) \,,$$

from which conserved Einstein-like curvature can be also constructed,

$$G_{AB} := 2(P_{AC} \bar{P}_{BD} - \bar{P}_{AC} P_{BD}) S^{CD} - \tfrac{1}{2} \mathcal{J}_{AB} (S_{pq}{}^{pq} - S_{\bar{p}\bar{q}}{}^{\bar{p}\bar{q}}) \,, \qquad \nabla_A G^{AB} = 0 \,.$$

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- Type II $D = 10$ *Maximally Supersymmetric Double Field Theory* Jeon-Lee-JHP-Suh 2012

$$\mathcal{L}_{\mathrm{Max}} = e^{-2d}\Big[\tfrac{1}{8}(P^{AB}P^{CD} - \bar{P}^{AB}\bar{P}^{CD})S_{ACBD} + \tfrac{1}{2}\mathrm{Tr}(\mathcal{F}\bar{\mathcal{F}}) + i\bar{\rho}\mathcal{F}\rho' + i\bar{\psi}_{\bar{p}}\gamma_q\mathcal{F}\bar{\gamma}^{\bar{p}}\psi'^q$$
$$+ i\tfrac{1}{2}\bar{\rho}\gamma^p\mathcal{D}_p\rho - i\bar{\psi}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho - i\tfrac{1}{2}\bar{\psi}^{\bar{p}}\gamma^q\mathcal{D}_q\psi_{\bar{p}} - i\tfrac{1}{2}\bar{\rho}'\bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho' + i\bar{\psi}'^p\mathcal{D}_p\rho' + i\tfrac{1}{2}\bar{\psi}'^p\bar{\gamma}^{\bar{q}}\mathcal{D}_{\bar{q}}\psi'_p\Big]$$

  - Due to the twofold spin groups, $\mathbf{Spin}(1,9)_L \times \mathbf{Spin}(9,1)_R$, the theory unifies the conventional IIA and IIB SUGRAs. Namely the theory is chiral w.r.t. both spin groups and hence unique. IIA and IIB appear as two distinct types of solutions.

    Further, it admits non-Riemannian solutions (type IIC) where the Riemannian metric, $g_{\mu\nu}$, is not defined.

  - Maximal $16+16$ SUSY [*full order, i.e. quartic, construction realizing '1.5 formalism'*],

$$\delta\rho = -\gamma^p\mathcal{D}_p\varepsilon\,, \quad \delta\rho' = -\bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\varepsilon'\,, \quad \delta\psi_{\bar{p}} = \mathcal{D}_{\bar{p}}\varepsilon + \mathcal{F}\bar{\gamma}_{\bar{p}}\varepsilon'\,, \quad \delta\psi'_p = \mathcal{D}'_p\varepsilon' + \bar{\mathcal{F}}\gamma_p\varepsilon\,.$$

  - Euler-Lagrange equations include the 'stringy' Einstein equation:

$$\underbrace{S_{p\bar{q}}}_{\text{curvature}} = \underbrace{\mathrm{Tr}(\gamma_p\mathcal{F}\bar{\gamma}_{\bar{q}}\bar{\mathcal{F}}) + \text{fermions}}_{\text{matters}}\,,$$

c.f. Coimbra-Strickland-Constable-Waldram, Hohm-Kwak-Zwiebach

- *Stringy geometry: Riemannian IIA/IIB vs. non-Riemannian IIC*

With $\tilde{\partial}^{\mu} \equiv 0$, the $\mathbf{O}(D,D)$ covariant vielbeins and dilaton can be generically parametrized by a pair of ordinary vierbeins, $e_{\mu}{}^{p}$, $\bar{e}_{\mu}{}^{\bar{p}}$, B-field and string dilaton, ϕ:

$$V_{Mp} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} (e^{-1})_{p}{}^{\mu} \\ (B+e)_{\nu p} \end{pmatrix}, \qquad \bar{V}_{M\bar{p}} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} (\bar{e}^{-1})_{\bar{p}}{}^{\mu} \\ (B+\bar{e})_{\nu\bar{p}} \end{pmatrix}, \qquad e^{-2d} \equiv \sqrt{|g|}e^{-2\phi},$$

where the two ordinary vierbeins must correspond to the same Riemannian metric,

$$e_{\mu}{}^{p} e_{\nu}{}^{q} \eta_{pq} = -\bar{e}_{\mu}{}^{\bar{p}} \bar{e}_{\nu}{}^{\bar{q}} \bar{\eta}_{\bar{p}\bar{q}} \equiv g_{\mu\nu},$$

such that the DFT-metric assumes the well-known form,

$$\mathcal{H}_{MN} := P_{AB} - \bar{P}_{AB} = V_{Ap} V_{B}{}^{p} - \bar{V}_{A\bar{p}} \bar{V}_{B}{}^{\bar{p}} \equiv \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}.$$

