Aspects of Gauge-Strings Duality

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Outline

1. I will comment on recent work in the area of non-Abelian T-duality. The focus will be on the general ideas and outcomes.
2. The knowledge of field theory results at strong coupling allows us to say things about a geometry (ranges of coordinates, smoothing-out of singularities, etc).
3. What I will discuss today will be in the context of an $N = 2$ SCFT in four dimensions and a Matrix Model with sixteen supercharges.
4. This talk is a schematic selection of topics taken from two papers with: Yolanda Lozano (2016) and Yolanda Lozano and Salomón Zacarías in March 2017. Previous and ongoing works with Georgios Itsios, C. Whitting, Jesús Montero, N. Macpherson, D. Zoakos, K. Sfetsos, D. Thompson, E. Caceres, V. Rodgers, L. Pando Zayas, inspire this talk.

Let me clarify some assumptions I made in preparing this talk

I assume that this audience has a good understanding of dualities in general (Condensed Matter, QFT, String Theory). In particular, some familiarity with non-Abelian T-duality, the generalisation of Buscher's procedure to non-Abelian isometries.

All I need you to know is that there is a well defined systematic procedure carried out in the string sigma-model. Supplemented by the work of Sfetsos and Thompson (2010), one can transform any configuration of 10-d Supergravity theories. For the purposes of this talk, I wish to think about non-Abelian T-duality as a *solution generating technique*.

Many interesting geometrical aspects of this generating technique have been studied —G-structures, calibrated cycles, SUSY preservation, backgrounds that avoid known classifications, etc.

Of course, this transformation is not free of conceptual issues.

Whilst precise formulas have been developed for the NS and the RR fields on interesting classes of backgrounds
Various things are unclear about this non-Abelian version of T-duality.

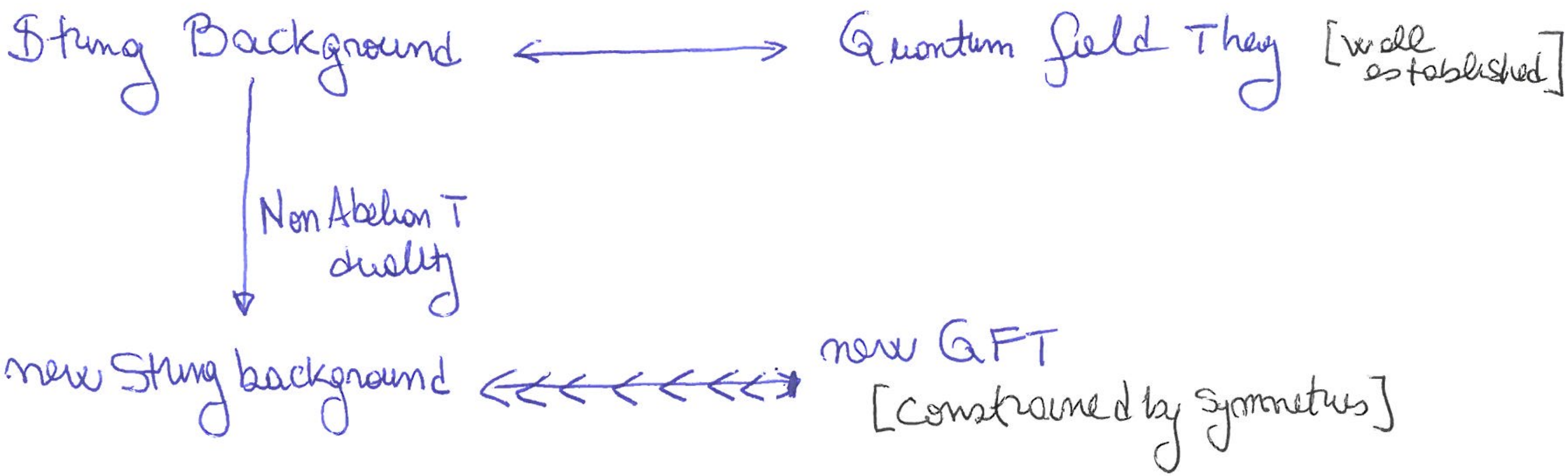
- What is the range of the dual coordinates (Lagrange multipliers)?
- What happens if we perform this duality on a world-sheet with genus $g > 0$?
- How does the initial world-sheet CFT relates to the final one?
- When is the generated background singular? Can these singularities be resolved?
- How we dualise 'back'?

These issues are not totally well understood. There were various papers in the 1990's addressing them.

