AdS_5 non-Abelian T-dual of Klebanov-Witten as a $\mathcal{N} = 1$ linear quiver

Based on: 1705.09661

In collaboration with:
G. Itsios, Y. Lozano, C. Nuñez

Universidad de Oviedo

Jesús Montero

Recent Advances in T/U-dualities and Generalized Geometries workshop
Rudjer Bošković Institute (Zagreb). June 7, 2017.

Motivation

Solution $\equiv (g, B_2, \Phi, F_{RR})$ satisfying EOMs of 10D Supergravity

AdS/CFT: IIA(B) solution / CFT $\overset{\text{NATD}}{\longrightarrow}$ IIB(A) solution / CFT$'$

NATD not proven to be a string theory symmetry

- Pro: Solution generating technique for AdS/CFT pairs.
- Con: lacks global properties of the geometry / field theory properties.

Lozano-Núñez ('16)

- NATD of $AdS_5 \times S^5$ provides a "zoomed-in" solution of $\mathcal{N} = 2$ Gaiotto-Maldacena theories.
- AdS/CFT: Managed to "complete" the geometry using field theory input.

Our purpose

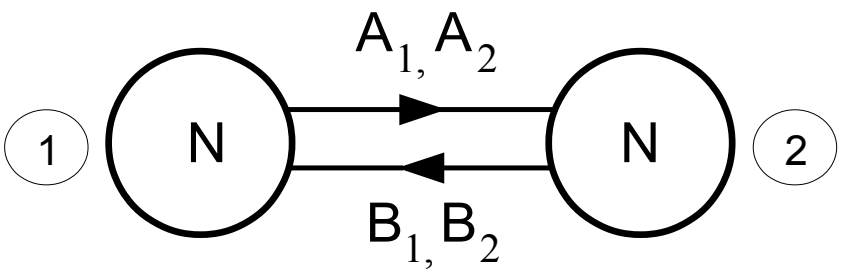
Propose a 4D $\mathcal{N}=1$ linear quiver gauge theory dual to the NATD of Klebanov-Witten, inspired by its (IIA) D4-NS5-NS5' set-up and the following conjecture,

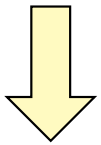
$$\begin{array}{lcc} \mathcal{N}=2: & AdS_5\times S^5/\mathbb{Z}_2 & \longrightarrow \text{ NATD of } AdS_5\times S^5/\mathbb{Z}_2 \\ & \Big\downarrow \text{mass} & \Big\downarrow \text{mass} \\ \mathcal{N}=1: & AdS_5\times T^{1,1} & \longrightarrow \text{NATD of } AdS_5\times T^{1,1} \end{array}$$

Outline

Klebanov-Witten theory ('97)

$SU(N) \times SU(N)$ gauge group with bifundamental matter fields A_1, A_2 and B_1, B_2, transforming in the $(N, \bar{N})$ and $(\bar{N}, N)$ representations of $SU(N)$, respectively:




AdS/CFT

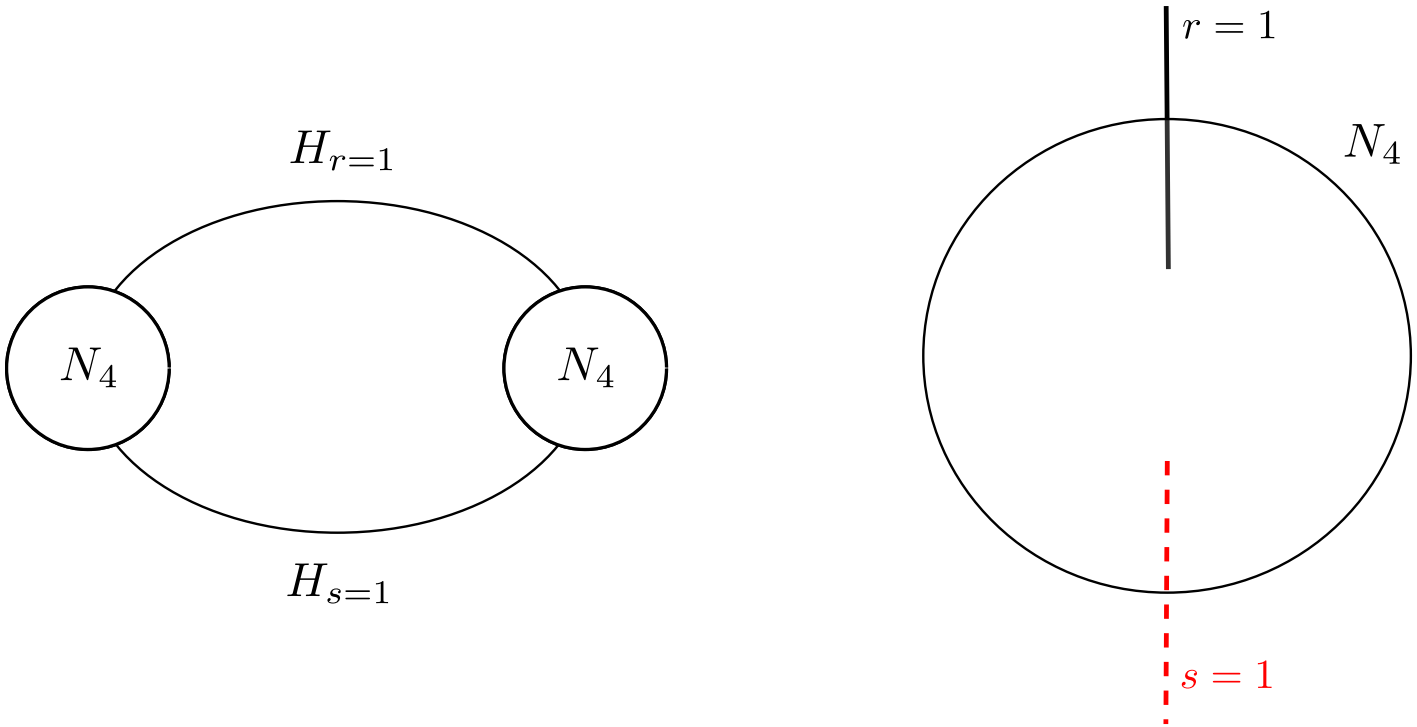
N D3-branes at the tip of the conifold with base $T^{1,1} \approx SU(2) \times SU(2)/U(1)$.
Near-horizon yields:

$$ds^2 = ds^2_{AdS_5} + L^2\, ds^2_{T^{1,1}}\,, \qquad ds^2_{AdS_5} = \frac{r^2}{L^2}\, dx^2_{1,3} + \frac{L^2}{r^2}\, dr^2\,,$$

$$ds^2_{T^{1,1}} = \lambda_1^2 ds^2(S_1^2) + \lambda_2^2 ds^2(S_2^2) + \lambda^2 (d\psi + \cos\theta_1\, d\phi_1 + \cos\theta_2\, d\phi_2)^2$$

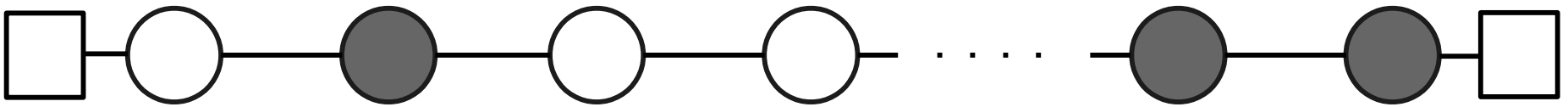
Abelian T-dual of KW

Type IIA description: D4-branes, NS5 and NS5′-branes (NS-NS field $B_2 \neq 0$) [Uranga '98], [Dasgupta, Mukhi '98]:



- NS5 $\perp$ NS5′ (rotation $\sim$ mass deformation of $\mathcal{N} = 2$ leading to $\mathcal{N} = 1$).
- Subtlety: Hopf-fiber T-dual has *Supersymmetry without supersymmetry* in the SUGRA approximation [Duff, Lu, Pope '97]. Use azimuthal T-dual.

