T-DUALITY IN HETEROTIC STRING THEORY

Rudjer Bošković Institute
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IMPA, Rio de Janeiro

OUTLINE

1. Review the global approach to T-duality in type II string theory.

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2. Describe a global approach to T-duality in heterotic string theory.

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Joint work with David Baraglia (ATMP, **19**, 2015, 613–672)

Why a global approach?

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Step 2: Obtain the **Buscher rules** in a unified and systematic manner:

- (g, B, ϕ) via generalised metrics and densities on Courant algebroids,
- *RR*-fields and *D*-brane charges via twisted cohomology and K-theory.

A TOY EXAMPLE OF T-DUALITY

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Note that

$$\widehat{T} \cong Hom(\pi_1(T), U(1)), \qquad T \cong Hom(\pi_1(\widehat{T}), U(1)).$$

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More explicitly:

$$\mathcal{T}(\alpha) = \int_T p^*(\alpha) \wedge e^{c_1(\mathcal{P})}$$

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A GENERAL APPROACH TO T-DUALITY

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2. Principal T-bundle $X \to M$.
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Need the **Neveu-Schwarz flux**

$$[H] \in H^3(X, \mathbb{Z})$$

to construct a non-trivial T-duals.

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$n = 1$: there always exists a unique pair $(\widehat{X}, \widehat{H})$, satisfying

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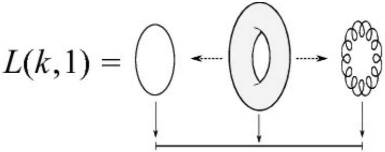
$n > 1$: **if** there exists a T-dual $(\widehat{X}, \widehat{H})$, it is **not** unique in general.

EXAMPLE OF TYPE II T-DUALS

- **Lens space** $L(k;q) = S^3/\mathbb{Z}_k$ is a S^1-bundle over $\mathbb{CP}^1$ (for a suitable q)



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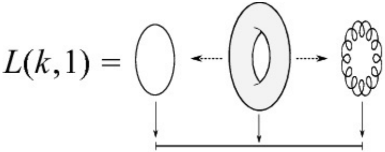

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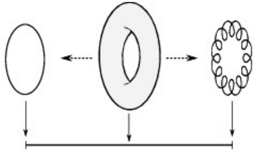
$L(k,1) =$ 

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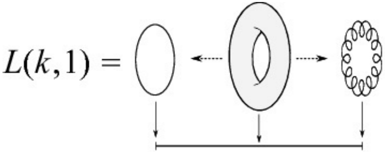

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- In particular, $(S^3, 0)$ is T-dual to $(S^2 \times S^1, 1)$.

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- Pick connections $\theta, \widehat{\theta}$ on $X, \widehat{X}$, with curvatures $F, \widehat{F}$ respectively. Then

$$H = h + \widehat{F} \wedge \theta, \qquad \widehat{H} = h + F \wedge \widehat{\theta} \qquad p^* H - \widehat{p}^* \widehat{H} = d\mathcal{F},$$

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- Isomorphism between twisted K-theory groups:

$$\mathcal{T} \colon K^*(X, H) \to K^{n-*}(\widehat{X}, \widehat{H}), \quad \beta \mapsto \widehat{p}_*(p^*(\beta) \otimes \mathcal{P})$$

  where $\mathcal{P}$ is a **twisted Poincaré line bundle**.

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The map $\Phi\colon E^T \to \widehat{E}^{\widehat{T}}$ defined by

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The Buscher rules for $(\widehat{g}, \widehat{B})$ is read off from $\widehat{G} = \Phi(G)$.

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HETEROTIC STRING THEORY

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- A **string structure** on X consists of a principal G-bundle $\sigma \colon P \to X$ and $[H] \in H^3(P, \mathbb{Z})$ which restricts fiberwise to the generator of $H^3(G, \mathbb{Z})$.

Heterotic T-duality

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- **Idea**: apply ordinary type II T-dualiy upstairs!
- The existence of a T-dual $(\widehat{P}, \widehat{H})$ imposes the usual constraints on H.

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- No guaranty that the G-action on P_0 lifts to an action on $\widehat{P}$ commuting with the $\widehat{T}$-action.
- In fact, there is a topological obstruction measured by $\kappa \in H^2(G, (\mathbb{Z}^n)^*)$.

STEP 1: Thus, $\widehat{X} \to M$ exists and the diagram commutes if and only if $\kappa = 0$:

$$
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- If $\kappa = 0$, then $\widehat{X}$ is unique and $(\widehat{P}, \widehat{H})$ is a dual string structure on $\widehat{X}$.

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$$\alpha : \mathfrak{g} \to \Gamma(E), \quad v \mapsto \psi(v) + \xi(v),$$

where ψ denotes the infinitesimal G-action on TP, and $\xi(\cdot) = k(A, \cdot)$ with k the Killing form.

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- The reduction

$$E^{red} = im(\alpha)^{\perp}/G \cong TX \oplus ad(P) \oplus T^*X$$

  is a transitive Courant algebroid on X.

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THEOREM

The Cavalcanti-Gualtieri isomorphism

$$\Phi\colon E^T \to \widehat{E}^{\widehat{T}}$$

exchanges the extended actions α and $\widehat{\alpha}$, so we have

$$(E_{red})^T \cong (E^T)_{red} \cong (\widehat{E}^{\widehat{T}})_{red} \cong (\widehat{E}_{red})^{\widehat{T}}$$

Heterotic T-duality

Step 2: The triple (g, H, A) defines a generalised metric on

$$E_{red} = TX \oplus ad(P) \oplus T^*X.$$

Theorem

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The heterotic Buscher rules are obtained via reduction of generalised metrics.

Examples of heterotic T-duals

- The first Chern classes of X and $\widehat{X}$ satisfy:

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- **Example 1**: The lens spaces $S^{2n+1}/\mathbb{Z}_c$ and $S^{2n+1}/\mathbb{Z}_{\widehat{c}}$ are heterotic T-duals with $P_0 \to \mathbb{CP}^n$ the spin bundle, if $n > 3$ odd and $c\widehat{c} = \frac{1}{2}(n+1)$.

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Thank you!