Compactifications on stringy-size tori from double field theory

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In collaboration with

G. Aldazabal, S. Iguri, M. Mayo, C. Nuñez, A. Rosabal — arXiv:1510.07644

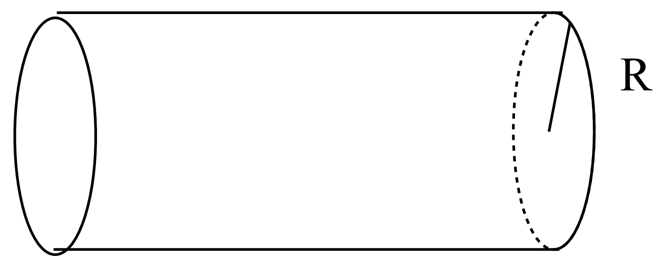
Y. Cagnacci, S. Iguri, C. Nuñez — arXiv:1704.04242

"Recent advances in T/U dualities and generalized geometries"
Zagreb, June 2017

Motivation

Closed string

ℓ_s



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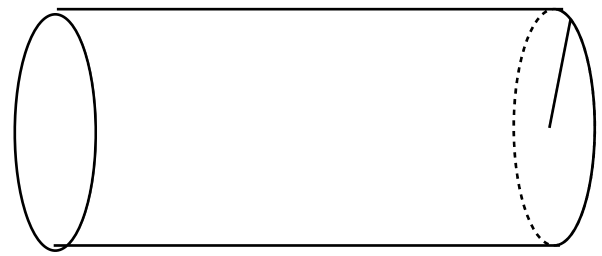

$R \gg \ell_s$

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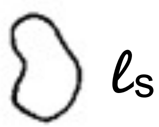

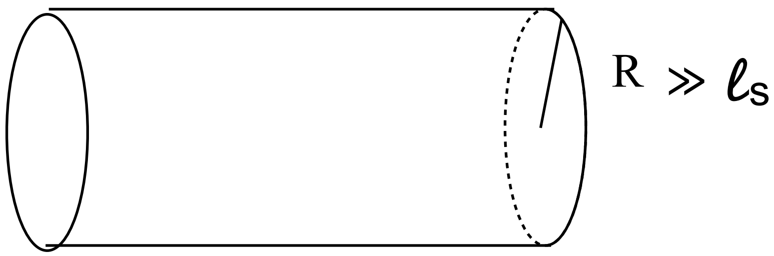

effective theory from
KK reduction of 10d sugra

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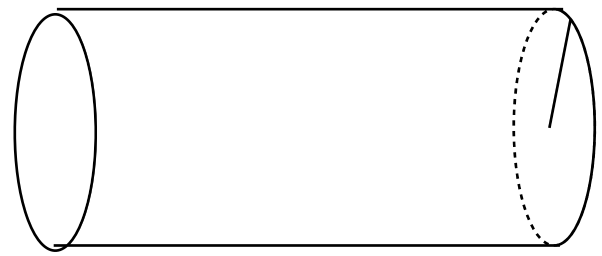

effective theory from
KK reduction of 10d sugra

Keep only zero modes of KK tower $\rightarrow$ effective description valid at $E << \frac{1}{R} << \frac{1}{\sqrt{\alpha'}}$

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R $= \ell_s = \sqrt{\alpha'}$

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~~effective theory from KK reduction of 10d sugra~~

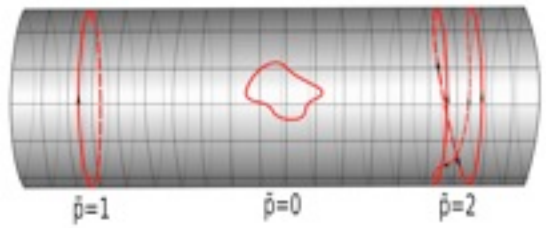
Keep only zero modes of KK tower $\rightarrow$ effective description valid at $E << \frac{1}{R} << \frac{1}{\sqrt{\alpha'}}$

Can we get an effective description valid at $E << \frac{1}{R} \sim \frac{1}{\sqrt{\alpha'}}$

Strings on S^1 at $R \sim \sqrt{\alpha'}$ (Closed bosonic string)

Strings on S[1] at $R \sim \sqrt{\alpha'}$ (Closed bosonic string)

$$\tilde{R} = \frac{\alpha'}{R}$$



momentum # winding #

Hamiltonian $$M^2 = \frac{2}{\alpha'}(N + \bar{N} - 2) + \frac{p^2}{R^2} + \frac{\tilde{p}^2}{\tilde{R}^2}$$

Level-matching $$\bar{N} - N = p\tilde{p}$$

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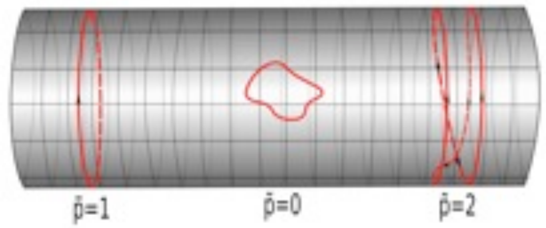

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Besides $N = \bar{N} = 1$ kept in sugra, at $R = \tilde{R} = \sqrt{\alpha'}$

Extra massless states for ex: $\bar{N} = 1, N = 0 \quad p = \tilde{p} = \pm 1$

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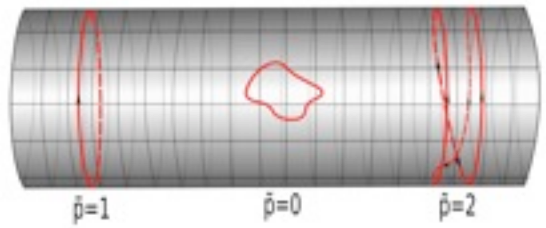

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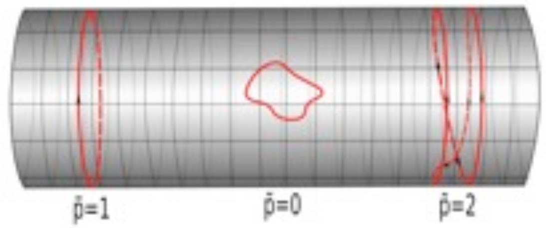

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Double Field Theory gives a good effective description of the physics including these modes at $E << \frac{1}{R} \sim \frac{1}{\sqrt{\alpha'}}$

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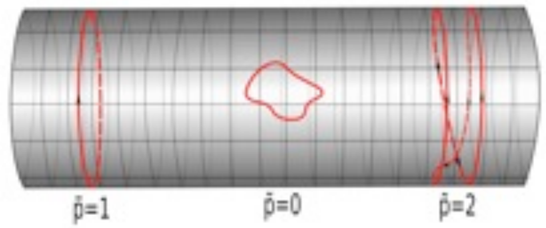

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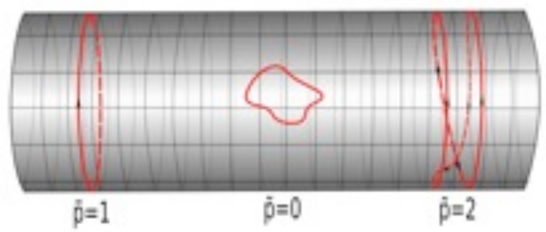

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Effective action for compactifications of bosonic string on stringy-size T^d from DFT