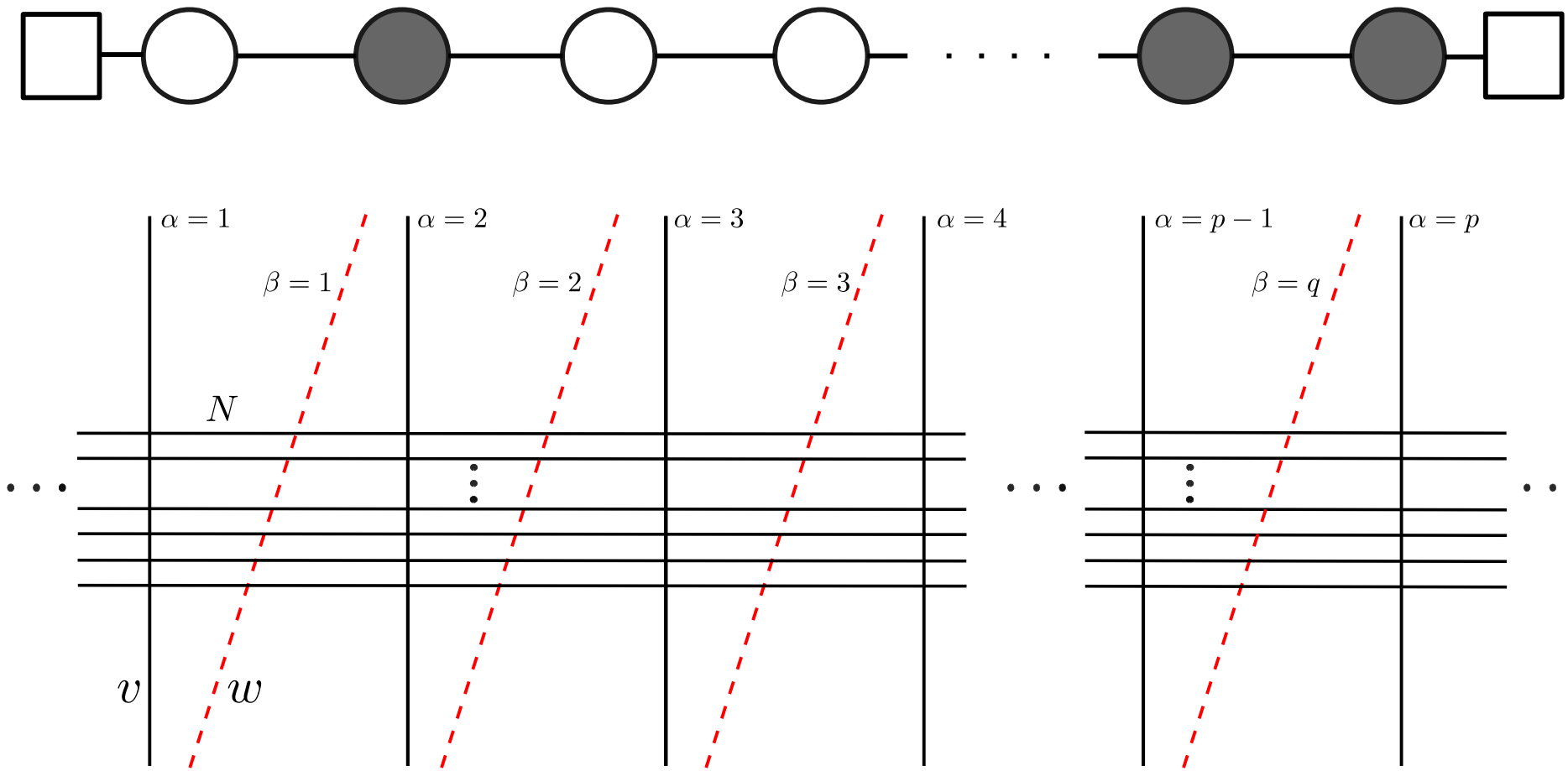
Bah-Bobev ('13) linear quivers



- $\ell - 1$ copies of $SU(N)$ gauge groups. $\mathcal{N} = 1$ (shaded) or $\mathcal{N} = 2$ (unshaded) vector multiplets.
- Lines between them represent bifundamentals of $SU(N) \times SU(N)$.
- The boxes at the two ends represent N $SU(N)$ fundamentals.
- In total $\ell - 1 = n_1 + n_2$ gauge groups and ℓ matter multiplets.
- The total global symmetry is,

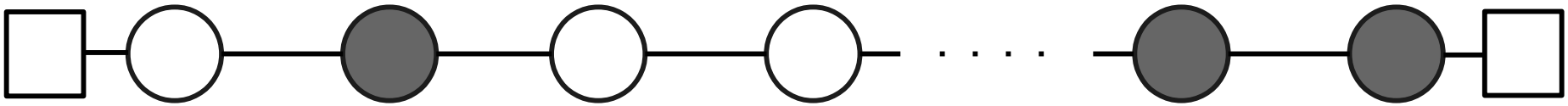
$$SU(N) \times SU(N) \times U(1)^{\ell+n_2} \times U(1)_R$$

Bah-Bobev linear quivers: Dual IIA brane set-up



N D4-branes (along x_6) extended between transversal NS5 (along v-plane) and NS5$'$ (along w-plane), being NS5 $\perp$ NS5$'$.

Bah-Bobev linear quivers: some properties



- $\mathcal{N}=2$ nodes connect same kind of hypers, $\mathcal{N}=1$ nodes connect different hypers. Twist parameter:

$$z = \frac{p-q}{\ell} \in [-1,1]$$

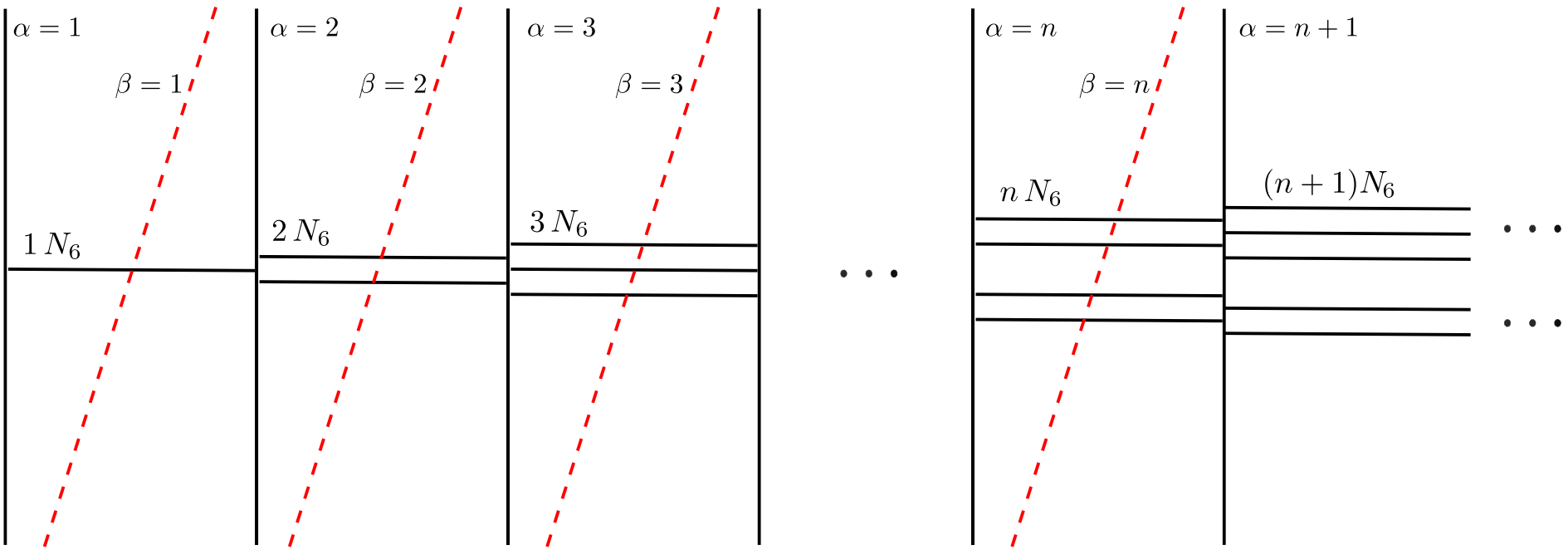
$$z=\pm 1 \implies \mathcal{N}=2 \text{ theory (only one kind of hypers/5-brane)},$$
$$z=0 \implies \mathcal{N}=1 \text{ theory (same number of both kinds of hypers)}.$$

- Conjecture: flows to IR CFT depending only on $\{N,\ell,z,\kappa\}$. Moduli space of 5-brane positions/gauge couplings of the vector mutliplets!