It follows that $(e^{-1}\bar{e})_{p}{}^{\bar{p}}$ is a Lorentz rotation, and hence, Jeon-Lee-JHP-Suh 2012

$$\det(e^{-1}\bar{e}) = +1 \ : \ \textbf{type IIA} \qquad \textbf{vs.} \qquad \det(e^{-1}\bar{e}) = -1 \ : \ \textbf{type IIB}$$

Diagonal gauge fixing, $e_{\mu}{}^{p} \equiv \bar{e}_{\mu}{}^{\bar{p}}$, leads to the conventional SUGRA.

However, the above is not the most general parametrization.

- Stringy Gravity encompasses novel geometries, namely **type IIC**, which do not allow any Riemannian interpretation, *e.g.* $\mathcal{H}_{MN} = \mathcal{J}_{MN}$. Lee-JHP 2013, JHP 2016
- A less simple example of non-Riemannian geometry realizes the Gomis-Ooguri 'non-relativistic' string theory. Ko-Melby-Thompson-Meyer-JHP 2015

- ## Stringy geometry: Riemannian IIA/IIB vs. non-Riemannian IIC

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- *Yang-Mills* Jeon-Lee-JHP 2011

  - Completely covariant Yang-Mills field strength is given by

    $$P_A{}^M \bar{P}_B{}^N \mathcal{F}_{MN}$$

    where $\mathcal{F}_{MN}$ is the semi-covariant field strength of a YM potential, $\mathcal{V}_M$,

    $$\mathcal{F}_{MN} := \nabla_M \mathcal{V}_N - \nabla_N \mathcal{V}_M - i\,[\mathcal{V}_M, \mathcal{V}_N]\ .$$

  - It is fully covariant w.r.t. all the symmetries of Stringy Gravity and the YM gauge symmetry,

    $$\mathcal{V}_M \quad \longrightarrow \quad \mathbf{g}\mathcal{V}_M\mathbf{g}^{-1} - i(\partial_M\mathbf{g})\mathbf{g}^{-1}\ .$$

  - We can freely impose $\mathbf{O}(D,D)$ & YM gauge covariant conditions on the potential:

    $$\mathcal{V}_M\mathcal{V}^M = 0\,, \qquad\qquad \mathcal{V}^M\partial_M = 0\,,$$

    in order *not* to double the physical degrees.

- *Coupling to the Standard Model* Choi-JHP 2015 [PRL]

$D=4$ **Stringy Gravity naturally, or minimally, couples to the Standard Model, dictated by the symmetry principle:**

- $\mathbf{O}(4,4)$ T-duality
- Twofold local Lorentz symmetry, $\mathbf{Spin}(1,3)_L \times \mathbf{Spin}(3,1)_R$
- Doubled-yet-gauged diffeomorphisms
- $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ gauge symmetry

$$\mathcal{L}_{\mathrm{SM-DFT}} = e^{-2d}\left[\begin{array}{l} \frac{1}{16\pi G_N}(P^{AB}P^{CD} - \bar{P}^{AB}\bar{P}^{CD})S_{ACBD} \\ +\sum_{\mathcal{V}} P^{AB}\bar{P}^{CD}\mathrm{Tr}(\mathcal{F}_{AC}\mathcal{F}_{BD}) + \sum_{\psi}\bar{\psi}\gamma^{a}\mathcal{D}_{a}\psi + \sum_{\psi'}\bar{\psi}'\bar{\gamma}^{\bar{a}}\mathcal{D}_{\bar{a}}\psi' \\ -\mathcal{H}^{AB}(\mathcal{D}_A\phi)^{\dagger}\mathcal{D}_B\phi - V(\phi) + y_d\,\bar{q}\cdot\phi\,d + y_u\,\bar{q}\cdot\tilde{\phi}\,u + y_e\,\bar{l}'\cdot\phi\,e' \end{array}\right]$$