Double field theory

Hull & Zwiebach 06

Double field theory

Hull & Zwiebach 06

Field theory incorporating T-duality

momentum	$p \longleftrightarrow y$		compact coordinate
winding	$\tilde{p} \longleftrightarrow \tilde{y}$		new, dual coordinate

Double field theory

Hull & Zwiebach 06

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However, it requires constraints

Level matching condition $$\underbrace{\bar{N} - N}_{=0 \text{ in usual massless states}} = \underbrace{\overset{\partial_y}{p}\,\overset{\partial_{\tilde{y}}}{\tilde{p}}}_{=0} \quad \Rightarrow \quad \partial_y \partial_{\tilde{y}}(\quad) = 0$$

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$$\eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

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$$\eta^{MN} \partial_M \partial_N (\quad) = 0 \qquad \text{weak constraint}$$

$M, N = 1, ..., 2D$

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$$\partial_{y^L} = \partial_y + \partial_{\tilde{y}}$$
$$\partial_{y^R} = \partial_y - \partial_{\tilde{y}}$$

$$\eta^{LR} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Level matching condition $\underbrace{\bar{N} - N}_{=0} = \underbrace{p\tilde{p}}_{=0}$ $\Rightarrow$ $\partial_y \partial_{\tilde{y}}(\quad) = 0$

=0 in usual massless states

$p \leftarrow \partial_y$, $\tilde{p} \leftarrow \partial_{\tilde{y}}$

$$\eta^{MN}\partial_M \partial_N(\quad) = 0$$ weak constraint

$M = 1, \ldots, 2D$

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$1, ..., 2D$

weak constraint

Weak constraint not enough $\Rightarrow$ $\partial_M(\quad)\partial^M(\quad) = 0$

strong constraint
or
section condition

Double field theory

Hull & Zwiebach 06

Field theory incorporating T-duality

At special points in the torus moduli space there are extra massless states with momentum or winding.
We will not include them here

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Level matching condition

$$\partial_y \qquad \partial_{\tilde{y}}$$

$$\underbrace{\bar{N} - N}_{=0 \text{ in usual massless states}} = \underbrace{p\tilde{p}}_{=0} \quad \Rightarrow \quad \begin{aligned} &\partial_y \partial_{\tilde{y}}(\quad) = 0 \\ &\eta^{MN}\partial_M \partial_N(\quad) = 0 \end{aligned}$$

weak constraint

$M, N = 1, \dots, 2D$

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Include winding modes here

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Level matching condition $\underbrace{\bar{N} - N}_{\neq 0 \text{ in usual massless states}} = \underbrace{\overset{\partial_y}{p}\,\overset{\partial_{\tilde{y}}}{\tilde{p}}}_{\neq 0}$ $\Rightarrow$

$$\partial_y \partial_{\tilde{y}}(\quad) = 0$$

$$\eta^{MN} \partial_M \partial_N (\quad) = 0 \qquad M, N = 1, ..., 2D$$

~~weak constraint~~

Weak constraint not enough $\Rightarrow$ $\partial_M(\quad)\partial^M(\quad) = 0$

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$1, \ldots, 2D$

~~weak constraint~~

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Include winding modes here , violating weak constraint

(though satisfying level matching condition)

Strong constraint sufficient but not necessary

Efforts in trying to get consistency while relaxing strong constraint

- Necessary and sufficient conditions for closure of algebra — M.G., Marques 12

Interpretation in a generic context obscure...

But in the context of "Generalized Sherk-Schwarz reductions" (leading to gauged maximal or half-maximal sugra)

Closure of algebra $\Leftrightarrow$ quadratic constraints of gauged sugra

weaker than strong constraint (also weak $\Leftrightarrow$ strong in GSS)

Bosonic string on S^1

Massless states at $R = \tilde{R} = 1$

Mass $M^2 = 2(N + \bar{N} - 2) + \dfrac{p^2}{R^2} + \dfrac{\tilde{p}^2}{\tilde{R}^2}$

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Vectors $\bar{N}_x = 1$

- $N_y = 1 \quad (g_{\mu y} + B_{\mu y})$
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Level-matching $\bar{N} - N = p\tilde{p}$

Vectors $\bar{N}_x = 1$

- $N_y = 1 \quad (g_{\mu y} + B_{\mu y})$ $\qquad V \sim J^3(z) \cdot (\bar{\partial} X^\mu e^{ikX})$
- $N_y = 0 \quad p = \tilde{p} = \pm 1 \quad (k_L = \pm 2)$ $\qquad V \sim J^\pm(z) \cdot (\bar{\partial} X^\mu e^{ikX})$

$$\boxed{\begin{aligned} J^3(z) &= \partial Y^L(z) \\ J^\pm(z) &= \mathrm{e}^{\pm 2iY^L(z)} \end{aligned}}$$

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- SU(2)$_L$ Vectors $\bar{N}_x = 1$

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- SU(2)$_R$ Vectors $N_x = 1 \qquad A^i \to \bar{A}^i$

$J^i(z) \to \bar{J}^i(\bar{z}) \qquad Y^L(z) \to Y^R(\bar{z})$

$$J^3(z) = \partial Y^L(z)$$

$$J^\pm(z) = \mathrm{e}^{\pm 2iY^L(z)}$$

$$J^i(z) J^j(0) \sim \frac{\delta^{ij}}{z^2} + \frac{i\,\epsilon^{ijk}}{z} J^k(0)$$

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- Scalars $N_x = \bar{N}_x = 0$

$$N_y = 1, \bar{N}_y = 1 \ \ (g_{yy})$$

$$N_y = 1, p = -\tilde{p} = \pm 1 \ (\bar{k} = \pm 2)$$

$$\bar{N}_y = 1, p = \tilde{p} = \pm 1 \ (k = \pm 2)$$

$$p = \pm 2, \tilde{p} = 0 \ (k = \bar{k} = \pm 2)$$

$$p = 0, \tilde{p} = \pm 2 \ (k = -\bar{k} = \pm 2)$$

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Massless states at $R = \tilde{R} = 1$

- SU(2)$_L$ Vectors $\bar{N}_x = 1$

  - $N_y = 1 \quad (g_{\mu y} + B_{\mu y}) \qquad : A^3_\mu \qquad V \sim J^3(z) \cdot (\bar{\partial} X^\mu e^{ikX})$
  - $N_y = 0 \quad p = \tilde{p} = \pm 1 \quad (k_L = \pm 2) : A^\pm_\mu \qquad V \sim J^\pm(z) \cdot (\bar{\partial} X^\mu e^{ikX})$

- SU(2)$_R$ Vectors $N_x = 1 \qquad A^i \to \bar{A}^i \qquad J^i(z) \to \bar{J}^i(\bar{z}) \qquad Y^L(z) \to Y^R(\bar{z})$