- M-theory origin: M5-branes wrapping a Riemann surface with $\kappa=\frac{1}{2}(\sigma_0+\sigma_\ell)\in\{-1,0,+1\}$.

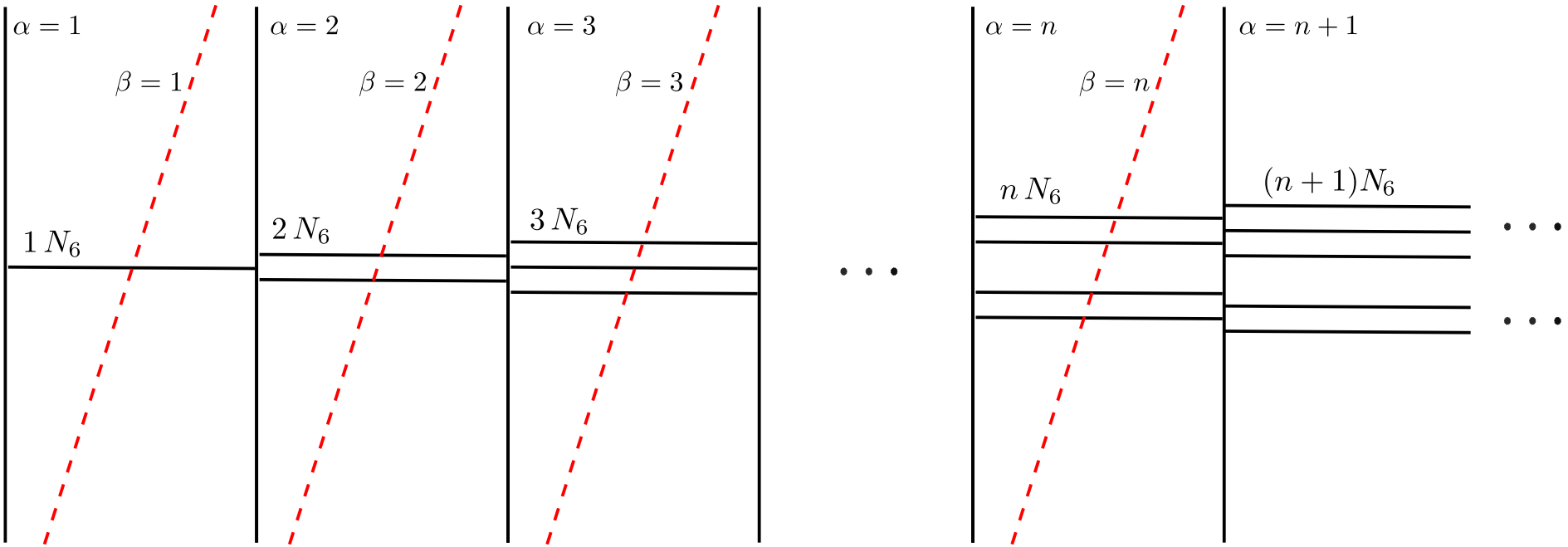
Outline

Brane set-up of the NATD of KW



- Solution originally derived in [Itsios, Núñez, Sfetsos, Thompson '12].
- (D4,NS5,NS5') brane set-up from quantized charges of the RR-fluxes.

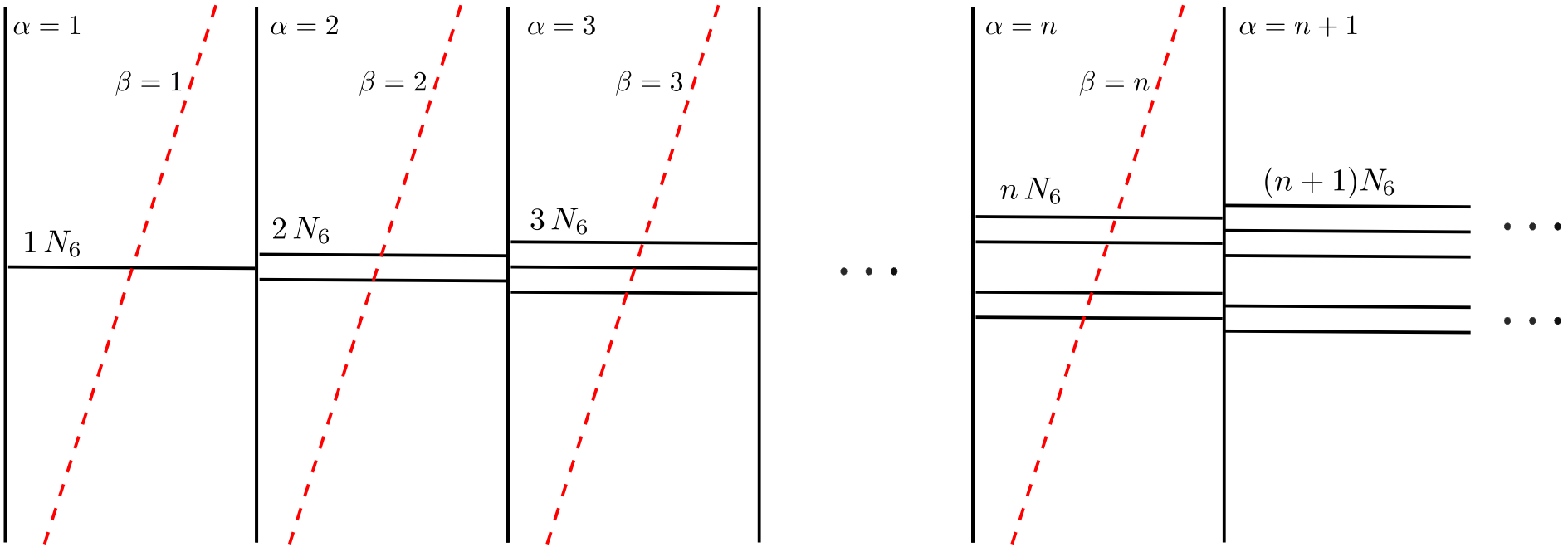
Brane set-up of the NATD of KW



- $N_4 = m\,N_6$ due to large gauge transformations each time an NS5-brane is crossed (remark: just NS5, not NS5$'$!) :

