Precision Holography with Wilson Loops

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1706.xxxx W. Muck, L. Pando Zayas, V. Rathee

Motivation

- Localization: A plethora of exact results for supersymmetric field theories.
- AdS/CFT beyond the leading order:

$$Z_{Field\ Theory}(\phi_0) = Z_{String} \approx \exp\left(-S(\phi \to \phi_0)_{Grav}\right).$$

- What can we learn about string perturbation theory given the "experimental data" provided by localization?
- Is our semiclassical intuition and its toolbox correct?

Outline

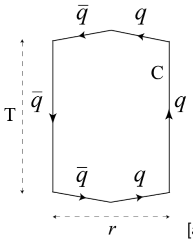
- Introduction to half BPS Wilson loops in $\mathcal{N} = 4$ SYM
- Reviving an old puzzle: The WL in the fundamental representation beyond the leading order
- Precision test with $1/4$ BPS Wilson Loops
- ABJM Wilson Loops beyond the leading order
- Higher dimensional representations in ABJM (D6 brane)
- Logarithmic correction to D6-brane Wilson loop
- Conclusions and Open Directions

Introduction

- Wilson Loops (WL) are an important class of gauge invariant non-local operators.

$$W_R\,(\mathrm{C}) = tr_R\,\mathcal{P}\,\exp\,\oint_C \left(A_\mu\,dx^\mu\right)$$

- The expectation value measures the effective action of an external particle; order parameter for confinement, $V_{q\bar{q}} \propto r$.



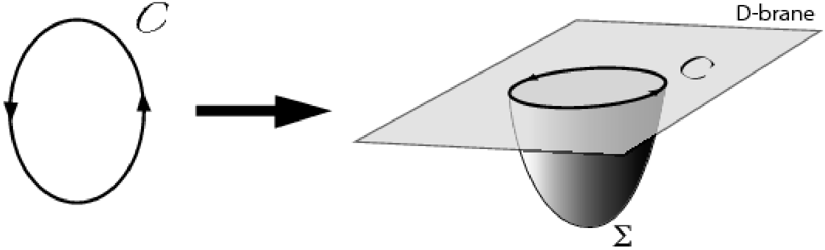
[arxiv:1412.8008]

- Wilson loop in Gauge/Gravity Correspondence
- The contour becomes a surface in higher dimensions.
- Expectation value:

$$\langle W(\mathcal{C})\rangle = Z_{\text{string}}\left(\partial\Sigma = \mathcal{C}\right) \tag{1}$$

- Right regime

$$Z_{\text{string}}\left(\partial\Sigma = \mathcal{C}\right) = e^{-S(\mathcal{C})} \tag{2}$$



[arxiv:1310.4319]

Half BPS Wilson Loops in $\mathcal{N} = 4$ SYM

- Coupling of an external probe to the multiplet $\left(A_\mu, \phi^I, \lambda^A_\alpha\right)$:
  - Curve C in superspace, parameterized by $\left(x^\mu(s), y^I(s), \theta^\alpha_A(s)\right)$,
  - Representation R of $SU(N)$, corresponding to the charge of the particle.
- We consider only bosonic operators with $\theta^\alpha_A = 0$:

$$W_R(C) = \mathrm{Tr}_R P \exp\left(i \int_C ds \left(A_\mu \dot{x}^\mu + \phi_I \dot{y}^I\right)\right)$$

- SUSY forces C to be a timelike straight line or a circle.

$$W_R = \mathrm{Tr}_R P \exp\left(i \int dt \left(A_0 + \phi_1\right)\right)$$

Localization: Pestun

- Want $\int \exp(S)$:

$$
\begin{aligned}
\int \exp(S) \quad &\longrightarrow \quad \int \exp\left(S + tQV\right) \\
\frac{d}{dt}\int \exp\left(S + tQV\right) \quad &= \quad 0
\end{aligned} \tag{3}
$$

- Independence of t, take $t \to \infty$ $\quad \longrightarrow \quad$ Classical plus one-loop.
- $\mathcal{N} = 4$ SYM localizes on a Gaussian action.