While coupling to SM, one has to decide the spin group for each fermion, as it a prediction of Stringy Gravity that **the spin group is intrinsically twofold:**

$$\mathbf{Spin}(1,3)_L \quad \textbf{vs.} \quad \mathbf{Spin}(3,1)_R\,.$$

Conjecture: **the quarks and the leptons may belong to the distinct spin groups.**

- *Problems with* $\mathrm{d}x^M$

In doubled-yet-gauged spacetime, the usual infinitesimal one-form, $\mathrm{d}x^M$, is neither covariant under diffeomorphisms,

$$\delta x^M = \xi^M \,, \qquad\qquad \delta(\mathrm{d}x^M) = \mathrm{d}x^N\partial_N\xi^M \;\neq\; (\partial_N\xi^M - \partial^M\xi_N)\mathrm{d}x^N \,,$$

nor invariant under coordinate gauge symmetry,

$$\mathrm{d}x^M \quad\longrightarrow\quad \mathrm{d}\big(x^M + \Phi_1\partial^M\Phi_2\big) \;\neq\; \mathrm{d}x^M \,.$$

$\Rightarrow$ The naive contraction with the DFT-metric, $\mathrm{d}x^M\mathrm{d}x^N\mathcal{H}_{MN}$, is not a scalar, and thus cannot give any sensible definition of 'proper length' in doubled-yet-gauged spacetime.

- *Gauged infinitesimal one-form, Dx^M*

The problems can be all cured by gauging the infinitesimal one-form,

$$Dx^M := \mathrm{d}x^M - \mathcal{A}^M\,.$$

The gauge potential should satisfy the same property as the coordinate gauge symmetry generator, *i.e.* derivative-index-valued vector, $\Delta^M = \Phi_1\partial^M\Phi_2$, such that

$$\mathcal{A}^M\partial_M = 0\,, \qquad \mathcal{A}_M\mathcal{A}^M = 0\,.$$

Essentially, half of the components are trivial, *e.g.* with $\tilde{\partial}_\mu \equiv 0$,

$$\mathcal{A}^M = A_\lambda\partial^M x^\lambda = (A_\mu\,,\,0)\;, \qquad Dx^M = (\mathrm{d}\tilde{x}_\mu - A_\mu\,,\,\mathrm{d}x^\nu)\;.$$

With the appropriate transformations of the gauge potential, the coordinate gauge symmetry invariance and the diffeomorphism covariance can be assured for Dx^M,

$$\begin{array}{lll} \delta_{\mathrm{C.G.}}\,x^M = \Phi_1\partial^M\Phi_2\,, & \delta_{\mathrm{C.G.}}\,\mathcal{A}^M = \mathrm{d}\left(\Phi_1\partial^M\Phi_2\right)\,, & \delta_{\mathrm{C.G.}}\,(Dx^M) = 0\,;\\ \delta x^M = \xi^M\,, & \delta\mathcal{A}^M = \partial^M\xi_N(\mathrm{d}x^N - \mathcal{A}^N)\,, & \delta(Dx^M) = (\partial_N\xi^M - \partial^M\xi_N)Dx^N\,. \end{array}$$

- *Proper length in doubled-yet-gauged spacetime*

With the gauged infinitesimal one-form, $Dx^M = \mathrm{d}x^M - \mathcal{A}^M$, we propose to define the **proper length through a path integral,**

$$\textbf{Length} := -\ln\left[\int \mathcal{D}\mathcal{A}\,\exp\left(-\int\sqrt{Dx^M Dx^N \mathcal{H}_{MN}}\right)\right].$$

For Riemannian DFT-metric, we get a useful relation,

$$Dx^M Dx^N \mathcal{H}_{MN} \equiv \mathrm{d}x^\mu \mathrm{d}x^\nu g_{\mu\nu} + (\mathrm{d}\tilde{x}_\mu - A_\mu + \mathrm{d}x^\rho B_{\rho\mu})(\mathrm{d}\tilde{x}_\nu - A_\nu + \mathrm{d}x^\sigma B_{\sigma\nu})\, g^{\mu\nu}.$$

Hence, after integrating out the gauge potential, A_μ, the above $\mathbf{O}(D,D)$ covariant path integral definition of the proper length reduces to the conventional one,

$$\text{Length} \implies \int\sqrt{\mathrm{d}x^\mu \mathrm{d}x^\nu g_{\mu\nu}}\,.$$

Apparently, being $\tilde{x}_\mu$-independent, it measures the distance between two gauge orbits rather than two points in $\mathbb{R}^{D+D}$, which is of course a desired feature.