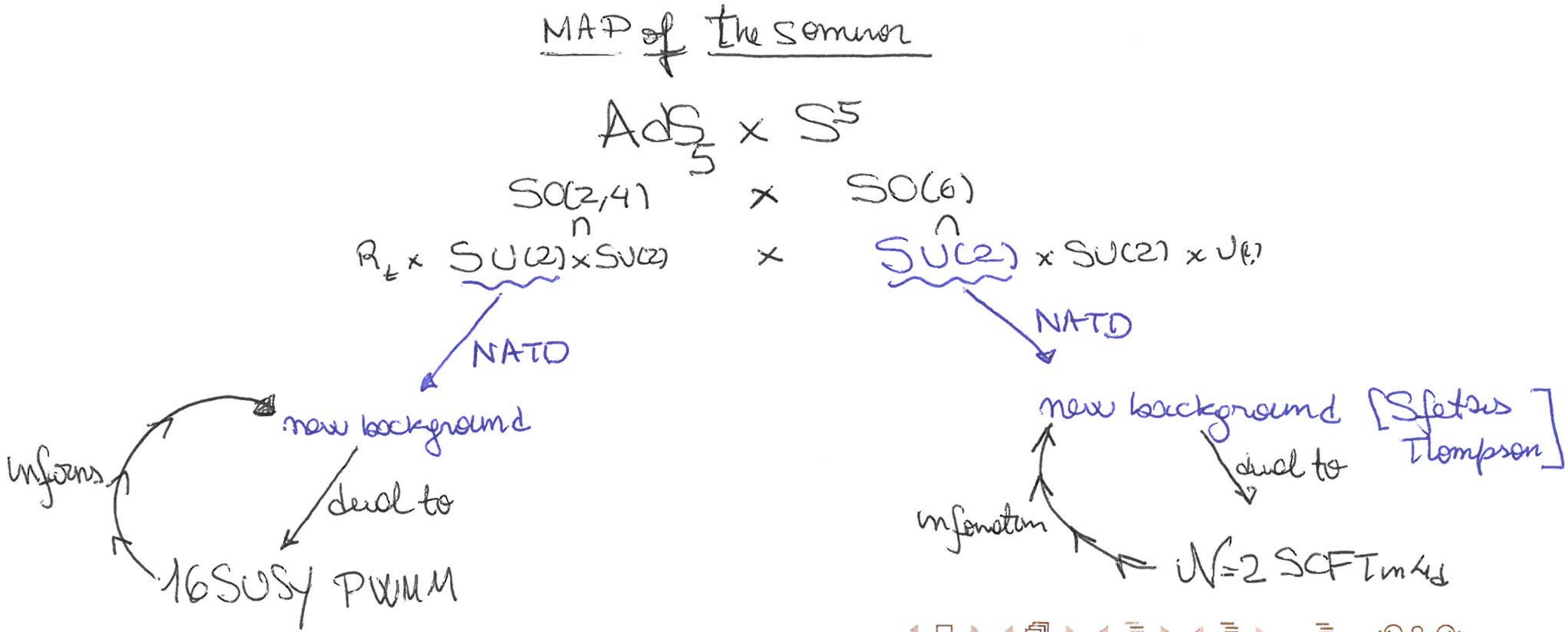
The point of this talk is to contribute to the understanding of some of the problematic issues mentioned above.

The approach taken will be to use the interplay between non-Abelian T-duality and Holography (the Maldacena Conjecture).

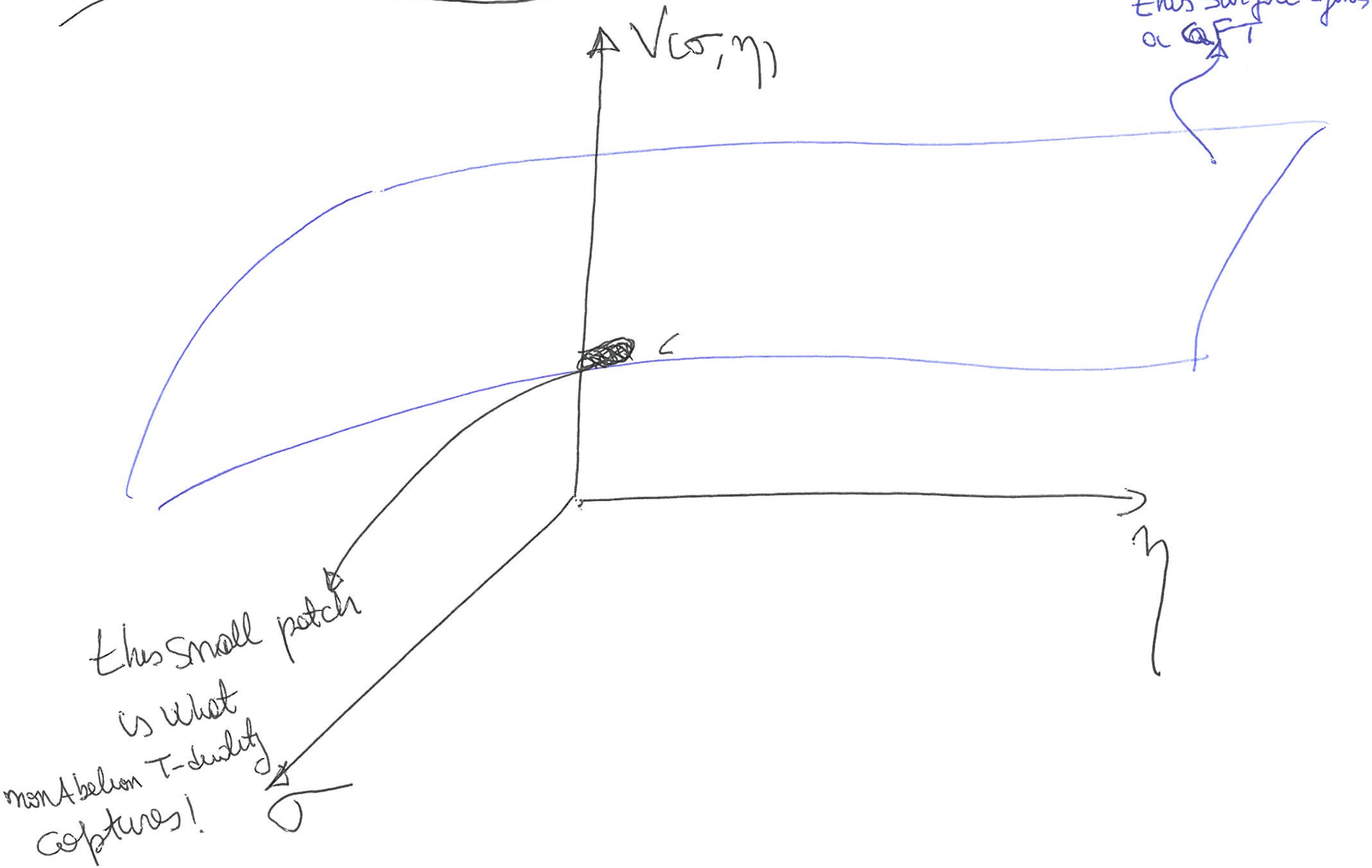
Given a background with a well established string dual, to produce a new background using non-Abelian T-duality. This new background should be symmetric enough so that its dual field theory can be determined. Then, use this new QFT to gain information about the dual background.