Matrix Model: Fundamental

- The Matrix Model computation gives the exact answer, for any N and λ, in terms of Laguerre polynomial,

$$
\begin{aligned}
\langle W_{\Box} \rangle_{\text{circle}} &= \frac{1}{N} L^1_{N-1}\left(-\frac{\lambda}{4N}\right) e^{\lambda/8N} \\
&\approx \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) + \frac{\lambda}{38N^2} I_2(\sqrt{\lambda}) + \frac{\lambda^2}{1280N^4} I_4(\sqrt{\lambda}) + \ldots \\
&\approx \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) \\
&\approx \exp\left(\sqrt{\lambda} - \frac{3}{4}\ln\lambda - \frac{1}{2}\ln\frac{\pi}{2} + \ldots\right)
\end{aligned}
$$

Gravity Side: Beyond the leading order

- Forste-Ghoshal-Theisen 9903042, Drukker-Gross-Tseytlin 0001204, Kruczenski-Tirziu 0803.0315, Buchbinder-Tseytlin 1404.4952.

$$\begin{aligned} \langle W \rangle &= \exp(-\Gamma), \qquad \Gamma = \Gamma_0 + \Gamma_1, \\ \Gamma_1 &= \frac{1}{2} \ln \frac{[\det(-\nabla^2 + 2)]^3 [\det(-\nabla^2)]^5}{[\det(-\nabla^2 + \frac{1}{4} R^{(2)} + 1)]^8} \end{aligned} \tag{4}$$

- Five massless modes (S^5); three massive modes $AdS_2 \subset AdS_5$.

$$\langle W_\Box \rangle_{\mathrm{circle}} = \exp\left(\sqrt{\lambda} - \frac{1}{2} \ln(2\pi) + \ldots \right)$$

WL beyond the leading order: Problem/Opportunity

$$\langle W_\Box \rangle_{\text{circle}} = \exp\left(\sqrt{\lambda} - \frac{3}{4}\ln\lambda - \frac{1}{2}\ln\frac{\pi}{2} + \ldots\right)$$

$$\langle W_\Box \rangle_{\text{circle}} = \exp\left(\sqrt{\lambda} - \frac{1}{2}\ln(2\pi) + \ldots\right)$$

- Missing the $\ln(\lambda)$ term on the gravity side (zero modes, more later..).
- Numerical discrepancy is not an error: Drukker-Gross-Tseytlin 0001204, Kruczenski-Tirziu 0803.0315, Buchbinder-Tseytlin 1404.4952.

Toward a Precision Computation

- The AdS/CFT formula:

$$< W(C) >_{CFT} = < V(\Sigma \to C) >_{String} \tag{5}$$

- What we are doing:

$$Z_{string} = exp(-S_{classical}) \times Z_{1-loop} \times (\text{Topological Sector}) \tag{6}$$

- We are missing aspects of string perturbation theory: Ghost zero modes, etc.
- $\Longrightarrow$ Compare configurations with the same world sheet topology!
- The $1/4$ BPS Wilson loop beyond the leading order.

The $1/4$ BPS Wilson loop

$$W(C) = \frac{1}{N} \mathrm{Tr} P \exp \oint (i A_\mu \dot{x}^\mu + |\dot{x}| \Theta_I \Phi_I) d\tau. \quad (7)$$

- Contour

$$\begin{aligned} x^\mu(\tau) &= (a \cos\tau, a \sin\tau, 0, 0), \\ \Theta_I &= (\sin\theta_0 \cos\tau, \sin\theta_0 \sin\tau, \cos\theta_0, 0, 0, 0). \end{aligned} \quad (8)$$

- In the planar limit

$$\langle W(C) \rangle = \frac{2}{\sqrt{\lambda'}} I_1(\sqrt{\lambda'}), \quad (9)$$

where $\lambda' = \lambda \cos^2\theta_0$.