- Scalars (3,3) $N_x = \bar{N}_x = 0$

$N_y = 1, \bar{N}_y = 1 \;\; (g_{yy}) \qquad : \quad M^{33}$

$N_y = 1, p = -\tilde{p} = \pm 1 \;\; (\bar{k} = \pm 2) \qquad : \quad M^{3\pm}$

$\bar{N}_y = 1, p = \tilde{p} = \pm 1 \;\; (k = \pm 2) \qquad : \quad M^{\pm 3}$

$p = \pm 2, \tilde{p} = 0 \;\; (k = \bar{k} = \pm 2) \qquad : \quad M^{\pm\pm}$

$p = 0, \tilde{p} = \pm 2 \;\; (k = -\bar{k} = \pm 2) \qquad : \quad M^{\pm\mp}$

$$V^{ij} \sim J^i J^j \, e^{ikX}$$

$$\boxed{\begin{aligned} J^3(z) &= \partial Y^L(z) \\ J^\pm(z) &= \mathrm{e}^{\pm 2 i Y^L(z)} \end{aligned}}$$

$$J^i(z) J^j(0) \sim \frac{\delta^{ij}}{z^2} + \frac{i\,\epsilon^{ijk}}{z} J^k(0)$$

Symmetry enhancement (recap)

$$R \neq 1 (\neq \tilde{R})$$

$$\color{red}{U(1)} \times \color{blue}{U(1)}$$

$\color{red}{A}$ $\color{blue}{\bar{A}}$

2 vectors

$(g_{\mu y} \pm B_{\mu y})$

M

1 scalar

(g_{yy})

Symmetry enhancement (recap)

$$R \neq 1 (\neq \tilde{R}) \qquad\qquad R = \tilde{R} = 1$$

$$U(1) \times U(1) \longrightarrow SU(2) \times SU(2)$$

A^3 $\bar{A}^3$ A^i $\bar{A}^i$ $i = \pm, 3$

$i = \pm, 3$

2 vectors

$(g_{\mu y} \pm B_{\mu y})$

6 vectors

M

1 scalar

(g_{yy})

Symmetry enhancement (recap)

$R \neq 1 (\neq \tilde{R})$ $\qquad$ $R = \tilde{R} = 1$

$$U(1) \times U(1) \longrightarrow SU(2) \times SU(2)$$

A^3 $\quad$ $\bar{A}^3$ $\qquad$ A^i $\quad$ $\bar{A}^i$ $\qquad$ $i = \pm, 3$

$i = \pm, 3$

2 vectors

$(g_{\mu y} \pm B_{\mu y})$

6 vectors

M^{33}

1 scalar

(g_{yy})

M^{ij}

9 scalars

Effective action from string theory

Computing 3-point functions <VVV> we read off

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} F^{i}_{\mu\nu} F^{i\mu\nu} + \frac{1}{4} \bar{F}^{i}_{\mu\nu} \bar{F}^{i\mu\nu}$$

$$+ \frac{1}{4} M^{ij} F^{i}_{\mu\nu} \bar{F}^{j\mu\nu} + D_{\mu} M^{ij} D^{\mu} M^{ij} - \det M$$

Effective action from string theory

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$$\mathcal{L} \;=\; R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \frac{1}{4} \bar{F}^i_{\mu\nu} \bar{F}^{i\mu\nu}$$

$$+\frac{1}{4} M^{ij} F^i_{\mu\nu} \bar{F}^{j\mu\nu} + D_\mu M^{ij} D^\mu M^{ij} - \det M$$

$$H = dB + A^i \wedge F^i + \epsilon_{ijk} A^i \wedge A^j \wedge A^k$$

$$- \bar{A}^i \wedge \bar{F}^i - \epsilon_{ijk} \bar{A}^i \wedge \bar{A}^j \wedge \bar{A}^k$$

$$F^i = dA^i + \epsilon^{ijk} A^j \wedge A^k$$

$$D_\mu M^{ii} = \partial_\mu M^{ii} + f^{ijk} A^j_\mu M^{ki} + f^{ijk} \bar{A}^j_\mu M^{ik}$$

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Higgs mechanism

$$M^{ij} \to \epsilon\, \delta^{ij}_{33} + M'^{ij}$$

Effective action from string theory

Computing 3-point functions <VVV> we read off

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \frac{1}{4} \bar{F}^i_{\mu\nu} \bar{F}^{i\mu\nu}$$

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$$- \bar{A}^i \wedge \bar{F}^i - \epsilon_{ijk} \bar{A}^i \wedge \bar{A}^j \wedge \bar{A}^k$$

$A^{\pm}$, $\bar{A}^{\pm}$ acquire mass[2] $= \epsilon^2$

$$F^i = dA^i + \epsilon^{ijk} A^j \wedge A^k$$

$$D_\mu M^{ii} = \partial_\mu M^{ii} + f^{ijk} A^j_\mu M^{ki} + f^{ijk} \bar{A}^j_\mu M^{ik}$$

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Effective action from string theory

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$$- \bar{A}^i \wedge \bar{F}^i - \epsilon_{ijk} \bar{A}^i \wedge \bar{A}^j \wedge \bar{A}^k$$

$A^{\pm}$

$\bar{A}^{\pm}$ acquire mass^2 $= \epsilon^2$

$$F^i = dA^i + \epsilon^{ijk} A^j \wedge A^k$$

SU(2) x SU(2) → U(1) x U(1)

$$D_\mu M^{ii} = \partial_\mu M^{ii} + f^{ijk} A^j_\mu M^{ki} + f^{ijk} \bar{A}^j_\mu M^{ik}$$

Higgs mechanism

$$M^{ij} \to \epsilon\, \delta^{ij}_{33} + M'^{ij}$$

Effective action from string theory

Computing 3-point functions <VVV> we read off

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \frac{1}{4} \bar{F}^i_{\mu\nu} \bar{F}^{i\mu\nu}$$

$$+ \frac{1}{4} M^{ij} F^i_{\mu\nu} \bar{F}^{j\mu\nu} + D_\mu M^{ij} D^\mu M^{ij} - \det M$$

$M^{\pm\pm}, M^{\pm\mp}$

acquire mass^2 $= \epsilon$

$$H = dB + A^i \wedge F^i + \epsilon_{ijk} A^i \wedge A^j \wedge A^k$$

$$- \bar{A}^i \wedge \bar{F}^i - \epsilon_{ijk} \bar{A}^i \wedge \bar{A}^j \wedge \bar{A}^k$$

$A^\pm$

acquire mass^2 $= \epsilon^2$

$\bar{A}^\pm$

$$F^i = dA^i + \epsilon^{ijk} A^j \wedge A^k$$

SU(2) x SU(2) $\rightarrow$ U(1) x U(1)

$$D_\mu M^{ii} = \partial_\mu M^{ii} + f^{ijk} A^j_\mu M^{ki} + f^{ijk} \bar{A}^j_\mu M^{ik}$$

Higgs mechanism

$$M^{ij} \rightarrow \epsilon\, \delta^{ij}_{33} + M'^{ij}$$

Bosonic string on T^d

Narain 86

Massless states:

$g_{\mu m}, B_{\mu m}$ 2d vectors: $U(1)^d \times U(1)^d$

g_{mn}, B_{mn} d^2 scalars

Bosonic string on T^d

Narain 86

Massless states:

$g_{\mu m}, B_{\mu m}$ 2d vectors: $U(1)^d \times U(1)^d$

g_{mn}, B_{mn} d^2 scalars

+

lots of extra vectors & scalars
with mom & winding at points of
enhancement where

$$\mathcal{H} = \mathcal{H}^{-1}$$

2dx2d

$$\mathcal{H} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}$$

$$\mathcal{H}^{-1} = \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

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$$\mathcal{H} = \mathcal{H}^{-1}$$

(up to SL(k, $\mathbb{Z}$) and
B → B + n)