The example chosen for this talk is that of $AdS_5 \times S^5$.
Studying the non-Abelian T dual background, we will find it can be put in correspondence with either an $\mathcal{N} = 2$ SCFT or a Matrix Model with 16 SUSYs.
This will depend on "which $SU(2)$ we dualise". We have a powerful formalism to study the field theory and learn things about the background.
A 'map of this seminar' and a 'snapshot summary' are



"Snapshot" of the seminar



Let us consider $AdS_5 \times S^5$ in $g_s = 1$ units, written as

$$ds^2 = 4L^2 \left(AdS_5 + S^5\right)$$

change variables

$$ds^2_{\mathrm{AdS5}} = \left[\frac{d\sigma^2}{(\sigma^2-1)} - \sigma^2 dt^2 + (\sigma^2-1)d\Omega_3^2\right], \qquad (\sigma = \cosh r). \to \text{usual global } AdS_5$$

$$d\Omega_5 = \left[\frac{d\sigma^2}{(1-\sigma^2)} + \sigma^2 d\beta^2 + (1-\sigma^2)d\tilde{\Omega}_3^2\right], \qquad (\sigma = \sin\alpha). \to \text{usual } S^5$$

$$d\Omega_3^2 = \sum_{i=1}^{3} \frac{\omega_i^2}{4}, \qquad d\tilde{\Omega}_3^2 = \sum_{i=1}^{3} \frac{\tilde{\omega}_i^2}{4}.$$

$$F_5 = \frac{4}{L}(\mathrm{Vol}_{AdS_5} - \mathrm{Vol}_{S^5}), \quad N_{D3} = \frac{\int F_5}{2\kappa_{10}^2 T_{D3}} \to \frac{L^4}{\alpha'^2} = 4\pi N_{D3}.$$

$$\sum_{i=1}^{3} \omega_i^2 = (d\psi + \cos\theta d\varphi)^2 + d\theta^2 + \sin^2\theta d\varphi^2.$$

We will calculate the non-Abelian T-dual on this background, using the SU(2) inside $SO(2,4)$ or $SO(6)$, realised in ω_i's .

This two different non-Abelian T-dualities will generate two different new solutions of Type IIA with the following characteristics.

- Both solutions are singular. Close to the singularity, the space asymptotes to NS5 branes.
- Both solutions will preserve SUSY.
- Both solutions are supported by NS flux. There is also charge of D_p and D_{p+2} branes (for $p = 4$ or $p = 0$).

For the purposes of this presentation, let me set $L = \alpha' = g_s = 1$. You will only miss the nice quantisations conditions that are properly written in our papers, namely $\frac{L^4}{\alpha'^2} \sim N_p$. Also, allow me to quote only the NS-fields

The generated backgrounds read,

$$(\Theta, \varphi, \psi) \longrightarrow (\eta, \chi, \xi)$$

$$(\tilde{\Theta}, \tilde{\varphi}, \tilde{\psi}) \longrightarrow (\eta, \chi, \xi)$$

Non-abelian T-duality on $SU(2)$ inside $SO(6)$, $(\tilde{\theta}, \tilde{\varphi}, \tilde{\psi}) \to (\eta, \chi, \xi)$.

$$ds^2_{IIA,st} = 4AdS_5 + 4\left[\frac{d\sigma^2 + d\eta^2}{(1-\sigma^2)} + \sigma^2 d\beta^2\right] + \frac{4\eta^2(1-\sigma^2)}{(4\eta^2 + (1-\sigma^2)^2)} d\Omega_2(\chi, \xi).$$

$$B_2 = \frac{8\eta^3}{(4\eta^2 + (1-\sigma^2)^2)} d\Omega_2(\chi, \xi),\ e^{-2\Phi} = (1-\sigma^2)(4\eta^2 + (1-\sigma^2)^2).$$

Non-abelian T-duality on $SU(2)$ inside $SO(2,4)$, $(\theta, \varphi, \psi) \to (\eta, \chi, \xi)$.

$$ds^2_{IIA,st} = 4d\Omega_5 + 4\left[\frac{d\sigma^2 + d\eta^2}{(\sigma^2 - 1)} - \sigma^2 dt^2\right] + \frac{4\eta^2(\sigma^2 - 1)}{(4\eta^2 + (\sigma^2 - 1)^2)} d\Omega_2(\chi, \xi).$$

$$B_2 = \frac{8\eta^3}{(4\eta^2 + (\sigma^2 - 1)^2)} d\Omega_2(\chi, \xi),\ e^{-2\Phi} = (\sigma^2 - 1)(4\eta^2 + (\sigma^2 - 1)^2).$$

Both non-Abelian T-dual backgrounds are complemented by F_2 and $F_4 = B_2 \wedge F_2$. They are both singular at $\sigma = 1$. $(*)$
Which is understandable, since the size of the (θ, φ, ψ)-cycle before the duality vanishes at $\sigma = 1$.