Holographic $1/4$ BPS WL

- The $EAdS_5$ metric is written as a foliation over $EAdS_2 \times S^2$:

$$\frac{ds^2_{AdS_5}}{L^2} = \cosh^2(u)\left(d\rho^2 + \sinh^2\rho\, d\psi^2\right) + \sinh^2(u)\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right) + du^2 \,.$$

- The metric on S^5 is taken to be

$$\frac{d\Omega_5^2}{L^2} = d\theta^2 + \sin^2\theta\, d\phi^2 + \cos^2\theta\left(d\chi^2 + \cos^2\chi\, d\alpha^2 + \sin^2\chi\, d\beta^2\right) \,.$$

Holographic $1/4$ BPS WL

- Classical solution – Ansatz:

$$
\begin{aligned}
\psi &= \tau, \qquad \sinh\rho = \frac{1}{\sinh\sigma}\,, \\
u &= 0 \\
\phi &= \tau, \qquad \sin\theta = \frac{1}{\cosh(\sigma_0+\sigma)}\,,
\end{aligned}
$$

- Classical solution – World-sheet metric:

$$
ds^2 = \left(\frac{1}{\sinh^2\sigma} + \frac{1}{\cosh^2(\sigma_0-\sigma)}\right)(d\tau^2 + d\sigma^2). \tag{10}
$$

Fluctuations

- A. Faraggi, L. Pando Zayas, G. Silva and D. Trancanelli; see also Forini-Giangreco-Griguolo-Seminaro-Vescovi.
- The quadratic action for the bosonic fluctuations is

$$S^{2,3,4} = \frac{\sqrt{\lambda}}{4\pi} \int d\tau d\sigma \sqrt{g} \left(g^{ab} \partial_a \chi \partial_b \chi + \frac{2}{\sqrt{g}} \chi^2 \right) , \tag{11}$$

$$S^{5,6} = \frac{\sqrt{\lambda}}{2\pi} \int d\tau d\sigma \sqrt{g} \left(g^{ab} D_a \chi (D_b \chi)^\dagger - \frac{2m^2}{\sqrt{g}} |\chi|^2 \right) , \tag{12}$$

$$S^{7,8,9} = \frac{\sqrt{\lambda}}{4\pi} \int d\tau d\sigma \sqrt{g} \left(g^{ab} \partial_a \chi \partial_b \chi - \frac{2 \sin^2 \theta}{\sqrt{g}} \chi^2 \right) . \tag{13}$$

- In the second line, χ is a complex scalar field defined as $\chi = \frac{1}{\sqrt{2}} \left(\chi^{5} + i \chi^{6} \right)$, and the σ-dependent mass term reads

$$m = \frac{1}{\cosh\left(2\sigma + \sigma_0\right)} , \tag{14}$$

One-loop effective action

$$e^{-\Gamma^{1-\text{loop}}_{\text{effective}}} = \frac{\left(\text{Det}\,\mathcal{O}^{+}\right)^{\frac{4}{2}}\ \left(\text{Det}\,\mathcal{O}^{-}\right)^{\frac{4}{2}}}{\left(\text{Det}\,\mathcal{O}^{2,3,4}\right)^{\frac{3}{2}}\ \left(\text{Det}\,\mathcal{O}^{5,6}\right)^{\frac{2}{2}}\ \left(\text{Det}\,\mathcal{O}^{7,8,9}\right)^{\frac{3}{2}}}\,, \tag{16}$$

$$\ln\left(\text{Det}\,\mathcal{O}\right) = \sum_{E} \ln\left(\text{Det}\,\mathcal{O}_E\right)\ , \tag{17}$$

- $\mathcal{O}_E$ is the corresponding one-dimensional operator acting on a specific Fourier mode.