2dx2d

$$\mathcal{H} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}$$

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Bosonic string on T^d

Narain 86

S^1 $$M^2 = 2(N + \bar{N} - 2) + \frac{p^2}{R^2} + \frac{\tilde{p}^2}{\tilde{R}^2}$$
$$0 = N - \bar{N} + p\tilde{p}$$

Massless states:

$g_{\mu m}, B_{\mu m}$ 2d vectors: $U(1)^d \times U(1)^d$

g_{mn}, B_{mn} d^2 scalars

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lots of extra vectors & scalars with mom & winding at points of enhancement where

$$\mathcal{H} = \mathcal{H}^{-1}$$ (up to $SL(k, \mathbb{Z})$ and $B \to B + n$)

2dx2d

$$\mathcal{H} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}$$

$$\mathcal{H}^{-1} = \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

Mass $$M^2 = 2(N + \bar{N} - 2) + Z^t \mathcal{H} Z \qquad Z = \begin{pmatrix} p_m \\ \tilde{p}^m \end{pmatrix}$$

Level-matching $$0 = (N - \bar{N}) + \tfrac{1}{2} Z^t \eta Z$$

Bosonic string on T^d

Narain 86

S^1
$$M^2 = 2(N+\bar{N}-2) + \frac{p^2}{R^2} + \frac{\tilde{p}^2}{\tilde{R}^2}$$
$$0 = N - \bar{N} + p\tilde{p}$$

Massless states:

$g_{\mu m}, B_{\mu m}$ 2d vectors: U(1)^d x U(1)^d

g_{mn}, B_{mn} d^2 scalars

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lots of extra vectors & scalars with mom & winding at points of enhancement where

$$\mathcal{H} = \mathcal{H}^{-1}$$

2dx2d

(up to SL(k, $\mathbb{Z}$) and B → B + n)

$$\mathcal{H} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}$$

$$\mathcal{H}^{-1} = \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

Mass
$$M^2 = 2(N+\bar{N}-2) + Z^t \underbrace{\mathcal{H}}_{E^T E} Z \qquad Z = \begin{pmatrix} p_m \\ \tilde{p}^m \end{pmatrix}$$

Level-matching
$$0 = (N - \bar{N}) + \tfrac{1}{2} Z^t \eta Z$$

Bosonic string on T^d

Narain 86

S[1] $$M^2 = 2(N + \bar{N} - 2) + \frac{p^2}{R^2} + \frac{\tilde{p}^2}{\tilde{R}^2}$$

$$0 = N - \bar{N} + p\tilde{p}$$

Massless states:

$g_{\mu m}, B_{\mu m}$ 2d vectors: $U(1)^d \times U(1)^d$

g_{mn}, B_{mn} d^2 scalars

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lots of extra vectors & scalars
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$$\mathcal{H} = \mathcal{H}^{-1}$$ (up to $SL(k, \mathbb{Z})$) and $B \to B + n$)

2dx2d

$$\mathcal{H} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}$$

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Mass $$M^2 = 2(N + \bar{N} - 2) + Z^t \underbrace{\mathcal{H}}_{E^T E} Z \qquad Z = \begin{pmatrix} p_m \\ \tilde{p}^m \end{pmatrix}$$

Level-matching $$0 = (N - \bar{N}) + \tfrac{1}{2} Z^t \eta Z$$

$$p = EZ$$

Bosonic string on T^d

Narain 86

S^1 $$M^2 = 2(N+\bar{N}-2) + \frac{p^2}{R^2} + \frac{\tilde{p}^2}{\tilde{R}^2}$$
$$0 = N - \bar{N} + p\tilde{p}$$

Massless states:

$g_{\mu m}, B_{\mu m}$ 2d vectors: $U(1)^d \times U(1)^d$

g_{mn}, B_{mn} d^2 scalars

+

lots of extra vectors & scalars with mom & winding at points of enhancement where

$$\mathcal{H} = \mathcal{H}^{-1}$$ (up to $SL(k,\mathbb{Z})$) and B → B + n)

2dx2d

$$\mathcal{H} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}$$

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Mass $$M^2 = 2(N+\bar{N}-2) + Z^t \underbrace{\mathcal{H}}_{E^T E} Z \qquad Z = \begin{pmatrix} p_m \\ \tilde{p}^m \end{pmatrix}$$

Level-matching $$0 = (N-\bar{N}) + \tfrac{1}{2} Z^t \eta Z$$

$$p = EZ \qquad \begin{pmatrix} p_L \\ p_R \end{pmatrix} = \begin{pmatrix} e_a^m \left[p_m + (g_{mn}+B_{mn})\tilde{p}^n\right] \\ e_a^m \left[p_m - (g_{mn}-B_{mn})\tilde{p}^n\right] \end{pmatrix}$$

Bosonic string on T^d

Narain 86

S^1 $$M^2 = 2(N+\bar N-2)+\frac{p^2}{R^2}+\frac{\tilde p^2}{\tilde R^2}$$
$$0 = N-\bar N + p\tilde p$$

Massless states:

$g_{\mu m}, B_{\mu m}$ 2d vectors: U(1)^d x U(1)^d

g_{mn}, B_{mn} d^2 scalars

+

lots of extra vectors & scalars with mom & winding at points of enhancement where

$$\mathcal{H} = \mathcal{H}^{-1}$$ (up to SL(k, $\mathbb{Z}$) and B → B + n)

2dx2d

$$\mathcal{H} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g-Bg^{-1}B \end{pmatrix}$$

$$\mathcal{H}^{-1} = \begin{pmatrix} g-Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}$$

Mass $$M^2 = 2(N+\bar N-2)+Z^t\underbrace{\mathcal{H}}_{E^TE}Z \qquad Z = \begin{pmatrix} p_m \\ \tilde p^m \end{pmatrix}$$
$$p_L^2+p_R^2$$

Level-matching $$0 = (N-\bar N)+\tfrac{1}{2}\,Z^t\underbrace{\eta}_{E^T\eta E}Z$$
$$p_L^2-p_R^2$$

$$p = EZ \qquad \begin{pmatrix} p_L \\ p_R \end{pmatrix} = \begin{pmatrix} e_a^m\left[p_m+(g_{mn}+B_{mn})\tilde p^n\right] \\ e_a^m\left[p_m-(g_{mn}-B_{mn})\tilde p^n\right] \end{pmatrix}$$