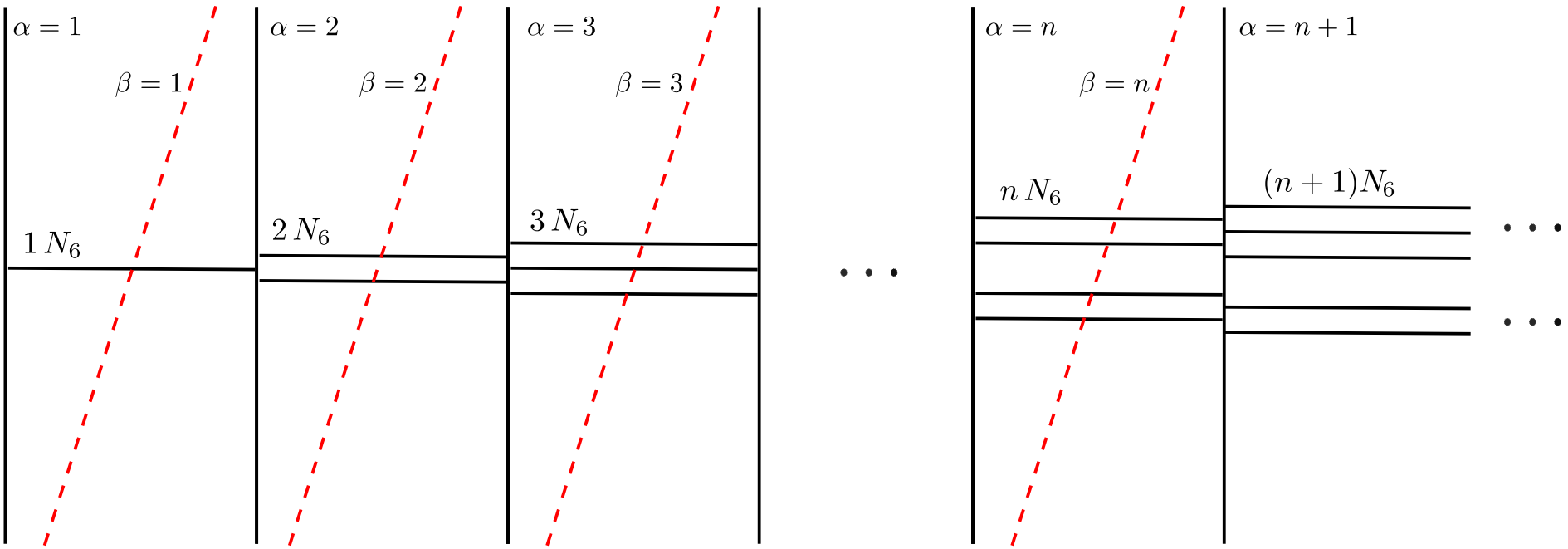
$$0 \leq \frac{1}{4\pi^2}\int_{M_2} |B_2| \leq 1 \qquad \Leftarrow \qquad B_2 \to B_2 - \alpha'\, m\, \pi\, Vol(M_2)$$

Brane set-up of the NATD of KW



- Generalizes the linear quiver gauge theories of Bah-Bobev.
- Uplift to M-theory: solution shown to fit in the general classification of $\mathcal{N} = 1$ theories with AdS_5 duals of [GMSW '04].

Brane set-up of the NATD of KW



- A priori infinite brane set-up and quiver! Needs regularization!
- Simplest option for the regularized quiver doesn't work: vanishing scaling dimensions, not an exact mass deformation of the $\mathcal{N} = 2$ theory.

Figuring out the $\mathcal{N} = 1$ quiver

$$\begin{array}{lcc} \mathcal{N}=2: & AdS_5 \times S^5/\mathbb{Z}_2 & \longrightarrow \text{ NATD of } AdS_5 \times S^5/\mathbb{Z}_2 \\ & \Big\downarrow \text{mass} & \Big\downarrow \text{mass} \\ \mathcal{N}=1: & AdS_5 \times T^{1,1} & \longrightarrow \text{NATD of } AdS_5 \times T^{1,1} \end{array}$$

Holographically:

$$\frac{c_{\mathcal{N}=1}}{c_{\mathcal{N}=2}} \approx \frac{27}{32} \longrightarrow \frac{c_{NA\ AdS_5 \times T^{1,1}}}{c_{NA\ AdS_5 \times S^5/\mathbb{Z}_2}} \approx \frac{27}{32}$$

- Consider first the completed quiver associated to the NATD of $AdS_5 \times S^5$ [Sfetsos, Thompson '10], [Lozano, Núñez '16]:

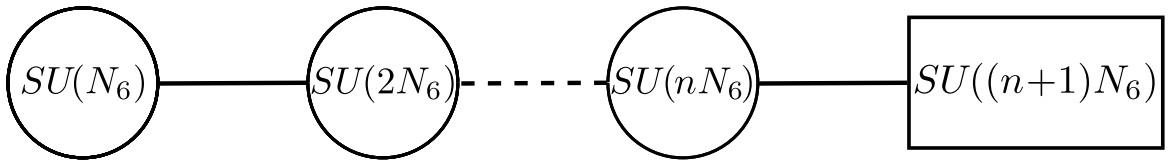
We need its $\mathbb{Z}_2$-modded version!