Ratio of 1-loop effective action

- For $1/4$ BPS dependence on the value of σ_0 that characterizes the classical string solution. The 1/2 BPS is $\sigma_0 \to \infty$.

$$\Omega_E^{2,3,4}(\sigma_0) = \ln \left[\frac{\mathrm{Det}\, \mathcal{O}_E^{2,3,4}(\sigma_0)}{\mathrm{Det}\, \mathcal{O}_E^{2,3,4}(\infty)} \right] , \tag{18}$$

$$\Omega_E^{5,6}(\sigma_0) = \ln \left[\frac{\mathrm{Det}\, \mathcal{O}_E^{5,6}(\sigma_0)}{\mathrm{Det}\, \mathcal{O}_E^{5,6}(\infty)} \right] , \tag{19}$$

$$\Omega_E^{7,8,9}(\sigma_0) = \ln \left[\frac{\mathrm{Det}\, \mathcal{O}_E^{7,8,9}(\sigma_0)}{\mathrm{Det}\, \mathcal{O}_E^{7,8,9}(\infty)} \right] , \tag{20}$$

$$\Omega_E^{\alpha}(\sigma_0) = \ln \left[\frac{\mathrm{Det}\, \mathcal{O}_E^{\alpha}(\sigma_0)}{\mathrm{Det}\, \mathcal{O}_E^{\alpha}(\infty)} \right] \tag{21}$$

- Each ratio is to be computed using the Gelfand-Yaglom (Coleman) method.

- Forini-Giangreco-Griguolo-Seminaro-Vescovi, Faraggi-PZ-Silva-Trancanelli

$$\begin{aligned} \Delta\Gamma^{1-\text{loop}}_{\text{effective}}(\sigma_0) &= \frac{1}{2}\sum_{E\in\mathbb{Z}}\Big(3\Omega^{2,3,4}_E(\sigma_0) + 2\Omega^{5,6}_E(\sigma_0) + 3\Omega^{7,8,9}_E(\sigma_0)\Big) \\ &- \frac{4}{2}\sum_{E\in\mathbb{Z}+\frac{1}{2}}\left(\Omega^{+}_E(\sigma_0) + \Omega^{-}_E(\sigma_0)\right) . \end{aligned}$$

- Result

$$\begin{aligned} \Delta\Gamma^{1-\text{loop}}_{\text{effective}} &= \frac{3}{2}\ln\tanh\sigma_0 - \ln\sqrt{\frac{1+\tanh\sigma_0}{2}} \\ &= \frac{3}{2}\ln\cos\theta_0 - \ln\cos\frac{\theta_0}{2} , \end{aligned}$$

The first term gives the predicted result from the gauge theory. It comes from the $E = 0$ mode of the $\Omega^{7,8,9}_E$ determinant.

Conclusions I

- Hints of string localization?
- The field theory answer is obtained by the $E=0$ mode of the $\Omega_E^{7,8,9}$ determinant associated to the fields charged under the $SU(2)_B$ factor of the supergroup preserved by the $1/4$ BPS latitude.
- Similar phenomena seems to take place in ABJM!
- Explicit hints of a localization structure? What are these modes the localization locus of ?
- Take this lessons to computing corrections in QCD-like models.

Revisions

- Forini-Tseytlin-Vescovi have proposed a perturbative computation that matches the field theory answer [1702.02164]. "Perturbative computation of string one-loop corrections to Wilson loop minimal surfaces in $AdS_5 \times S^5$ "
- Hard to compute heat kernels in non-homogeneous spaces. Expand the heat kernel around $\theta_0 = 0$.
- What does it teach us? The Gelfand-Yaglom method might be imposing very strict conditions.
- Back to the drawing board analyzing the space of boundary conditions.

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- Back to the drawing board analyzing the space of boundary conditions.
- Stay tuned ... coming up this summer!

Why go to ABJM ?

ABJM theory is a Chern-Simons theory in three dimensions of the form $U(N)_k \times U(N)_{-k}$.

- Extra knob, k, (Chern-Simons level).

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ABJM theory is a Chern-Simons theory in three dimensions of the form $U(N)_k \times U(N)_{-k}$.