form a lattice Lorentzian (d,d), even, self-dual

root lattice of enhanced simply-laced gauge group

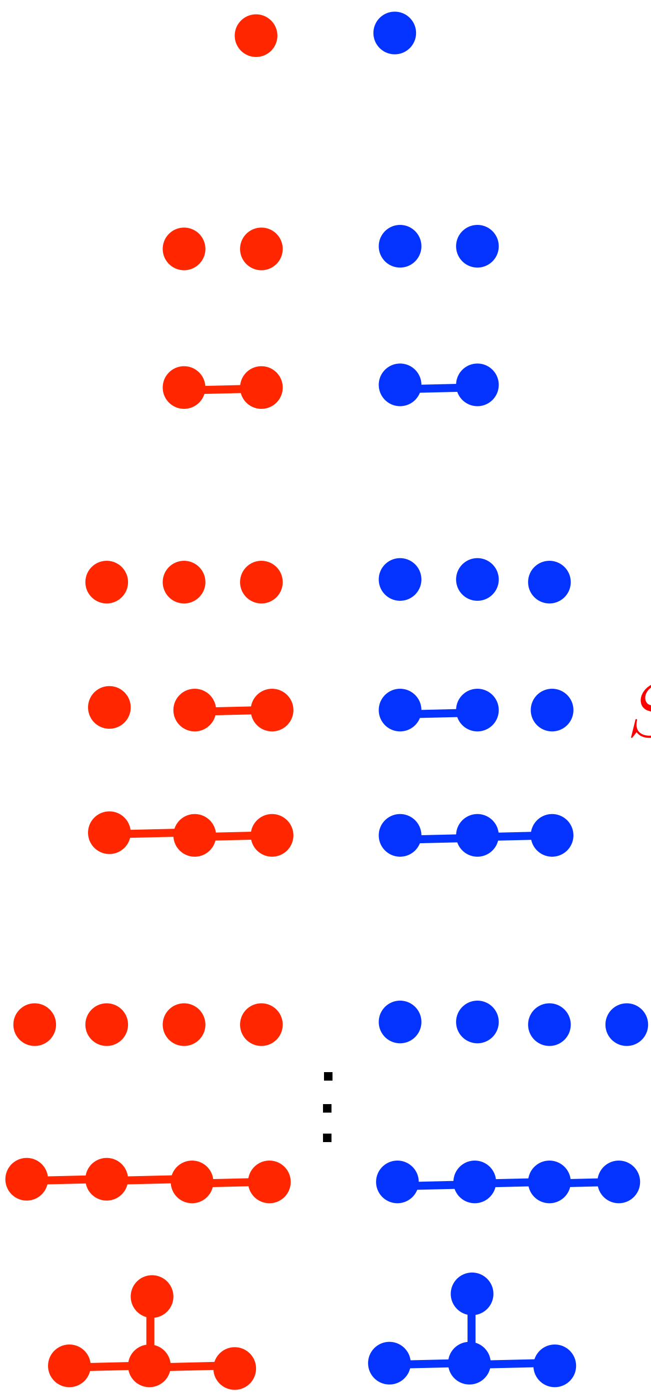
$$G\times G$$

rank d d

Symmetry enhancement, bosonic string on T^d

G simply-laced Lie algebra rank d, dim n G

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$



d=1 $\quad SU(2) \times SU(2) \quad A_1 \times A_1$

d=2 $\quad SU(2)^2 \times SU(2)^2$

$SU(3) \times SU(3) \quad A_2 \times A_2$

d=3 $\quad SU(2)^3 \times SU(2)^3$

$SU(2) \times SU(3) \times SU(3) \times SU(2)$

$SU(4) \times SU(4) \quad A_3 \times A_3$

d=4 $\quad SU(2)^4 \times SU(2)^4$

$\vdots$

$SU(5) \times SU(5) \quad A_4 \times A_4$

$SO(8) \times SO(8) \quad D_4 \times D_4$

maximal enhancement

ADE series

Fields of reduced theory $\mathcal{M}_D \times T^d$

$U(1)^d \times U(1)^d$

Fields of reduced theory $\mathcal{M}_D \times T^d$

$U(1)^d \times U(1)^d$

$A^m \quad \bar{A}^m$

2d vectors

$g_{\mu m} \pm B_{\mu m}$

Fields of reduced theory $\mathcal{M}_D \times T^d$

rank d dim n   rank d dim n

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

$A^m \quad \bar{A}^m$

2d vectors

$$g_{\mu m} \pm B_{\mu m}$$

Fields of reduced theory $\quad \mathcal{M}_D \times T^d$

0= $M^2 = 2(N + \bar{N} - 2) + (p_L^2 + p_R^2)$

rank d rank d
dim n dim n

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

LMC $\quad 0 = 2(N - \bar{N}) + (p_L^2 - p_R^2)$

$p = EZ$

Vectors $\quad N = 0, \; \bar{N} = 1$

positive root

negative root

$A^m \quad \bar{A}^m$

$A^m, A^\alpha, A^{-\alpha} \qquad A^m, A^\alpha, A^{-\alpha}$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ M²=0

2d vectors

2n vectors

$g_{\mu m} \pm B_{\mu m}$

Fields of reduced theory $\mathcal{M}_D \times T^d$

0= $M^2 = 2(N + \bar{N} - 2) + (p_L^2 + p_R^2)$

rank d
dim n

rank d
dim n

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

LMC $0 = 2(N - \bar{N}) + (p_L^2 - p_R^2)$

$p = EZ$

Vectors $N = 0,\ \bar{N} = 1$

positive root

negative root

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$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ M²=0

2d vectors

2n vectors

$g_{\mu m} \pm B_{\mu m}$

M^{mn}

Scalars $\bar{N}_y = N_y = 1$

d² scalars

$g_{mn} + B_{mn}$

Fields of reduced theory $\mathcal{M}_D \times T^d$

0= $M^2 = 2(N + \bar{N} - 2) + (p_L^2 + p_R^2)$

rank d rank d
dim n dim n

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

LMC $0 = 2(N - \bar{N}) + (p_L^2 - p_R^2)$

$p = EZ$

Vectors $N = 0,\ \bar{N} = 1$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ $M^2=0$

positive root
negative root

$A^m \quad \bar{A}^m$

$A^m, A^\alpha, A^{-\alpha} \quad A^m, A^\alpha, A^{-\alpha}$

2d vectors

2n vectors

$g_{\mu m} \pm B_{\mu m}$

M^{mn}

$$\underbrace{M^{mn} \quad M^{\alpha n} \quad M^{m\beta} \quad M^{\alpha\beta}}_{M^{ab}}$$

$a = 1, ..., n$

d^2 scalars

n^2 scalars

$g_{mn} + B_{mn}$

Scalars $\bar{N}_y = N_y = 1 \quad M^{mn}$

$N_y = 1, \bar{N} = 0 \quad M^{m\beta}$

$N = 0, \bar{N}_y = 0 \quad M^{\alpha n}$

$N = \bar{N} = 0 \quad M^{\alpha\beta}$

$p_L^2 - p_R^2 = 2$

$p_L^2 + p_R^2 = 4$

$p_L^2 = p_R^2 = 2$

Fields of reduced theory $\mathcal{M}_D \times T^d$

$$0= M^2 = 2(N+\bar{N}-2) + (p_L^2 + p_R^2)$$

LMC $0 = 2(N-\bar{N}) + (p_L^2 - p_R^2)$

$p = EZ$

rank d, dim n; rank d, dim n

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

Vectors $N=0,\ \bar{N}=1$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ M²=0

positive root

negative root

$A^m \quad \bar{A}^m$

2d vectors

$A^m, A^\alpha, A^{-\alpha} \qquad A^m, A^\alpha, A^{-\alpha}$

2n vectors

$g_{\mu m} \pm B_{\mu m}$

M^{mn}

d² scalars

$g_{mn} + B_{mn}$

$$\boxed{\mathcal{H} \in \frac{O(d,d)}{O(d)\times O(d)}}$$

$$\underbrace{M^{mn} \quad M^{\alpha n} \quad M^{m\beta} \quad M^{\alpha\beta}}_{M^{ab}}$$