- *Doubled-yet-gauged point particle action* Ko-JHP-Suh 2016

The definition of the proper length readily gives

$$\mathcal{S}_{\rm particle} = \int {\rm d}\tau \left[e^{-1} D_\tau X^M D_\tau X^N \mathcal{H}_{MN}(X) - \tfrac{1}{4} m^2 e \right],$$

where e is an einbein and m is the mass of the particle.

With the Riemannian DFT-metric substituted, after integrating out e and $\mathcal{A}^M$, the above action reduces to the conventional one in string frame:

$$S_{\rm particle} \equiv \int {\rm d}\tau \;\; -m\sqrt{-\dot{X}^\mu \dot{X}^\nu g_{\mu\nu}}\,.$$

This implies that the particle follows the geodesic path defined in the string frame.

This preferred choice of the frame, *i.e.* String frame over Einstein frame, is due to the Symmetry Principle, such as $\mathbf{O}(D,D)$, coordinate gauge symmetry, *etc.*

\# Newton mechanics can be also formulated in the doubled-yet-gauged Euclidean space,

$$\mathcal{L}_{\rm Newton} = \tfrac{1}{2} m \, D_t X^M D_t X^N \, \delta_{MN} - V(X)\,,$$

where $M, N = 1, 2, \cdots, 6$, and the potential, $V(X)$, satisfies the section condition.

- *Doubled-yet-gauged string action* Lee-JHP 2013 [*c.f.* Hull 2006]

$$\frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma\ \mathcal{L}_{\rm string}\,, \qquad \mathcal{L}_{\rm string} = -\tfrac{1}{2}\sqrt{-h}\,h^{ij}D_iX^MD_jX^N\mathcal{H}_{MN}(X) - \epsilon^{ij}D_iX^M\mathcal{A}_{jM}\,,$$

where $i,j=0,1$ and $D_iX^M = \partial_iX^M - \mathcal{A}_i^M$.

For an arbitrary curved DFT-metric, $\mathcal{H}_{MN}(X)$, the above action is *fully symmetric*:

- worldsheet diffeomorphisms plus Weyl symmetry
- $\mathbf{O}(D,D)$ T-duality
- Douled-yet-gauged target spacetime diffeomorphisms
- the coordinate gauge symmetry : $X^M \sim X^M + \Phi_1\partial^M\Phi_2$

- With the Riemannian DFT-metric, after integrating out $\mathcal{A}^M$, the doubled-yet-gauged string action reduces to the conventional one,

$$\frac{1}{4\pi\alpha'}\mathcal{L}_{\text{string}} \equiv \frac{1}{2\pi\alpha'}\left[-\tfrac{1}{2}\sqrt{-h}h^{ij}\partial_i X^\mu \partial_j X^\nu g_{\mu\nu}(X) + \tfrac{1}{2}\epsilon^{ij}\partial_i X^\mu \partial_j X^\nu B_{\mu\nu}(X) + \tfrac{1}{2}\epsilon^{ij}\partial_i \tilde{X}_\mu \partial_j X^\mu\right],$$

plus the bonus of the topological term introduced by Giveon-Rocek; Hull.

  - The EOM of $\mathcal{A}_i^M$ implies **self-duality** in the full doubled spacetime,

$$\mathcal{H}^M{}_N D^i X^N + \frac{1}{\sqrt{-h}}\epsilon^{ij} D_j X^M = 0\,,$$

    which relates X^μ and $\tilde{X}_\mu$.

  - The EOM of X^M is identified as the **Stringy Geodesic Equation:**

$$\frac{1}{\sqrt{-h}}\partial_i\left(\sqrt{-h}\,\mathcal{H}_{LM} D^i X^M\right) + \Gamma_{LMN}\,(\bar{P}^M{}_A D_i X^A)(P^N{}_B D^i X^B) = 0\,.$$

  - With the conjugate momentum, $P_M = \frac{\delta \mathcal{L}_{\text{string}}}{\delta\,\partial_0 X^M}$,

$$P_M P^M = 0 \quad (\text{level matching})\,, \qquad\qquad P_M P_N \mathcal{H}^{MN} = 0 \quad (\text{mass shell})\,.$$

- On the other hand, upon non-Riemannian backbrounds, the doubled-yet-gauged string action leads to chiral or non-Relativistic string theory *a la* Siegel or Gomis-Ooguri.