Let us look at the isometries of both backgrounds. Before the duality we have

$$SO(2,4) \times SO(6), \; 32 \text{ SUSY} \sim (R_t \times SO(4)) \times (SO(2) \times SO(4)\,).$$

When the non-Abelian T-duality acts on an $SU(2)$ inside $SO(6)$, we find

$$SO(2,4) \times SO(3) \times U(1), \quad 16 \text{ SUSY}.$$

When the non-Abelian T-duality acts on an $SU(2)$ inside $SO(2,4)$, we find

$$SO(6) \times SO(3) \times R_t, \quad 16 \text{ SUSY}.$$

This very large isometry group implies strong restrictions on the backgrounds. There exist classifications and our backgrounds are new solutions within those classifications. We will use this shortly.
We can calculate the charges associated to each of these backgrounds.
This give valuable information about the dual QFT.

In calculating the Page charges, we find, for the case in which we dualise on the S^5 (we have magnetic F_2, F_4),

$\int_\eta \xrightarrow{\text{range}}$?

$$Q_{NS5} = \frac{1}{2\kappa_{10}^2 T_{NS5}} \int_{\eta,\chi,\xi} H_3,$$

$$Q_{D_6} = \frac{1}{2\kappa_{10}^2 T_{D_6}} \int_{\sigma,\beta} F_2, \quad Q_{D_4} = \frac{1}{2\kappa_{10}^2 T_{D4}} \int_{\sigma,\beta,\chi,\xi} F_4 - B_2 \wedge F_2.$$

In the example in which the duality goes on AdS_5 (we have magnetic F_6, F_8),

$$Q_{NS5} = \frac{1}{2\kappa_{10}^2 T_{NS5}} \int_{\eta,\chi,\xi} H_3,$$

$$Q_{D_2} = \frac{1}{2\kappa_{10}^2 T_{D_2}} \int_{\eta,\Omega_5} F_6, \quad Q_{D_0} = \frac{1}{2\kappa_{10}^2 T_{D0}} \int_{\eta,\Omega_5,\Omega_2} F_8 - B_2 \wedge F_6,$$

Where $2\kappa_{10}^2 T_p = (2\pi)^{7-p}$ in the conventions I am using here.

This calculation is one place where some of the unknown things about non-Abelian T-duality, shows itself.

What is the range of the coordinate η?

A very nice observation was made by Lozano+Macpherson in 2014:

Bound $0 < \frac{1}{4\pi^2} \oint_{\chi,\xi} B_2 = \frac{\eta}{\pi} < 1$.

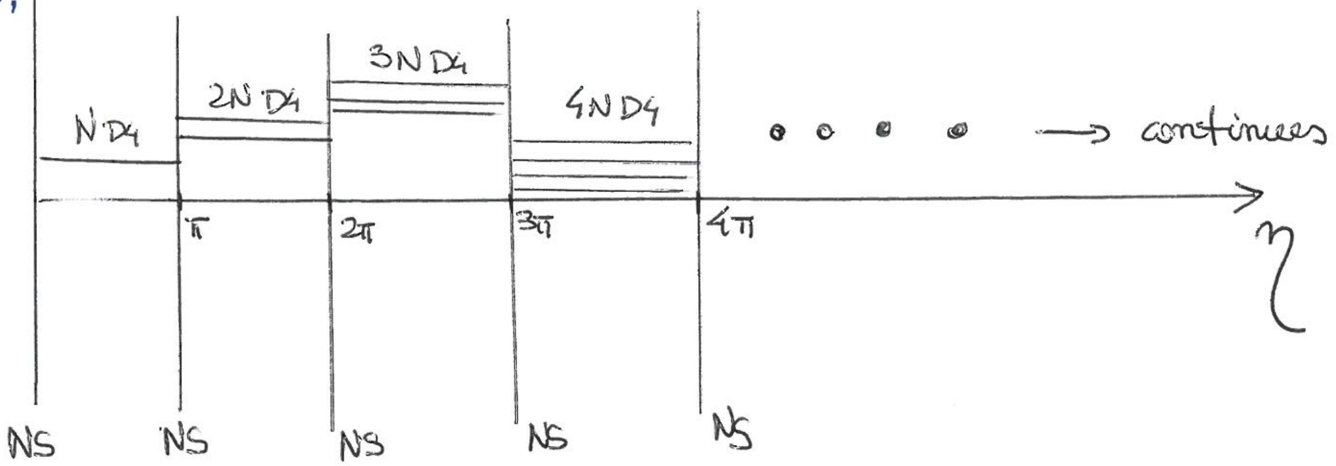
This has some very interesting consequences

- The coordinate η, naturally divides itself in intervals of size π.
- Each time we cross $\eta = k\pi$, a large gauge transformation that restores the integral above to $[0, 1]$ should be performed.
- This large gauge transformation of B_2 generates charge of NS5 branes. Also, it changes the Page charges due to the $B_2 \wedge F_p$ terms.
- The η-coordinate turns out to be a sort of 'theory space" coordinate. All the QFT information will be encoded by a function of η.

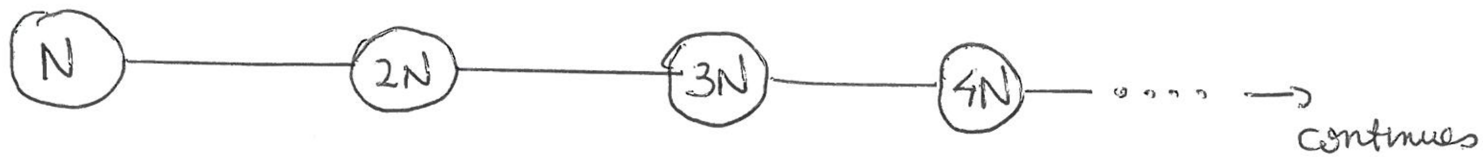
Let us briefly describe the dual QFTs. I will be very sketchy in the arguments that lead to these proposals

The dual field theories can be 'induced' by a combination metric, fluxes, SUSY study. This gives clues about the branes involved, their relative positions in space, etc.