$a = 1, \ldots, n$

n² scalars

Scalars $\bar{N}_y = N_y = 1 \quad M^{mn}$

$N_y = 1, \bar{N} = 0 \quad M^{m\beta}$

$N = 0, \bar{N}_y = 0 \quad M^{\alpha n}$

$N = \bar{N} = 0 \quad M^{\alpha\beta}$

$p_L^2 - p_R^2 = 2$

$p_L^2 + p_R^2 = 4$

$p_L^2 = p_R^2 = 2$

Fields of reduced theory $\mathcal{M}_D \times T^d$

0= $M^2 = 2(N + \bar{N} - 2) + (p_L^2 + p_R^2)$

rank d dim n rank d dim n

$U(1)^d \times U(1)^d \longrightarrow G \times G$

LMC $0 = 2(N - \bar{N}) + (p_L^2 - p_R^2)$

$p = EZ$

Vectors $N = 0,\ \bar{N} = 1$

positive root

negative root

$A^m \quad \bar{A}^m$

$A^m, A^\alpha, A^{-\alpha} \quad A^m, A^\alpha, A^{-\alpha}$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ $M^2=0$

2d vectors

2n vectors

$g_{\mu m} \pm B_{\mu m}$

M^{mn}

$\underbrace{M^{mn} \quad M^{\alpha n} \quad M^{m\beta} \quad M^{\alpha\beta}}$

Scalars $\bar{N}_y = N_y = 1 \quad M^{mn}$

$N_y = 1, \bar{N} = 0 \quad M^{m\beta}$

d^2 scalars

$g_{mn} + B_{mn}$

M^{ab}

n^2 scalars

$a = 1, \ldots, n$

$N = 0, \bar{N}_y = 0 \quad M^{\alpha n}$

$N = \bar{N} = 0 \quad M^{\alpha\beta}$

$p_L^2 - p_R^2 = 2$

$p_L^2 + p_R^2 = 4$

$p_L^2 = p_R^2 = 2$

$$\boxed{\mathcal{H} \in \frac{O(d,d)}{O(d)\times O(d)}} \qquad \boxed{\mathcal{H} \in \frac{O(n,n)}{O(n)\times O(n)}}$$

Fields of reduced theory $\mathcal{M}_D \times T^d$

0= $M^2 = 2(N + \bar{N} - 2) + \left(p_L^2 + p_R^2\right)$

LMC $0 = 2(N - \bar{N}) + \left(p_L^2 - p_R^2\right)$

$p = EZ$

$U(1)^d \times U(1)^d \longrightarrow G \times G$

rank d, dim n; rank d, dim n

Vectors $N = 0,\ \bar{N} = 1$

$p_L^2 - p_R^2 = 2$ LMC

$p_L^2 + p_R^2 = 2$ $M^2=0$

positive root

negative root

$A^m \quad \bar{A}^m$

$A^m, A^\alpha, A^{-\alpha} \qquad A^m, A^\alpha, A^{-\alpha}$

2dD dof

2d vectors

$g_{\mu m} \pm B_{\mu m}$

2n vectors

M^{mn}

$\underbrace{M^{mn} \quad M^{\alpha n} \quad M^{m\beta} \quad M^{\alpha\beta}}$

Scalars $\bar{N}_y = N_y = 1 \quad M^{mn}$

$N_y = 1, \bar{N} = 0 \quad M^{m\beta}$

$N = 0, \bar{N}_y = 0 \quad M^{\alpha n}$

$N = \bar{N} = 0 \quad M^{\alpha\beta}$

$p_L^2 - p_R^2 = 2$

$p_L^2 + p_R^2 = 4$

$p_L^2 = p_R^2 = 2$

d² scalars

$g_{mn} + B_{mn}$

M^{ab}

n² scalars

$a = 1, \ldots, n$

$\boxed{\mathcal{H} \in \frac{O(d,d)}{O(d)\times O(d)}}$

$\boxed{\mathcal{H} \in \frac{O(n,n)}{O(n)\times O(n)}}$

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rank d
dim n

rank d
dim n

$U(1)^d \times U(1)^d \longrightarrow G \times G$

LMC $0 = 2(N-\bar{N}) + \left(p_L^2 - p_R^2\right)$

D[2]
dof

tensors
$g_{\mu\nu}, B_{\mu\nu}$

tensors
$g_{\mu\nu}, B_{\mu\nu}$

$p = EZ$

Vectors $N=0,\ \bar{N}=1$

positive root

negative root

$A^m \quad \bar{A}^m$

$A^m, A^\alpha, A^{-\alpha} \qquad A^m, A^\alpha, A^{-\alpha}$

$p_L^2 - p_R^2 = 2$ LMC

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2dD
dof

2d vectors
$g_{\mu m} \pm B_{\mu m}$

2n vectors

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d[2] scalars
$g_{mn} + B_{mn}$

M^{ab}
n[2] scalars

$a = 1, \ldots, n$

$p_L^2 - p_R^2 = 2$

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Fields of reduced theory $\mathcal{M}_D \times T^d$

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

rank d, dim n — rank d, dim n

D² dof

tensors $g_{\mu\nu}, B_{\mu\nu}$ — tensors $g_{\mu\nu}, B_{\mu\nu}$

positive root

negative root

$A^m \quad \bar{A}^m$ — $A^m, A^\alpha, A^{-\alpha} \quad A^m, A^\alpha, A^{-\alpha}$

2dD dof

2d vectors $g_{\mu m} \pm B_{\mu m}$ — 2n vectors

M^{mn} — $\underbrace{M^{mn} \quad M^{\alpha n} \quad M^{m\beta} \quad M^{\alpha\beta}}$

d² scalars $g_{mn} + B_{mn}$ — M^{ab} n² scalars $a = 1, ..., n$

$\mathcal{H} \in \frac{O(d,d)}{O(d)\times O(d)}$ — $\mathcal{H} \in \frac{O(n,n)}{O(n)\times O(n)}$

$\mathcal{H} \in \frac{O(D+d,D+d)}{O(D+d)\times O(D+d)}$ — $\mathcal{H} \in \frac{O(D+n,D+n)}{O(D+n)\times O(D+n)}$

0= $M^2 = 2(N + \bar{N} - 2) + (p_L^2 + p_R^2)$

LMC $\quad 0 = 2(N - \bar{N}) + (p_L^2 - p_R^2)$

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rank d, dim n — rank d, dim n

$$U(1)^d \times U(1)^d \longrightarrow G \times G$$

	$U(1)^d \times U(1)^d$	$G \times G$
D^2 dof	tensors $g_{\mu\nu}, B_{\mu\nu}$	tensors $g_{\mu\nu}, B_{\mu\nu}$
	A^m $\bar{A}^m$	$A^m, A^\alpha, A^{-\alpha}$ $A^m, A^\alpha, A^{-\alpha}$
2dD dof	2d vectors $g_{\mu m} \pm B_{\mu m}$	2n vectors
	M^{mn}	$\underbrace{M^{mn} \quad M^{\alpha n} \quad M^{m\beta} \quad M^{\alpha\beta}}$
	d² scalars $g_{mn} + B_{mn}$	M^{ab} n² scalars, $a = 1, \ldots, n$
	$\boxed{\mathcal{H} \in \frac{O(d,d)}{O(d)\times O(d)}}$	$\boxed{\mathcal{H} \in \frac{O(n,n)}{O(n)\times O(n)}}$
	$\boxed{\mathcal{H} \in \frac{O(D+d,D+d)}{O(D+d)\times O(D+d)}}$	$\boxed{\mathcal{H} \in \frac{O(D+n,D+n)}{O(D+n)\times O(D+n)}}$