Figuring out the $\mathcal{N}=1$ quiver

$$\begin{array}{lcc} \mathcal{N}=2: & AdS_5\times S^5/\mathbb{Z}_2 & \longrightarrow \text{ NATD of } AdS_5\times S^5/\mathbb{Z}_2 \\ & \Big\downarrow \text{mass} & \Big\downarrow \text{mass} \\ \mathcal{N}=1: & AdS_5\times T^{1,1} & \longrightarrow \text{NATD of } AdS_5\times T^{1,1} \end{array}$$

Holographically: $\dfrac{c_{\mathcal{N}=1}}{c_{\mathcal{N}=2}}\approx\dfrac{27}{32}\longrightarrow\dfrac{c_{NA\ AdS_5\times T^{1,1}}}{c_{NA\ AdS_5\times S^5/\mathbb{Z}_2}}\approx\dfrac{27}{32}$

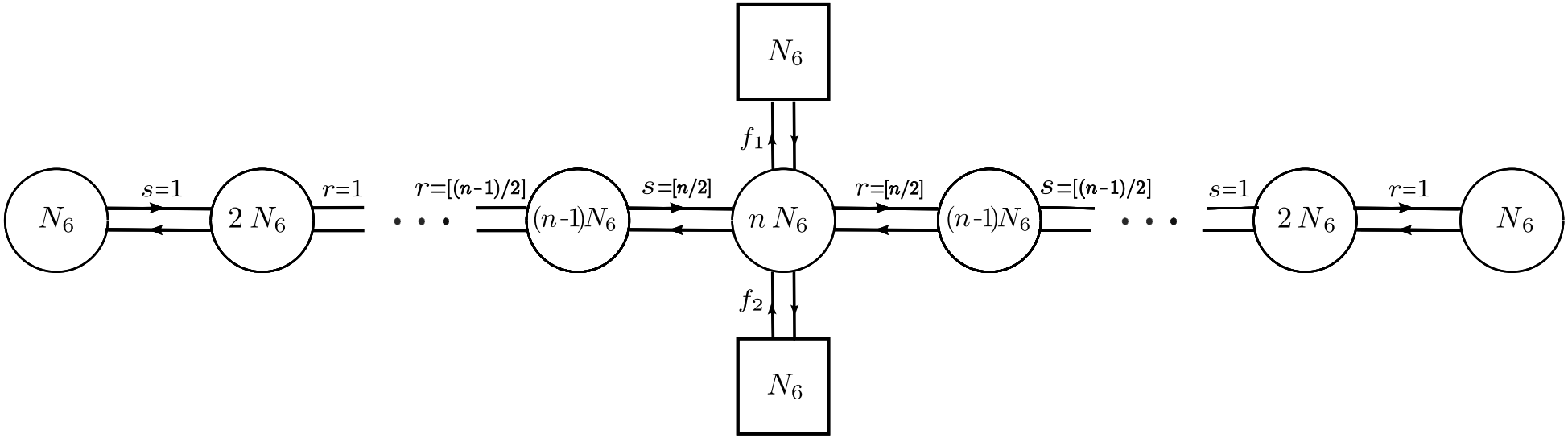
- Consider first the completed quiver associated to the NATD of $AdS_5\times S^5$ [Sfetsos, Thompson '10], [Lozano, Núñez '16]:



We need its $\mathbb{Z}_2$-modded version!

Complete linear quiver proposal for the NATD of KW

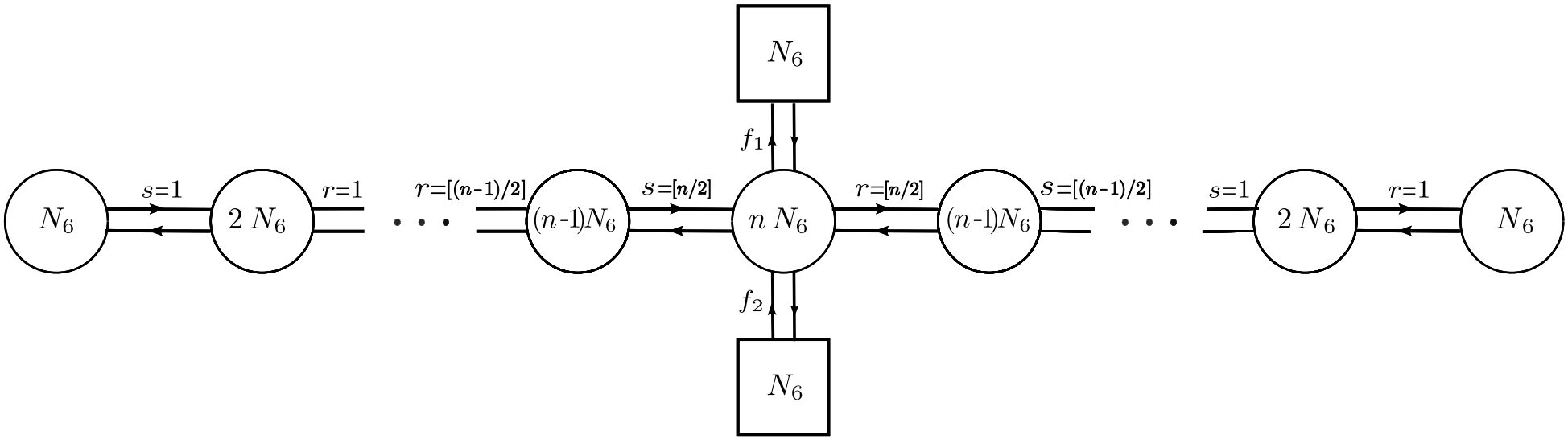
4D $\mathcal{N} = 1$ strongly coupled field theory proposal:



- Each tail $n-1$ hypers + two central hypers f_1 and f_2.
- $z = 0$: "r" hypers (NS5) paired with "s" hypers (NS5$'$), also f_1 and f_2.
- Added flavors: semi-infinite D4s or transversal D6s. They *complete* the quiver at finite n!
- Recover $\mathcal{N} = 2$: $z = \pm 1$ (i. e. all hypers either "r" or "s").