Lee-JHP 2013, Ko-Melby-Thompson-Meyer-JHP 2015

- *Doubled-yet-gagued Green-Schwarz superstring* **JHP 1609.04265**

$$\mathcal{S}_{\rm superstring} = \frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma\ \mathcal{L}_{\rm superstring}\,,$$

$$\mathcal{L}_{\rm superstring} = -\tfrac{1}{2}\sqrt{-h}h^{ij}\Pi_i^M\Pi_j^N\mathcal{H}_{MN} - \epsilon^{ij}D_iX^M\left(\mathcal{A}_{jM} - i\Sigma_{jM}\right)\,,$$

where

$$\Pi_i^M := D_iX^M - i\Sigma_i^M\,, \qquad \Sigma_i^M := \bar{\theta}\gamma^M\partial_i\theta + \bar{\theta}'\bar{\gamma}^M\partial_i\theta'\,,$$

and θ^α and $\theta'^{\bar{\alpha}}$ are respectively $\mathbf{Spin}(1,9)_L$ and $\mathbf{Spin}(9,1)_R$ Majorana-Weyl spinors.

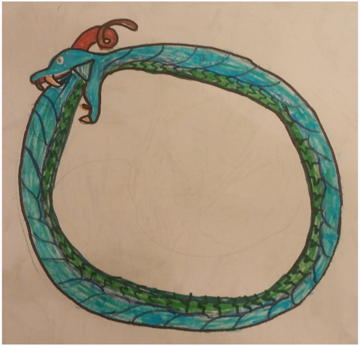
It enjoys symmetries:

- worldsheet diffeomorphisms plus Weyl symmetry
- $\mathbf{O}(D,D)$ T-duality
- Doubled-yet-gauged target spacetime diffeomorphisms
- coordinate gauge symmetry: $X^M \sim X^M + \Phi_1\partial^M\Phi_2$
- **twofold Lorentz symmetry, $\mathbf{Spin}(1,9)_L \times \mathbf{Spin}(9,1)_R$** $\Rightarrow$ **Unification of IIA & IIB**
- **Maximal 16+16 SUSY & kappa symmetries**

Further, it encompasses type IIC, or the supersymmetric extension of the Gomis-Ooguri non-relativistic string.

$*$ **String theory is better formulated on doubled-yet-gauged spacetime.**

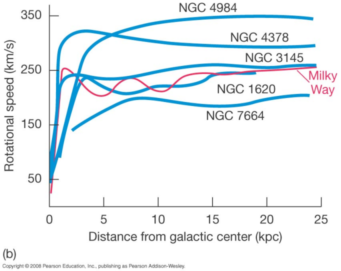
Gravitational Implication

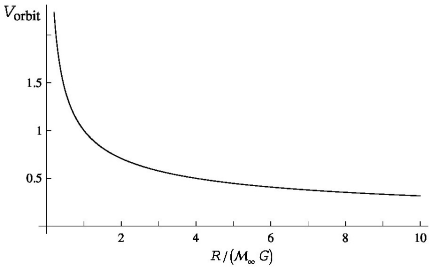


'Uroboros': an ancient Egyptian symbol for a serpent which eats its own tail.

"The rotation curve of a point particle in stringy gravity"
Ko-JHP-Suh 1606.09307 [JCAP]

Dark Matter Problem





Galaxy rotation curves : observation

Keplerian $\sqrt{MG/R}$ fall-off : GR or Newton

- The galaxy rotation curve is a plot of the orbital velocities of visible stars versus their radial distance from the galactic center.
- While Einstein gravity (GR), with Schwarzschild solution, predicts the Keplerian (inverse square root) monotonic fall-off of the velocities, $V = \sqrt{1/\mathbf{x}}$, $\mathbf{x} = R/MG$, observations however show rather 'flat' (~ 200 km/s) curves after a fairly rapid rise.
- The resolution of the discrepancy may call for 'dark matter', or modifications of the law of gravity, or perhaps both as is the case with Stringy Gravity.
- I will argue that **Stringy Gravity modifies GR at 'short' distance and may solve the dark matter and energy problems in 'Uroboros' manner.**

- *Darkness of Stringy Gravity*

i) Point-like particles couple to the string metric only,

$$\int \mathrm{d}\tau \left[e^{-1} D_\tau X^M D_\tau X^N \mathcal{H}_{MN}(X) - \tfrac{1}{4} m^2 e \right] \quad \Longrightarrow \quad \int \mathrm{d}\tau \; - m\sqrt{-\dot{X}^\mu \dot{X}^\nu g_{\mu\nu}}\,.$$