- Extra knob, k, (Chern-Simons level).
- N, k $\rightarrow \infty$ with $\lambda = \frac{N}{k}$ = fixed $\Rightarrow$ Dual to string theory in $AdS_4 \times \mathbb{CP}^3$

  N $\rightarrow \infty$, with k = fixed $\Rightarrow$ Dual to M - theory in $AdS_4 \times S^7/\mathbb{Z}_k$
- Wilson Loop in Higher Dimensional Representations

Configuration	Representation of SU(N)
String	Fundamental
D2	Symmetric
D6	Anti-symmetric

The ABJM $1/2$ BPS WL

- Klem-Mariño-Schiereck-Soroush 1207.0611:

$$\langle W_{\Box}^{1/2}\rangle = \frac{1}{4}\csc\left(\frac{2\pi}{k}\right)\frac{\mathrm{Ai}[C^{-1/3}(N-\frac{k}{24}-\frac{7}{3k})]}{\mathrm{Ai}[C^{-1/3}(N-\frac{k}{24}-\frac{1}{3k})]}, \tag{22}$$

- $C = 2/(\pi^2 k)$.
- This result is exact at all orders in $1/N$, up to exponentially small corrections in N (world sheet or membrane instanton corrections).
- The denominator in the expression above is the partition function of ABJM theory [Fuji-Hirano-Moriyama,Marino-Putrov].

Holographic WL

- Classical background:

$$
\begin{aligned}
ds^2 &= \frac{R^3}{4k}(ds^2_{AdS_4} + 4ds^2_{\mathbb{CP}^3}) \\
ds^2_{AdS_4} &= du^2 + \cosh^2 u ds^2_{AdS_2} + \sinh^2 u d\phi^2,
\end{aligned}
\tag{23}
$$

- $ds^2_{\mathbb{CP}^3}, e^{2\Phi}, F_4, F_2$
- Classical Solution: The solution is $u = 0$.

$$
S_{string,cl.} = \frac{R^3}{8\pi\, k}\int_0^{2\pi} d\psi \int_0^{\rho_0} d\rho \sinh\rho = \pi\sqrt{2\lambda}(\cosh\rho_0 - 1). \tag{24}
$$

- Classical action coincides! Large N, large λ always coincides.

Holographic ABJM WL effective action

- The one-loop effective action (six massless scalars - point on $\mathbb{CP}^3$ and two massive - $AdS_2 \subset AdS_4$):

$$e^{-\Gamma} = \frac{\det(-\nabla_F^2 - \frac{1}{2}) \det^3(-\nabla_F^2 + \frac{1}{2})}{\det(-\nabla^2 + 2) \det^3(-\nabla^2)}.$$

$$\begin{aligned}
\Gamma_1 &= -\frac{1}{2}\zeta_B'(0) + \frac{1}{2}\zeta_F'(0) \\
&= \frac{1}{2}\,[\frac{14}{3} + 16\ \ln\ A - 6\ \ln\,(2\pi) - \frac{14}{3} - 16\ \ln\ A + 6\ \ln\,(2\pi)] \\
\Gamma_1 &= 0
\end{aligned}$$

- Field theory: $\ln\langle W_{\Box}^{1/2}\rangle = \pi\sqrt{2\lambda} - \ln(8\pi\lambda) + \ldots$
- Confusion: Six massless mode from point in $\mathbb{CP}^3$ should be smeared?

Comparing the $1/6$ (latitude) versus the $1/2$ BPS [1706.xxxx]

$$
\begin{aligned}
S_{(2,3)} &= \frac{L^2}{\pi\,\alpha'}\int d\tau\, d\sigma\,\sqrt{g}\left(g^{ab}\,(\partial_a\chi^{\underline{23}})^*\,\partial_b\chi^{\underline{23}} + \frac{2\sinh^2\rho}{\sqrt{g}}\,|\chi^{\underline{23}}|^2\right),\\
S_{(4,5)} &= \frac{L^2}{\pi\,\alpha'}\int d\tau\, d\sigma\,\sqrt{g}\left(g^{ab}\,(\mathcal{D}^A_a\chi^{\underline{45}})^*\,\mathcal{D}^A_b\chi^{\underline{45}} - \frac{2m^2}{\sqrt{g}}\,|\chi^{\underline{45}}|^2\right),\\
S_{(6,7),(8,9)} &= \frac{L^2}{\pi\,\alpha'}\int d\tau\, d\sigma\,\sqrt{g}\left(g^{ab}\,(\mathcal{D}^B_a\chi)^*\,\mathcal{D}^B_b\chi - \frac{\sin^2\theta_1}{2\,\sqrt{g}}\,|\chi|^2\right),
\end{aligned}
\tag{25}
$$