In the case in which we perform the duality on the S^5, we find a system of D4, NS5 branes that correspond to a Hanany-Witten set-up,



That suggests an $\mathcal{N} = 2$ four dimensional quiver



In the case in which we dualised along the S^3 in AdS_5 we found a system containing D0 and D2 branes (Myers blown up D0's). The system is a Matrix Model, the Plane wave Matrix Model—a deformation of BFSS. The vacuum of this Matrix Model is in this case,

$$J^i = \begin{bmatrix} J^i_{j=0}\ \}\ \text{1 time} & & & & \\ & J^i_{j=1/2}\ \}\ \text{2 times} & & & \\ & & J^i_{j=1}\ \}\ \text{3 times} & & \\ & & & J^i_{j=3/2}\ \}\ \text{4 times} & \\ & & & & \ddots \ \text{continues} \end{bmatrix}$$

Different calculations support these claims. For example, the SUSY probe branes in the non-abelian T-dual backgrounds coincide with the structure we are suggesting.

Perhaps more definitive, is the fact that these quivers and matrix models have very precise string duals.

Indeed, due to the large isometry groups involved, both the QFTs and their respective gravity duals are well understood and classified.

Given the isometries we mentioned before, we can propose that backgrounds dual to QFTs with those global symmetries, must be

$$ds^2 \sim f_1 AdS_5 + f_2 d\Omega_2 + f_3 d\beta^2 + f_4 d\eta^2 + f_5 d\sigma^2; \quad f_i(\sigma,\eta)$$

for $SO(2,4) \times SO(3) \times U(1)$.

$$ds^2 \sim -g_1 dt^2 + g_2 d\Omega_5 + g_3 d\Omega_2 + g_4 d\eta^2 + g_5 d\sigma^2; \quad g_i(\sigma,\eta)$$

for $SO(6) \times SO(3) \times R_t$.

And a similar proposal for the fluxes. Imposing the preservation of 16 SUSYs, Lin, Lunin and Maldacena found an explicit expression for the $f_i(\sigma,\eta), g_i(\sigma,\eta)$.

There is a generic background geometry dual to an 4d-$\mathcal{N} = 2$ SCFT and Matrix Models with the isometries above. Lin, Lunin, Maldacena (2004), Lin, Maldacena (2005), Gaiotto, Maldacena (2009).

After some simplifications are made, the configuration is described in terms of a function $V(\sigma, \eta)$. Defining $\dot{V} = \sigma\partial_\sigma V$, $V' = \partial_\eta V$ the equation solved by $V(\sigma, \eta)$ is,

$$\boxed{\ddot{V} + \sigma^2 V'' = 0.}$$

This differential equations, valid for both the SCFT in 4d or the Matrix Model in (0+1)-d, needs to be supplemented by boundary conditions. This is what distinguishes between different dynamics.

dual to $\mathcal{N}=2$ $d=4$ SCFT $\leftarrow$ $\ddot{V}+\sigma^2 V''=0$ $\rightarrow$ PWMM

Boundary conditions — other Boundary conditions

For the 4d SCFTs, the metric reads, Gaiotto-Maldacena (2009)

$$ds^2_{IIA,st} = \left(\frac{2\dot{V}-\ddot{V}}{V''}\right)^{1/2}\Big[4AdS_5 + \frac{2V''\dot{V}}{\Delta}d\Omega_2^2(\chi,\xi) + \frac{2V''}{\dot{V}}(d\sigma^2+d\eta^2) + \frac{4V''\sigma^2}{2\dot{V}-\ddot{V}}d\beta^2\Big],$$
$$\Delta = (2\dot{V}-\ddot{V})V'' + (\dot{V}')^2.$$

For the case of the Plane Wave Matrix Model, Lin and Maldacena (2005) determined that,

$$ds^2_{IIA,st} = \left(\frac{-2\dot{V}+\ddot{V}}{-V''}\right)^{1/2}\Big[4d\Omega_5 + \frac{2V''\dot{V}}{\Delta}d\Omega_2^2(\chi,\xi) + \frac{-2V''}{\dot{V}}(d\sigma^2+d\eta^2) - \frac{4V''\sigma^2}{-2\dot{V}+\ddot{V}}dt^2\Big],$$
$$\Delta = (-2\dot{V}+\ddot{V})V'' - (\dot{V}')^2.$$

And analog expressions for the rest of the RR and NS fields.

As you imagine, we can find the expression for $V(\sigma, \eta)$, that satisfies the Laplace equation and gives the two non-Abelian T-dual backgrounds.