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$p_L^2 = p_R^2 = 2$

DFT

Effective action from string theory

Computing 3-point functions <VVV> at a point of enhancement we read off

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \frac{1}{4} \bar{F}^a_{\mu\nu} \bar{F}_a^{\mu\nu}$$

$$+\frac{1}{4} M_{aa'} F^a_{\mu\nu} \bar{F}^{a'\mu\nu} + D_\mu M_{aa'} D^\mu M^{aa'} - \frac{1}{12} f_{abc} \bar{f}_{a'b'c'} M^{aa'} M^{bb'} M^{cc'}$$

$$H = dB + A^a \wedge F_a + f_{abc} A^a \wedge A^b \wedge A^c$$

$$- \bar{A}^a \wedge \bar{F}_a - \bar{f}_{abc} \bar{A}^a \wedge \bar{A}^b \wedge \bar{A}^c$$

$$F^a = dA^a + f^a_{bc} A^b \wedge A^c$$

$$D_\mu M^{aa'} = \partial_\mu M^{aa'} + f^a_{bc} A^b_\mu M^{ca'} + f^{a'}_{b'c'} \bar{A}^{b'}_\mu M^{ac'}$$

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Higgs mechanism

Effective action from string theory

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Higgs mechanism

$$M^{mn} = \underbrace{v^{mn}}_{\text{deviation from point of enhancement}} + M'^{mn}$$

deviation from
point of enhancement $\delta(g+B)_{mn}$

Effective action from string theory

Computing 3-point functions <VVV> at a point of enhancement we read off

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \frac{1}{4} \bar{F}^a_{\mu\nu} \bar{F}_a^{\mu\nu}$$

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A^α
$\bar{A}^\alpha$ acquire mass^2 $\sim vv^t$

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A^α
$\bar{A}^\alpha$ acquire mass² $\sim vv^t$

$$F^a = dA^a + f^a_{bc} A^b \wedge A^c$$

G x G → $U^d(1)$ x $U^d(1)$

$$D_\mu M^{aa'} = \partial_\mu M^{aa'} + f^a_{bc} A^b_\mu M^{ca'} + f^{a'}_{b'c'} \bar{A}^{b'}_\mu M^{ac'}$$

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$$\delta(g+B)_{mn}$$

Effective action from string theory

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$M^{\alpha\beta}$

acquire mass$^2 \sim v$

$$+\frac{1}{4}M_{aa'}F^a_{\mu\nu}\bar{F}^{a'\mu\nu} + D_\mu M_{aa'}D^\mu M^{aa'} - \frac{1}{12}f_{abc}\bar{f}_{a'b'c'}M^{aa'}M^{bb'}M^{cc'}$$

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$\bar{A}^\alpha$ acquire mass^2 $\sim vv^t$

$$F^a = dA^a + f^a_{bc}\,A^b \wedge A^c$$

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Higgs mechanism

$$M^{mn} = \underbrace{v^{mn}}_{\text{deviation from point of enhancement}} + M'^{mn}$$

$\delta(g+B)_{mn}$

Effective action from string theory

Computing 3-point functions <VVV> at a point of enhancement we read off

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$M^{\alpha\beta}$ acquire mass$^2 \sim v$

$$+\frac{1}{4} M_{aa'} F^a_{\mu\nu} \bar{F}^{a'\mu\nu} + D_\mu M_{aa'} D^\mu M^{aa'} - \frac{1}{12} f_{abc} \bar{f}_{a'b'c'} M^{aa'} M^{bb'} M^{cc'}$$

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A^α
$\bar{A}^\alpha$ acquire mass$^2 \sim v v^t$

$$F^a = dA^a + f^a_{bc} A^b \wedge A^c$$

G x G → U^d(1) x U^d(1)

$$D_\mu M^{aa'} = \partial_\mu M^{aa'} + f^a_{bc} A^b_\mu M^{ca'} + f^{a'}_{b'c'} \bar{A}^{b'}_\mu M^{ac'}$$

Higgs mechanism

$$M^{mn} = \underbrace{v^{mn}}_{\substack{\text{deviation from} \\ \text{point of enhancement}}} + M'^{mn}$$

$\delta(g+B)_{mn}$

Can we get this action from DFT ??

DFT action

DFT action

DFT O(N,N) action

Hull & Zweibach 09

$$S = \int dX \left(-\partial_{MN}\mathcal{H}^{MN} + \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL} \right)$$

Equivalent to

$$S = \int dX \, \mathbb{R}$$

generalized Ricci scalar

Coimbra, Strickland-Constable, Waldram 09

DFT action

DFT O(N,N) action

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Equivalent to

$$S = \int dX\, \mathbb{R}$$

generalized Ricci scalar

Coimbra, Strickland-Constable, Waldram 09

Generalized Scherk-Schwarz reduction of DFT action

$$\mathcal{M}_{\bar{\mathsf{N}}-d} \times \mathcal{M}^d$$

$$\mathrm{O(N,N)} \longrightarrow \mathrm{O(N\text{-}d,N\text{-}d)} \times \mathrm{O(d,d)}$$

external internal

DFT action

DFT O(N,N) action

Hull & Zweibach 09

$$S = \int dX \left(-\partial_{MN}\mathcal{H}^{MN} + \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL} \right)$$

Equivalent to

$$S = \int dX\, \mathbb{R}$$

generalized Ricci scalar

Coimbra, Strickland-Constable, Waldram 09

Generalized Scherk-Schwarz reduction of DFT action

$$\mathcal{M}_{\bar{\mathsf{N}}-d} \times \mathcal{M}^d$$

$$x \;,\; y$$

$$\mathcal{H}^{MN} = \delta^{AB} E_A{}^M E_B{}^N \qquad E_A(x,y) = U_A{}^{A'}(x)\, E'_{A'}(y)$$

$$\text{O(N,N)} \longrightarrow \text{O(N-d,N-d)} \times \text{O(d,d)}$$

external internal

DFT action

DFT O(N,N) action

Hull & Zweibach 09

$$S=\int dX\left(-\partial_{MN}\mathcal{H}^{MN}+\frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL}-\frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL}\right)$$

Equivalent to

$$S=\int dX\,\mathbb{R}$$

generalized Ricci scalar

Coimbra, Strickland-Constable, Waldram 09

Generalized Scherk-Schwarz reduction of DFT action

$$\underset{x}{\mathcal{M}_{\bar{\mathsf{N}}-d}}\times\underset{y}{\mathcal{M}^d}$$

generalized parallelizable

$$\mathcal{H}^{MN}=\delta^{AB}E_A{}^M E_B{}^N \qquad E_A(x,y,\tilde{y})=U_A{}^{A'}(x)\,E'_{A'}(y,\tilde{y})$$

O(N,N) $\longrightarrow$ O(N-d,N-d) x O(d,d)

external internal

$$\partial_M=(\underbrace{\partial_\mu}_{\text{N-d}},\underbrace{\partial_m,\partial_{m}}_{\underset{I}{\text{2d}}},\cancel{\partial^\mu})$$