- The modes $(6,7,8,9)$ generate the field theory answer through the $E=0$ modes.
- $(4/2)\ln\cos\theta_0 = \ln(\lambda'/\lambda) = \ln\cos^2\theta_0$.
- The degeneracy of these modes - 4 - provides an argument against the ghost zero modes claim for $(3/4)\log\lambda$ used in $\mathcal{N}=4$.

Conclusions II

- ABJM provides another empiric example of bulk localization.
- Can this be made precise using a localizing supercharge or a equivariant localization?
- Is there a similar bulk localization mechanism for D-branes (corresponding to Wilson loops in higher dimensional representations)?

Dp as Wilson loops in Higher Dimensional Representations

- Wilson loops in the m-symmetric and m-antisymmetric representations correspond to D2 and D6 branes respectively.

$$S_{Dp}^{B} = -T_p \int d^{p+1}\xi \, e^{-\Phi} \sqrt{-\mathrm{det}(g_{ab} + \mathcal{F}_{ab})} + T_p \int e^{\mathcal{F}} \wedge \sum_q C_q \qquad (26)$$

$$\begin{aligned}
ds^2 &= \frac{R^3}{4k}(ds^2_{AdS_4} + 4ds^2_{\mathbb{CP}^3}) \\
ds^2_{AdS_4} &= du^2 + \cosh^2 u ds^2_{AdS_2} + \sinh^2 u d\phi^2, \\
4ds^2_{\mathbb{CP}^3} &= d\alpha^2 + \cos^2\frac{\alpha}{2}\,(d\theta_1^2 + \sin^2\theta_1\, d\phi_1^2) + \sin^2\frac{\alpha}{2}\,(d\theta_2^2 + \sin^2\theta_2\, d\phi_2^2) \\
&\quad + \sin^2\frac{\alpha}{2}\,\cos^2\frac{\alpha}{2}\,(d\chi + \cos\theta_1\, d\phi_1 - \cos\theta_2\, d\phi_2)^2,
\end{aligned}$$

D6 brane

- Classical Solution:
  The solution is $u = 0$ with $AdS_2 \subset AdS_4$ and $\tilde{T}^{1,1} \subset \mathbb{CP}^3$ at constant α.

$$\begin{aligned} S_{WL} &= S^B_{D6} - \frac{1}{\beta} mE \\ &= \frac{N}{4\beta}\left[\sin^3\alpha\,\sqrt{1-E^2} - E\left(\sin^2\alpha\ \cos\alpha + 2\cos\alpha - 2\right)\right] - \frac{1}{\beta} mE \end{aligned} \tag{27}$$

  EOM gives,

$$E = -\cos\alpha, \tag{28}$$

$$m = \beta\,\frac{\delta S^B_{D6}}{\delta E} = \frac{N}{2}\,(1-\cos\alpha). \tag{29}$$

$$S_{WL} = \frac{m\,(N-m)}{\beta N} \tag{30}$$

- D6-brane is 1/6 BPS and symmetric under $m \leftrightarrow N - m$.