Hence, the string dilaton, ϕ, and B-field are *dark* to point particles.

ii) Each SM fermion couples to Stringy Gravity as

$$\begin{aligned} e^{-2d}\,\bar{\psi}\gamma^A \mathcal{D}_A \psi &= e^{-2d}\,\bar{\psi}\gamma^p V^A{}_p(\partial_A\psi + \tfrac{1}{4}\Phi_{Apq}\gamma^{pq}\psi) \\ &\equiv \tfrac{1}{\sqrt{2}}\sqrt{-g}\,\bar{\chi}\gamma^\mu \left(\partial_\mu\chi + \tfrac{1}{4}\omega_{\mu pq}\gamma^{pq}\chi + \tfrac{1}{24}H_{\mu pq}\gamma^{pq}\chi\right) \end{aligned}$$

c.f. Coimbra-Strickland-Constable-Waldram

where $\chi \equiv e^{-\phi}\psi$. This field redefinition removes the string dilaton, ϕ, completely.

- The string dilaton, ϕ, is *dark* to the SM fermions, χ ;
- Like F1, χ can source the H-flux, and seems to remember it stringy origin!

iii) Each SM gauge boson couples to Stringy Gravity as

$$e^{-2d}\,\mathrm{Tr}\left(P^{AB}\bar{P}^{CD}\mathcal{F}_{AC}\mathcal{F}_{BD}\right) \equiv -\tfrac{1}{4}\sqrt{-g}e^{-2\phi}\,\mathrm{Tr}\left(g^{\kappa\lambda}g^{\mu\nu}F_{\kappa\mu}F_{\lambda\nu}\right)$$

- B-field, or 'axion' (dual scalar), is *dark* to the gauge bosons;
- Standard Model gauge bosons can source the string dilaton, ϕ.

- *Spherical symmetry in doubled-yet-gauged spacetime*

While ϕ and B-field are 'dark' to point particles, the self-interaction of the massless closed string sector, together with its coupling to the Standard Model, should let Stringy Gravity modify General Relativity.

This motivates us to look for spherically symmetric vacua of Stringy Gravity, especially $D = 4$.

  - Spherical solutions should admit three Killing vectors, V_a^A, $a = 1, 2, 3$,

$$\hat{\mathcal{L}}_{V_a}\mathcal{H}_{MN} = 0 \quad\Longleftrightarrow\quad (P\nabla)_M(\bar{P}V_a)_N - (\bar{P}\nabla)_N(PV_a)_M = 0$$

$$\hat{\mathcal{L}}_{V_a}(e^{-2d}) = 0 \quad\Longleftrightarrow\quad \nabla_M V_a^M = 0$$

which form an $\mathbf{so}(3)$ algebra in terms of the $\mathbf{C}$-bracket,

$$[V_a, V_b]_{\mathbf{C}} = \sum_c \epsilon_{abc} V_c \,.$$

JHP-Rey-Rim-Sakatani 2015

- *Asymptotically flat spherical vacuum:*

$$e^{2\phi} = \gamma_+ \left(\frac{r-\alpha}{r+\beta}\right)^{\frac{b}{\sqrt{a^2+b^2}}} + \gamma_- \left(\frac{r-\alpha}{r+\beta}\right)^{\frac{-b}{\sqrt{a^2+b^2}}}, \qquad B_{(2)} = h\cos\vartheta\, \mathrm{d}t \wedge \mathrm{d}\varphi\,,$$

$$\mathrm{d}s^2 = e^{2\phi}\left[-\left(\frac{r-\alpha}{r+\beta}\right)^{\frac{a}{\sqrt{a^2+b^2}}} \mathrm{d}t^2 + \left(\frac{r-\alpha}{r+\beta}\right)^{\frac{-a}{\sqrt{a^2+b^2}}} \left(\mathrm{d}r^2 + (r-\alpha)(r+\beta)\mathrm{d}\Omega^2\right)\right],$$

where a, b, h ($h^2 \leq b^2$) are three free parameters and

$$\alpha = \frac{a}{a+b}\sqrt{a^2+b^2}\,, \qquad \beta = \frac{b}{a+b}\sqrt{a^2+b^2}\,, \qquad \gamma_\pm = \frac{1}{2}\left(1 \pm \sqrt{1-h^2/b^2}\right).$$

In particular, the special case of $b = h = 0$ corresponds to the Schwarzschild geometry.