Indeed, for the case in which we dualised an $SU(2)$ insider S^5, one finds,

$$V_{CFT}(\sigma, \eta) = \eta(\log \sigma - \frac{\sigma^2}{2}) + \frac{1}{3}\eta^3.$$

. $\mathcal{N}=2$ SCFT

While, for the case in which we dualised on AdS_5, the function $V(\sigma, \eta)$ is,

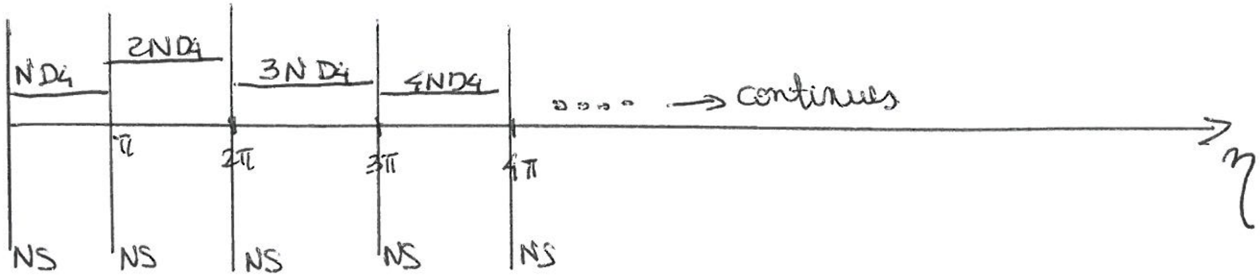
$$V_{MM}(\sigma, \eta) = \eta(-\log \sigma + \frac{\sigma^2}{2}) - \frac{1}{3}\eta^3.$$

. PWMM

The non-Abelian T-dual backgrounds are new solutions that fit the classifications. Both backgrounds are singular since correct boundary conditions are not satisfied. What is wrong with the Physics dual these non-Abelian T-dual backgrounds? → Question!

We can have an intuition for what goes wrong using 4-d CFT tools.

Consider the Hanany-Witten set-up corresponding to the $\mathcal{N}=2$ CFT. This CFT has infinite number of degrees of freedom. $\rightarrow$ $\boxed{\text{Infinite central charge (?)}}$



Defining linking numbers for NS5 and D6 branes as

$$\hat{L}_{NS5} = Net_{D4} - Right_{D6}, \quad L_{D6} = Net_{D4} + Left_{NS5}.$$

The linking numbers satisfy a relation— for P NS5-branes and D6 branes that appear with multiplicity d_i,

$$\sum_{i=1}^{P} \hat{L}_{NS,i} + \sum_{i=1}^{P-1} d_i L_{D6,i} = 0.$$

One learns from here that if we decide to limit or bound the size of the η-coordinate, we will need to add D6 branes so to satisfy the relation above.

QFT informs the geometry!

Something very similar happens in the matrix model case.
Indeed, there there is an ever-growing dimension of the matrix that represents the vacuum. This changes the correct D0 brane asymptotics into something else.
In some way, we need to 'stop' this growth in the representation-sizes. Same with the ever growing ranks of the quiver in the CFT case.

QFT "informs" the geometry.

The way to amend this problem is to find a new set of solutions that in some form 'reduce' to the non-Abelian T-dual ones, or such that they capture the same information.
Technically, the ways of finding solutions have been discussed in various papers, but I would like to emphasise the works of
Reid-Edwards and Stefanski (2010), Aharony Berkooz and Berdichevsky (2012), for the case of SCFTs
Bak, Siwach, Yee (2005), Shieh, van Anders, Van Raamsdonk (2007); Donos, Simon (2010)

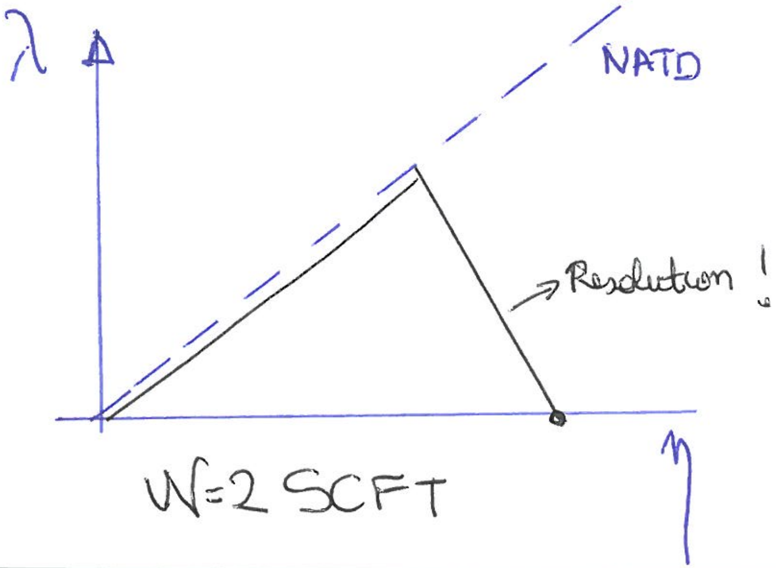
In other words, our procedure is to let the QFT inform the backgrounds what to do, how to resolve their singularities, how to bound its coordinates, if needed.

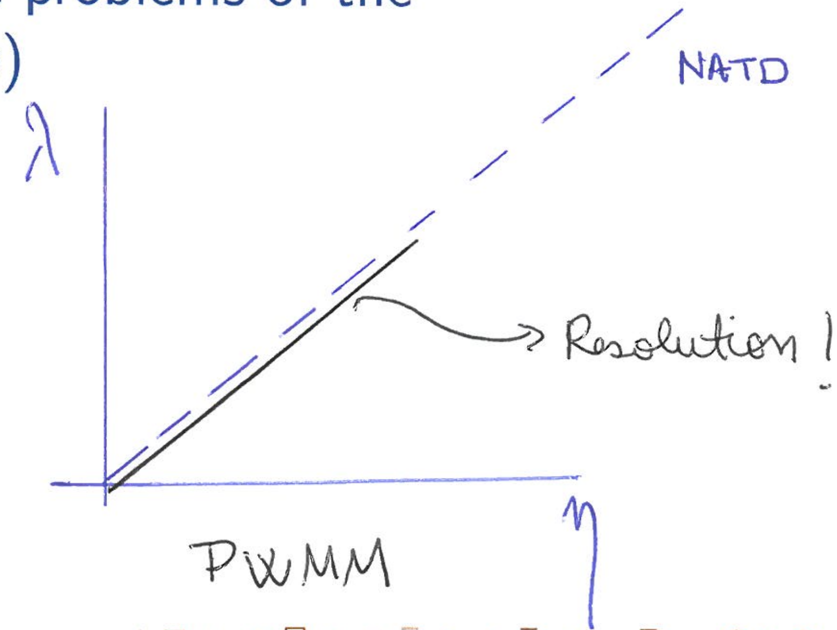
In other words, we ask a well understood 4d CFT to define for us the String Theory.