Fluctuations

- Embedding a D6: 3 scalar fields and a gauge fields in $AdS_2 \times \tilde{T}^{1,1}$.
- Bosonic symmetries : $OSp(4|2)$.

$$S^B_{D6} = -\frac{T_6}{\sin\alpha}\int d^7\xi\sqrt{-\tilde{g}}\left\{\frac{1}{2}\tilde{\nabla}^a\chi^{\underline{i}}\,\tilde{\nabla}_a\chi^{\underline{i}} + \frac{1}{\sin^2\alpha}[(\chi^{\underline{2}})^2 + (\chi^{\underline{3}})^2] + \frac{1}{4}\tilde{f}^{ab}\tilde{f}_{ab}\right.$$
$$\left. -\frac{3}{2\sin^2\alpha}\,(\chi^{\underline{4}})^2 + \frac{1}{\sin\alpha}e^{\nu}_{\underline{9}}\chi^{\underline{3}}\nabla_\nu\,\chi^{\underline{2}} - \frac{3}{\sin\alpha}\chi^{\underline{4}}\left(\frac{1}{2}\tilde{\epsilon}^{\alpha\beta}f_{\alpha\beta}\right) - \frac{1}{2}\cot\alpha\epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho\right\}$$

$$S^F_{D6} = \frac{T_6}{2\sin\alpha}\int d^7\,\xi\, e^{-\Phi}\sqrt{-g}\bar{\theta}_\pm\left[\tilde{\gamma}^a\,\tilde{\nabla}_a - \frac{\cot\alpha}{4}(\gamma^{\underline{569}} + \gamma^{\underline{789}})\right.$$
$$\left. \pm\frac{i\gamma^{\underline{9}}}{4\sin\alpha}\,(1 - 3\gamma^{\underline{5678}})\right]\theta_\pm$$

Logarithmic Correction to D6-brane Wilson loop

$$-\frac{1}{2}\ln \det{}' A = \left(\frac{1}{(4\pi)^{d/2}} \int d^d x \sqrt{g}\, a_{d/2}(x,x) - n_A^0 \right) \log L + \ldots\ldots \quad (31)$$

- n_A^0 are number of zero modes of A and $a_{d/2}$ are Seeley coefficients.
- In odd-dim spacetime, $a_{d/2}$ vanishes.

- Non-zero contribution from zero modes (n_A^0).

$$n_A^0 = \left(\sum_l |\phi_l^{(0)}(x)|^2 \right) \int d^d x \sqrt{g} \quad (32)$$

- M/2-forms on AdS_M spaces. Use by Bhatacharya-Grassi-Mariño-Sen in $AdS_4 \times S^7$.

$$\sum_l |\phi_l^{(0)}(x)|^2 = \frac{1}{2^M \pi^{M/2}} \frac{M!}{(M/2)!} \frac{1}{L^M} \quad (33)$$

- The one-loop effective action arises *only* from the contribution of a vector zero mode in AdS_2.
- The answer is clean.
- $\ln\left(T_{D6}/\sin\alpha\right) \sim \ln\left(N\sqrt{\lambda}\ \frac{N^2}{m(N-m)}\right)$. [1706.xxxx]

Partial Field Theory Results

- p-antisymmetric representation $f = p/N$ – fixed.

$$W_{1/6}^{A_m} \sim \exp\left[N\pi\sqrt{2\lambda}f(1-f) + \frac{1}{4}\ln\left(\frac{2\lambda}{N^2}\right)\right]. \tag{34}$$

- p-symmetric representation $f = p/N$ – fixed.

$$W_{1/6}^{S_m} \sim \exp\left[fN\pi\sqrt{2\lambda} - \ln(Nf^2 2\pi\sqrt{2\lambda}) + o(1)\right]. \tag{35}$$

- Not a match.
- Note that the D6 computes corrections in $1/(N\sqrt{\lambda})$, whereas the saddle point computes in 1/N.
- Need to improve on the matrix side.

Status and Open Directions

- Precision holography is a painstaking job and we are learning how to perform holographic computations.
- There are various venues in $\mathcal{N} = 4$ and ABJM.
- Similar techniques have been applied by Sen and collaboraters to asymptotically flat black holes. They should be applied to the entropy of asymptotically AdS black holes.
- Benini-Hristov-Zaffaroni: Localization side [Topologically Twisted Index] - Gravity side [Magnetically charged Asymptotically AdS_4 black holes.]

Thanks !