  - This is a rederivation of the solution by Burgess-Myers-Quevedo (1994) who generated the above solution by applying S-duality to the scalar-gravity solution of Fischer (1948), Janis-Newman-Winicour (1968). It solves the familiar action,

$$\int \mathrm{d}^4x\, \sqrt{-|g|}\, e^{-2\phi}\left(R + 4\,|\mathrm{d}\phi|^2 - \tfrac{1}{12}\,|\mathrm{d}B|^2\right).$$

  - Equivalently, it solves the EOMs of $D = 4$ DFT (*i.e.* pure Stringy Gravity):

$$(P^{AB}P^{CD} - \bar{P}^{AB}\bar{P}^{CD})S_{ACBD} \equiv 0\,, \qquad P_A{}^C\bar{P}_B{}^D S_{CD} \equiv 0\,.$$

  Thus, within the framework of Stringy Gravity, it should be identified as **the vacuum solution**, in analogy with the Schwarzschild solution in GR.

  Further, although it would be naked-singular from GR point of view, within Stringy Gravity it can be regular: no $\mathbf{O}(4,4)$ covariant curvature diverges.

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- *Orbital velocity*

Given the exact spherical solution, we define 'proper' radius, $R := \sqrt{g_{\vartheta\vartheta}(r)}$, which converts the string metric into a canonical form,

$$\mathrm{d}s^2 = g_{tt}\mathrm{d}t^2 + g_{RR}\mathrm{d}R^2 + R^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\,\mathrm{d}\varphi^2\right).$$

We then compute the 'orbital velocity' of circular geodesics,

$$V_{\mathrm{orbit}} = \left|R\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right| = \left[-\tfrac{1}{2}R\frac{\mathrm{d}g_{tt}}{\mathrm{d}R}\right]^{\frac{1}{2}},$$

as a function of $R/(M_\infty G)$ which is a dimensionless radial variable normalized by 'asymptotic' mass (Komar mass[1]),

$$M_\infty G := \lim_{R\to\infty}(RV^2_{\mathrm{orbit}}) = \tfrac{1}{2}\left(a + b\sqrt{1 - h^2/b^2}\right).$$

⋆ **Stringy Gravity reduces to Newton Gravity at spatial infinity,**

$$g_{tt} \rightarrow -1 + \frac{2M_\infty G}{R}, \qquad V_{\mathrm{orbit}} \rightarrow \sqrt{\frac{M_\infty G}{R}} \qquad \textbf{as} \quad R \rightarrow \infty.$$

⋆ Yet, **Stringy Gravity modifies GR at 'short' distance, in terms of $R/(M_\infty G)$.** Generically ($b \neq 0$), the orbital velocity is not monotonic: it features a maximum.

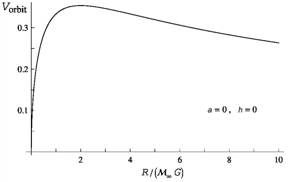
[1] *c.f.* ADM mass *a la* Wald, $\mathcal{Q}[\partial_t] = \frac{1}{4}\left[a + \left(\frac{a-b}{a+b}\right)\sqrt{a^2+b^2}\right]$ JHP-Rey-Rim-Sakatani, Blair 2015

- *Rotation curves*

– Specifically, if $b=0$ (and hence $h=0$), the solution reduces to the Schwarzschild metric, resulting in the Keplerian orbital velocity, $V_{\rm orbit}=\sqrt{\frac{M_\infty G}{R}}$.

– As long as $b\neq 0$, **rotation curves feature a maximum** and thus non-Keplerian over a finite range, while becoming asymptotically Keplerian at infinity.

For example, if $a=h=0$ and $b=2M_\infty G$, we reproduce the renowned orbital velocity formula, $V_{\rm orbit}=\sqrt{\frac{M_\infty R}{(R+2M_\infty G)^2}}$, by Hernquist :

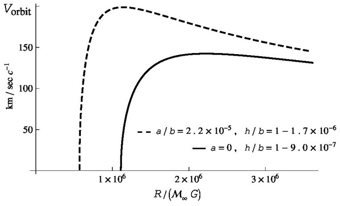


The orbital velocity in Hernquist model assumes its maximum, $\frac{1}{2\sqrt{2}}$, about 35% of the speed of light, at $R=2M_\infty G$.