Complete linear quiver proposal for the NATD of KW

4D $\mathcal{N} = 1$ strongly coupled field theory proposal:



- Brane re-ordering consistent with Seiberg self-duality and the vanishing of the beta functions and R-symmetry anomalies.
- Central charge consistent with the NATD of KW.
- Exact cc consistent with RG flow from UV mass-deformed $\mathcal{N} = 2$ theory (requires $\sigma_{f_1} = -\sigma_{f_2}$).

A hint: Tachikawa-Wecht UV-IR relations ['09]

Exact central charges obtained with a-maximization:

[Anselmi, Freedman, Grisaru, Johansen '97], [Intriligator, Wecht '03]

$$a = \frac{3}{32}\left(3\ \mathrm{Tr}\ R_\epsilon^3 - \mathrm{Tr}\ R_\epsilon\right), \qquad c = \frac{1}{32}\left(9\ \mathrm{Tr}\ R_\epsilon^3 - 5\ \mathrm{Tr}\ R_\epsilon\right) \qquad \left(R_\epsilon = R_0 + \tfrac{1}{2}\,\epsilon\,\mathcal{F}\right)$$

If 4D $\mathcal{N}=1$ IR of mass-deformed $\mathcal{N}=2$ UV, then:

$$a_{\mathcal{N}=1} = \frac{9}{32}\left(4\,a_{\mathcal{N}=2} - c_{\mathcal{N}=2}\right), \qquad c_{\mathcal{N}=1} = \frac{1}{32}\left(-12\,a_{\mathcal{N}=2} + 39\,c_{\mathcal{N}=2}\right)$$

AdS/CFT regime: large n (long quiver) limit

$$a_{\mathcal{N}=2} \approx c_{\mathcal{N}=2} \approx \frac{1}{6}\,n^3\,N_6^2 \approx c_{NA\ AdS_5\times S^5/\mathbb{Z}_2}$$

$$a_{\mathcal{N}=1} \approx c_{\mathcal{N}=1} \approx \frac{9}{64}\,n^3\,N_6^2 \approx c_{NA\ AdS_5\times T^{1,1}}$$

$\Rightarrow$ $\dfrac{c_{\mathcal{N}=1}}{c_{\mathcal{N}=2}} \approx \dfrac{27}{32}$ holographically (large n)!

A hint: Tachikawa-Wecht UV-IR relations ['09]

Exact central charges obtained with a-maximization:

[Anselmi, Freedman, Grisaru, Johansen '97], [Intriligator, Wecht '03]

$$a = \frac{3}{32}\left(3\ \mathrm{Tr}\ R_\epsilon^3 - \mathrm{Tr}\ R_\epsilon\right), \qquad c = \frac{1}{32}\left(9\ \mathrm{Tr}\ R_\epsilon^3 - 5\ \mathrm{Tr}\ R_\epsilon\right) \qquad \left(R_\epsilon = R_0 + \tfrac{1}{2}\,\epsilon\,\mathcal{F}\right)$$

If 4D $\mathcal{N}=1$ IR of mass-deformed $\mathcal{N}=2$ UV, then:

$$a_{\mathcal{N}=1} = \frac{9}{32}\left(4\,a_{\mathcal{N}=2} - c_{\mathcal{N}=2}\right), \qquad c_{\mathcal{N}=1} = \frac{1}{32}\left(-12\,a_{\mathcal{N}=2} + 39\,c_{\mathcal{N}=2}\right)$$

AdS/CFT regime: large n (long quiver) limit

$$a_{\mathcal{N}=2} \approx c_{\mathcal{N}=2} \approx \frac{1}{6}\,n^3\,N_6^2 \approx c_{NA\ AdS_5\times S^5/\mathbb{Z}_2}$$

$$a_{\mathcal{N}=1} \approx c_{\mathcal{N}=1} \approx \frac{9}{64}\,n^3\,N_6^2 \approx c_{NA\ AdS_5\times T^{1,1}}$$

$\Rightarrow$ $\dfrac{c_{\mathcal{N}=1}}{c_{\mathcal{N}=2}} \approx \dfrac{27}{32}$ holographically (large n)!

A hint: Tachikawa-Wecht UV-IR relations ['09]

Exact central charges obtained with a-maximization:

[Anselmi, Freedman, Grisaru, Johansen '97], [Intriligator, Wecht '03]

$$a = \frac{3}{32}\left(3\ \mathrm{Tr}\ R_\epsilon^3 - \mathrm{Tr}\ R_\epsilon\right), \qquad c = \frac{1}{32}\left(9\ \mathrm{Tr}\ R_\epsilon^3 - 5\ \mathrm{Tr}\ R_\epsilon\right) \qquad \left(R_\epsilon = R_0 + \tfrac{1}{2}\,\epsilon\,\mathcal{F}\right)$$

If 4D $\mathcal{N}=1$ IR of mass-deformed $\mathcal{N}=2$ UV, then:

$$a_{\mathcal{N}=1} = \frac{9}{32}\left(4\,a_{\mathcal{N}=2} - c_{\mathcal{N}=2}\right), \qquad c_{\mathcal{N}=1} = \frac{1}{32}\left(-12\,a_{\mathcal{N}=2} + 39\,c_{\mathcal{N}=2}\right)$$