As I mentioned before, a lot of information is contained in the η-coordinate. For example, one can define a 'charge density'

$$\lambda(\eta) = \sigma \partial_\sigma V(\sigma, \eta)|_{\sigma=0} = \eta.$$

We wish to find solutions that solve the problems of the non-Abelian T-duals, by 'bounding' $\lambda(\eta)$





For the case in which the non-Abelian duality acts on S^3 inside S^5, the solution for the $V(\sigma, \eta)$ that does what we ask above is

$$\dot{V}(\sigma, \eta) = \sum_{m=1}^{\infty} A_m \sigma K_1(\frac{m\pi\sigma}{N_5}) \sin(\frac{m\pi\eta}{N_5}).$$

Where A_m are the Fourier coefficients of $\lambda(\eta)$ defined in $[0, N_5]$ and odd-periodically extended.

For the case of the matrix model, one can find the potential by a 'method of images'

$$V(\sigma, \eta) = \int_{-\infty}^{\infty} dz \frac{\lambda(z)}{\sqrt{\sigma^2 + (z-\eta)^2}}.$$

Calculating explicitly one finds the full expressions for these potentials. From here, one finds new backgrounds. ← they "complete" NATD backgrounds.

In the case of the $\mathcal{N} = 2$ SCFT, the characteristics of the new solution are

- It is fully encoded in the function $\dot{V}(\sigma, \eta)$. A solution to a Laplace problem in the $[\sigma, \eta]$ plane, with charge density $\lambda(\eta)$.
- The background is smooth, except at the position of the D6 branes. They realise the global $SU(P)$ flavor symmetry.
- The coordinate η is now bounded—like the quiver, whose length is finite.
- The geometry that before was singular at $\sigma = 1$ is now smooth there. The CFT gave a completion to the originally singular metric

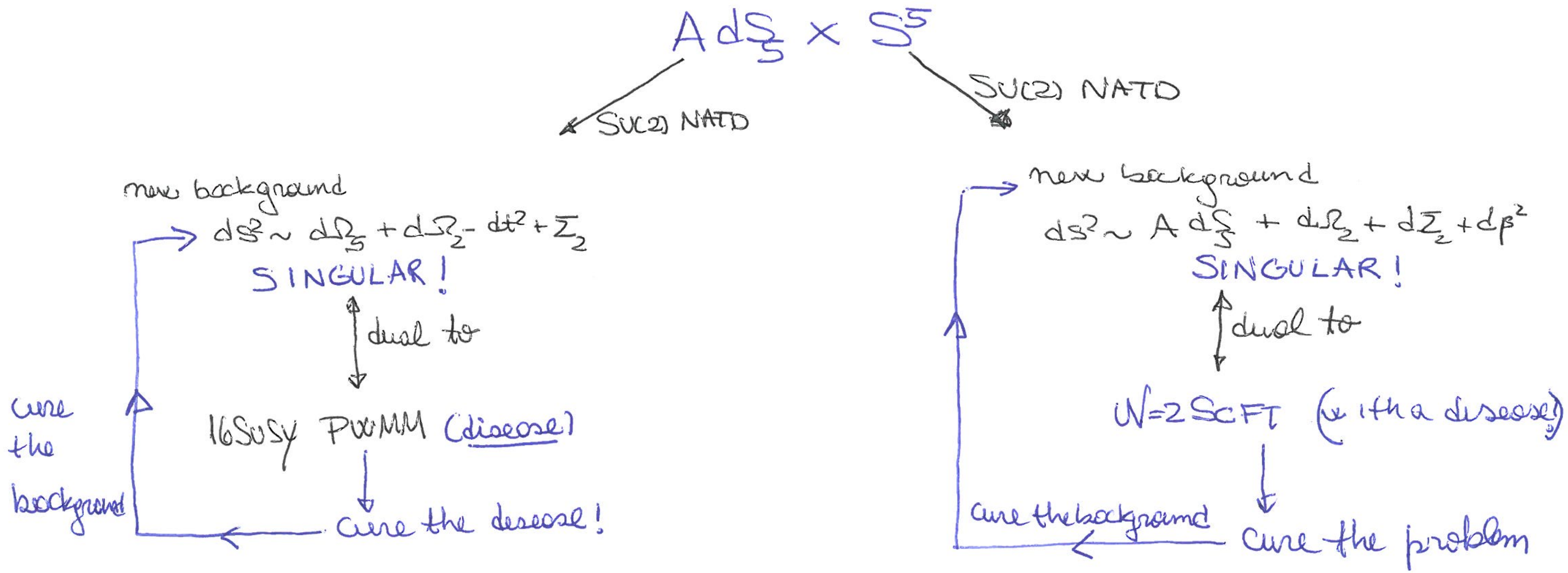
For the case of the matrix model, the new background's characteristics are:

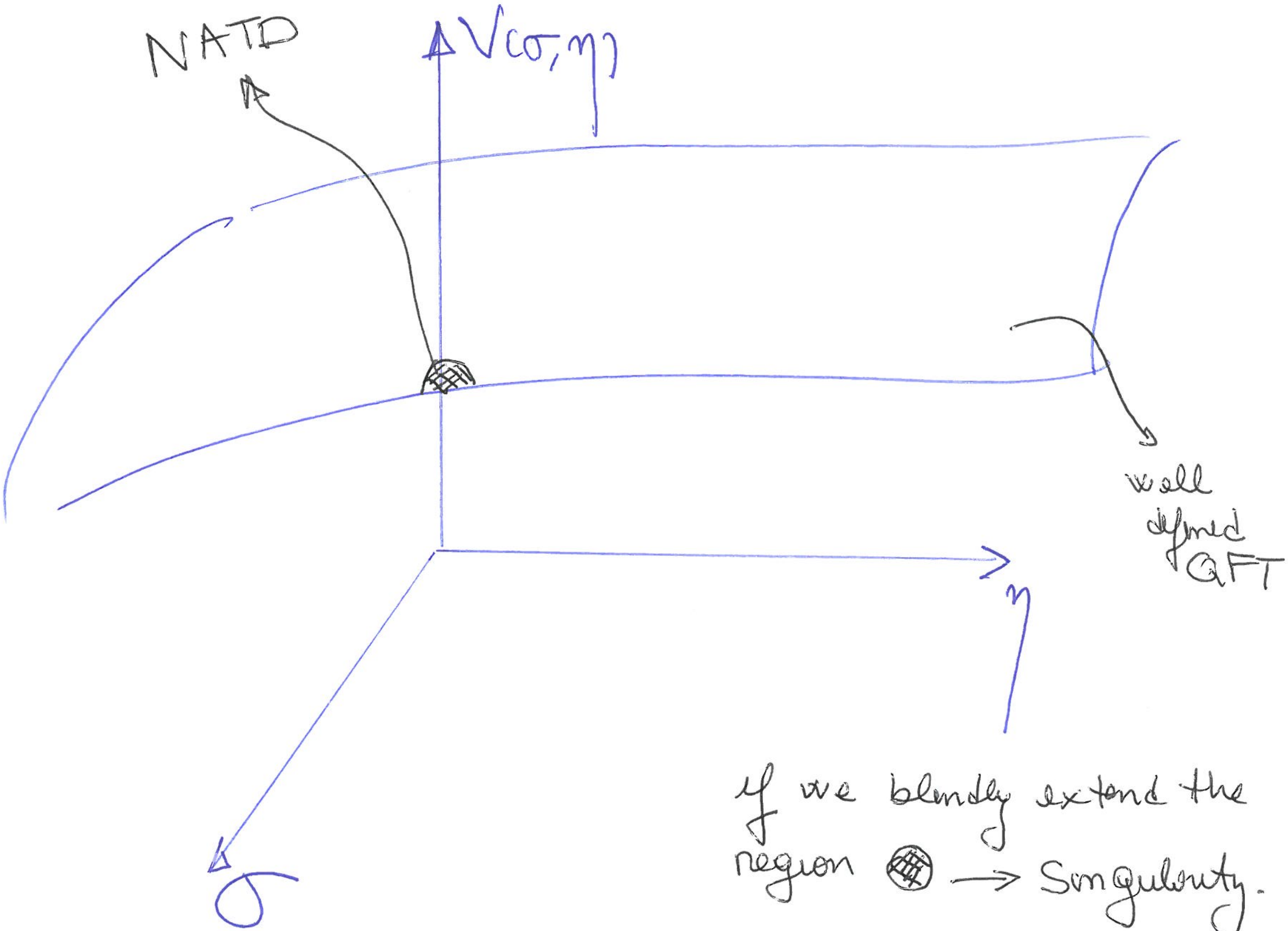
- It is fully encoded in the function $\dot{V}(\sigma, \eta)$. A solution to a Laplace problem in the $[\sigma, \eta]$ plane, with charge density $\lambda(\eta)$–bounded.
- The background is smooth, except at the position $\sigma = 0$, where a continuos charge distribution $\lambda(z)$ was placed.
- The coordinate η is not bounded, but the number of $SU(2)$ representations in the vacuum is finite, as read from $\lambda(\eta)$.
- The geometry that before was singular at $\sigma = 1$ is now smooth there. The QFT gave a completion to the originally singular metric

In both cases, one can see very explicitly that there is an scaling of the coordinates of the new 'completed' backgrounds, that gives as a result the non-Abelian T-dual ones,

In other words, we are finding either a 4d SCFT or a (0+1)d matrix model that 'complete' the ones that correspond with the non-Abelian T-dual backgrounds.

In a drawing, the idea could be expressed like this.





If we blindly extend the region ⊛ $\longrightarrow$ Singularity.

Here, we used the GFT $[V(\sigma,\eta)]$ to tell the geometry what to do!

Summary, Conclusions and Final Comments

The usual T-duality and its non-Abelian version can be seen as "solution generating techniques". Its 'stringy character' is less clear.

Using CFT information, we obtained information about the manifold generated by non-Abelian T-duality acting on $AdS_5 \times S^5$. Other examples follow a similar logic, the non-Abelian T-duality, 'focuses' on a patch of a more generic manifold.

Non-Abelian T-duality has been applied to a large variety of examples.

- Find new backgrounds, avoiding known classifications.
- Generate new backgrounds with 'dynamic' $SU(2)$-structure and other G-structures.
- Use these new backgrounds to 'define' new QFTs at strong coupling, by the calculation of the QFT observables.

The "QFT perspective" promoted in this talk may help to clarify problems and issues present in the points above.