However, this value seems too high compared to observations of galaxies.

– **More interesting cases turn out to include nontrivial H-flux ($h\neq 0$ and hence $b\neq 0$).**

– By tuning the variable, it is possible to make the maximal velocity arbitrarily small, such as about $150\,\mathrm{km/s}\,c^{-1}$, comparable to observations:



Rotation curves in Stringy Gravity (dimensionless, nonexhaustive).

For sufficiently small $R/(M_\infty G)$, the gravitational force can be *repulsive*.

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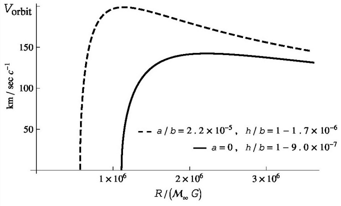

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- *Uroboros spectrum of $R/(M_\infty G)$*

	Electron ($R\simeq 0$)	Proton	Hydrogen Atom	Billiard Ball	Earth	Solar System ($1\mathrm{AU}/M_\odot G$)	Milky Way (visible)	Universe ($M_\infty\propto R^3$)
$R/(M_\infty G)$	0^+	7.1×10^{38}	2.0×10^{43}	2.4×10^{26}	1.4×10^{9}	1.0×10^{8}	1.5×10^{6}	0^+

'Uroboros' spectrum of the dimensionless radial variable normalized by mass in natural units. The orbital speed is also dimensionless, and depends on the single variable, $R/(M_\infty G)$.

– The observations of stars and galaxies far away, or the dark matter and the dark energy problems, are revealing the short-distance nature of gravity!

– The repulsive gravitational force at very short-distance, $R/(M_\infty G)\to 0^+$, may be responsible for the acceleration of the Universe.

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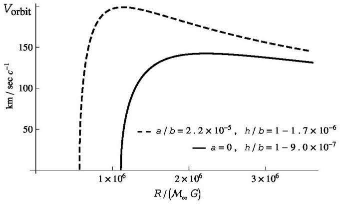

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Thank you.

This talk summarizes works in collaborations with Imtak Jeon, Kanghoon Lee, Soo-Jong Rey, Yuho Sakatani, Yoonji Suh, Wonyoung Cho, Jose Fernández-Melgarejo, Woohyun Rim, Sung Moon Ko, Charles Melby-Thompson, Rene Meyér, Minwoo Suh, Kang-Sin Choi, Chris Blair, Emanuel Malek and Xavier Bekaert.

- *Differential geometry with a projection: Application to double field theory* — 1011.1324 JHEP
- **Stringy differential geometry, beyond Riemann** — **1105.6294 PRD**
- *Incorporation of fermions into double field theory* — 1109.2035 JHEP
- *Ramond-Ramond Cohomology and O(D,D) T-duality* — 1206.3478 JHEP
- *Supersymmetric Double Field Theory: Stringy Reformulation of Supergravity* — 1112.0069 PRD
- **Stringy Unification of IIA and IIB Supergravities under $\mathcal{N}=2$ $D=10$ Supersymmetric Double Field Theory** — **1210.5078 PLB**
- *Supersymmetric gauged Double Field Theory: Systematic derivation by virtue of 'Twist'* — 1505.01301 JHEP
- **Comments on double field theory and diffeomorphisms** — **1304.5946 JHEP**
- *Covariant action for a string in doubled yet gauged spacetime* — 1307.8377 NPB
- **Green-Schwarz superstring on doubled-yet-gauged spacetime** — **1609.04265 JHEP**
- *Double field formulation of Yang-Mills theory* — 1102.0419 PLB
- **Standard Model as a Double Field Theory** — **1506.05277 PRL**
- **The rotation curve of a point particle in stringy gravity** — **1606.09307 JCAP**
- **O**(D,D) *Covariant Noether Currents and Global Charges in Double Field Theory* — 1507.07545 JHEP
- *Dynamics of Perturbations in Double Field Theory & Non-Relativistic String Theory* — 1508.01121 JHEP
- *Higher Spin Double Field Theory: A Proposal* — 1605.00403 JHEP
- *U-geometry: SL(5)* $\Rightarrow$ **U-gravity: SL(N)** — 1302.1652 JHEP/**1402.5027 JHEP**
- *M-theory and Type IIB from a Duality Manifest Action* — 1311.5109 JHEP