AdS/CFT regime: large n (long quiver) limit

$$a_{\mathcal{N}=2} \approx c_{\mathcal{N}=2} \approx \frac{1}{6}\,n^3\,N_6^2 \approx c_{NA\ AdS_5\times S^5/\mathbb{Z}_2}$$

$$a_{\mathcal{N}=1} \approx c_{\mathcal{N}=1} \approx \frac{9}{64}\,n^3\,N_6^2 \approx c_{NA\ AdS_5\times T^{1,1}}$$

$\Rightarrow$ $\dfrac{c_{\mathcal{N}=1}}{c_{\mathcal{N}=2}} \approx \dfrac{27}{32}$ holographically (large n)!

Outline

The INST-BBBW puzzle

[Itsios, Núñez, Sfetsos, Thompson '12 & '13] claimed NATD of KW solution belonged to class of [Bah, Beem, Bobev, Wecht '12], but $c_{NATD} \sim N_{D4}^2$ could not be reproduced from BBBW results.

The BBBW class of solutions

- M-theory solutions from wrapped M5-branes: $AdS_5 \times \Sigma_2 \times M_4$.
- Dual field theory built up from Gaiotto's T_N blocks. Characterized by parameters $\{N, g, z, \kappa\}$.

The NATD of KW

- Matching uplifted local solution for $\kappa = z = 1$.
- Gravity: not a fixed point of the BPS equations of BBBW.
- Field theory: ours is a linear quiver without Gaiotto's T_N building blocks.

The INST-BBBW puzzle

[Itsios, Núñez, Sfetsos, Thompson '12 & '13] claimed NATD of KW solution belonged to class of [Bah, Beem, Bobev, Wecht '12], but $c_{NATD} \sim N_{D4}^2$ could not be reproduced from BBBW results.

The BBBW class of solutions

- M-theory solutions from wrapped M5-branes: $AdS_5 \times \Sigma_2 \times M_4$.
- Dual field theory built up from Gaiotto's T_N blocks. Characterized by parameters $\{N, g, z, \kappa\}$.

The NATD of KW

- Matching uplifted local solution for $\kappa = z = 1$.
- Gravity: not a fixed point of the BPS equations of BBBW.
- Field theory: ours is a linear quiver without Gaiotto's T_N building blocks.

The INST-BBBW puzzle

[Itsios, Núñez, Sfetsos, Thompson '12 & '13] claimed NATD of KW solution belonged to class of [Bah, Beem, Bobev, Wecht '12], but $c_{NATD} \sim N_{D4}^2$ could not be reproduced from BBBW results.

The BBBW class of solutions

- M-theory solutions from wrapped M5-branes: $AdS_5 \times \Sigma_2 \times M_4$.
- Dual field theory built up from Gaiotto's T_N blocks. Characterized by parameters $\{N, g, z, \kappa\}$.

The NATD of KW

- Matching uplifted local solution for $\kappa = z = 1$.
- Gravity: not a fixed point of the BPS equations of BBBW.
- Field theory: ours is a linear quiver without Gaiotto's T_N building blocks.

Solving the INST-BBBW puzzle

About the NATD of KW:

- It does not belong to the class of solutions of BBBW (not BPS fixed point, quite different field theory).
- Neither is it a Bah-Bobev linear quiver theory, as it is a generalization thereof for the $\mathcal{N} = 1$ case.

Outline

Summary and conclusions

What we have done...

- Proposed completed 4D $\mathcal{N} = 1$ linear quiver theory dual to the NATD of KW.
- Shown NATD preserves the mass deformation relation between the original $\mathcal{N} = 2$ and $\mathcal{N} = 1$ theories (Tachikawa-Wecht holds for the central charges).
- Proposed a solution to the the INST-BBBW controversy.

...and some open problems

- Completion of the gravity solution: backreacted flavour D6-branes? Classifications of [Bah '13], [Tomasiello et al. '15 & '17].
- Understanding other IIA NATD solutions using field theories derived from our $\mathcal{N} = 1$ theory (cascade of Seiberg dualities).

Thank you for your attention!

Summary and conclusions

What we have done...

- Proposed completed 4D $\mathcal{N} = 1$ linear quiver theory dual to the NATD of KW.
- Shown NATD preserves the mass deformation relation between the original $\mathcal{N} = 2$ and $\mathcal{N} = 1$ theories (Tachikawa-Wecht holds for the central charges).
- Proposed a solution to the the INST-BBBW controversy.

...and some open problems

- Completion of the gravity solution: backreacted flavour D6-branes? Classifications of [Bah '13], [Tomasiello et al. '15 & '17].
- Understanding other IIA NATD solutions using field theories derived from our $\mathcal{N} = 1$ theory (cascade of Seiberg dualities).

Thank you for your attention!

Summary and conclusions

What we have done...

- Proposed completed 4D $\mathcal{N} = 1$ linear quiver theory dual to the NATD of KW.
- Shown NATD preserves the mass deformation relation between the original $\mathcal{N} = 2$ and $\mathcal{N} = 1$ theories (Tachikawa-Wecht holds for the central charges).
- Proposed a solution to the the INST-BBBW controversy.

...and some open problems

- Completion of the gravity solution: backreacted flavour D6-branes? Classifications of [Bah '13], [Tomasiello et al. '15 & '17].
- Understanding other IIA NATD solutions using field theories derived from our $\mathcal{N} = 1$ theory (cascade of Seiberg dualities).

Thank you for your attention!