Aldazabal, Baron, Marques, Nuñez 11
Geissbuhler 11

$$\mathcal{L} \;=\; R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \mathcal{H}_{IJ} F^{I\mu\nu} F^{J}_{\mu\nu} + (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ}$$
$$-\frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)$$

$$H \;=\; dB + F^I \wedge A_I$$

$$F^I \;=\; dA^I + \boxed{f^I{}_{JK}}\, A^J \wedge A^K$$

$$E_A(x, y, \tilde{y}) = \; U_A{}^{A'}(x)\, E'_{A'}(y, \tilde{y})$$

$$\boxed{[E'_J, E'_K]_C = f^I{}_{JK} E'_K}$$

$$\partial_M = (\underbrace{\partial_\mu,}_{\text{N-d}} \underbrace{\partial_m, \partial_m,}_{\substack{\text{2d} \\ I}} \cancel{\partial^\mu})$$

Aldazabal, Baron, Marques, Nuñez 11
Geissbuhler 11

$$\mathcal{L} \;=\; R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \mathcal{H}_{IJ} F^{I\mu\nu} F^{J}_{\mu\nu} + (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ}$$
$$- \frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)$$

$$H \;=\; dB + F^I \wedge A_I$$

$$F^I \;=\; dA^I + \boxed{f^I{}_{JK}}\, A^J \wedge A^K$$

$$E_A(x, y, \tilde{y}) = U_A{}^{A'}(x)\, E'_{A'}(y, \tilde{y})$$

$$\boxed{[E'_J, E'_K]_C = f^I{}_{JK} E'_K}$$

$$\partial_M = (\underbrace{\partial_\mu}_{\text{N-d}}, \underbrace{\partial_{\color{red}m}, \partial_{\color{blue}m}}_{\substack{\text{2d} \\ I}}, \cancel{\partial^\mu})$$

Claim: this action reproduces the string theory action compactifications on T^d close to enhancement point

For simplicity, do: d=1 (enhancement to SU(2) x SU(2))

Frame on $T\mathcal{M}_{\mathsf{N}} \oplus T^*\mathcal{M}_{\mathsf{N}}$

frame e_a

dual frame e^a

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

For simplicity, do: d=1 (enhancement to SU(2) x SU(2)) $\mathcal{M}_{\mathsf{N}-1} \times S^1$

Frame on $T\mathcal{M}_{\mathsf{N}} \oplus T^*\mathcal{M}_{\mathsf{N}}$

frame e_a

dual frame e^a

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

$$T\mathcal{M}_{\mathsf{N}} = T\mathcal{M}_{\mathsf{N}-1} \oplus TS^1$$

$$y \sim y + 2\pi$$

For simplicity, do: d=1 (enhancement to SU(2) x SU(2)) $\mathcal{M}_{\mathsf{N}-1} \times S^1$

Frame on $T\mathcal{M}_{\mathsf{N}} \oplus T^*\mathcal{M}_{\mathsf{N}}$

frame e_a

dual frame e^a

$$T\mathcal{M}_{\mathsf{N}} = T\mathcal{M}_{\mathsf{N}-1} \oplus TS^1$$

$$y \sim y + 2\pi$$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

$e^a \to e^{\hat{a}}$, $\phi\,(dy + V_1)$

$g_{\mu y}$

$\sqrt{g_{yy}} = R$

For simplicity, do: d=1 (enhancement to SU(2) x SU(2)) $\mathcal{M}_{\mathsf{N}-1} \times S^1$

Frame on $T\mathcal{M}_{\mathsf{N}} \oplus T^*\mathcal{M}_{\mathsf{N}}$

frame e_a dual frame e^a

$$T\mathcal{M}_{\mathsf{N}} = T\mathcal{M}_{\mathsf{N}-1} \oplus TS^1$$

$$y \sim y + 2\pi$$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

$\#$

$B_{\mu y}$

$\phi^{-1}(\partial_y + B_1)$

$e^{\hat{a}}$

$\phi\,(dy + V_1)$ $g_{\mu y}$

$\sqrt{g_{yy}} = R$

$$B_{\mu y}$$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix} \qquad \begin{array}{l} \# \\ \boxed{\begin{array}{c} \phi^{-1}(\partial_y + B_1) \\ \phi\,(dy + V_1) \end{array}} \quad g_{\mu y} \\ e^{\hat{a}} \end{array}$$

$$\sqrt{g_{yy}} = R$$

$B_{\mu y}$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

$e_a - \iota_{e_a} B \to \#$

$$\boxed{\begin{matrix} \phi^{-1}(\partial_y + B_1) \\ \phi\,(dy + V_1) \end{matrix}}$$

$g_{\mu y}$

$e^a \to e^{\hat{a}}$

$\sqrt{g_{yy}} = R$

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix}$$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

$\#$

$$B_{\mu y}$$

$$\boxed{\begin{matrix} \phi^{-1}(\partial_y + B_1) \\ \phi\,(dy + V_1) \end{matrix}} \quad g_{\mu y}$$

$$e^{\hat{a}}$$

$$\sqrt{g_{yy}} = R$$

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix} \xrightarrow[LR]{} \begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} U^+ & U^- \\ U^- & U^+ \end{pmatrix} \quad \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

$$U^{\pm} = \frac{1}{2}(\phi^{-1} \pm \phi) \qquad \begin{aligned} A &= V_1 + B_1 \quad J = \partial_y + dy \\ \bar{A} &= V_1 - B_1 \quad \bar{J} = \partial_y - dy \end{aligned}$$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

#

$$B_{\mu y}$$

$$\boxed{\begin{matrix} \phi^{-1}(\partial_y + B_1) \\ \phi\,(dy + V_1) \end{matrix}}$$

$$g_{\mu y}$$

$$e^{\hat{a}}$$

$$\sqrt{g_{yy}} = R = 1 + \tfrac{1}{2}v + \ldots \qquad v \sim \epsilon$$

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix} \xrightarrow[LR]{} \begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} U^+ & U^- \\ U^- & U^+ \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

$$U^\pm = \frac{1}{2}(\phi^{-1} \pm \phi)$$

$$A = V_1 + B_1 \qquad J = \partial_y + dy$$

$$\bar{A} = V_1 - B_1 \qquad \bar{J} = \partial_y - dy$$

$$B_{\mu y}$$

$$\#$$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix} \qquad \boxed{\begin{matrix} \phi^{-1}(\partial_y + B_1) \\ \phi\,(dy + V_1) \end{matrix}} \qquad g_{\mu y}$$

$$e^{\hat{a}}$$

$$\sqrt{g_{yy}} = R = 1 + \tfrac{1}{2}v + \ldots \qquad v \sim \epsilon$$

$$\langle M \rangle$$

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix} \xrightarrow[LR]{} \begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} U^+ & U^- \\ U^- & U^+ \end{pmatrix} \quad \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

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$B_{\mu y}$

$\#$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix} \qquad \begin{array}{l} \phi^{-1}(\partial_y + B_1) \\ \phi\,(dy + V_1) \end{array}$$

$g_{\mu y}$

$e^{\hat{a}}$

$$\sqrt{g_{yy}} = R = 1 + \tfrac{1}{2} v + \ldots \qquad v \sim \epsilon$$

$\langle M \rangle$

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix} \xrightarrow{\color{red}{L}\color{blue}{R}} \begin{pmatrix} E^{\color{red}{L}} \\ E^{\color{blue}{R}} \end{pmatrix} = \begin{pmatrix} U^+ & U^- \\ U^- & U^+ \end{pmatrix} \begin{pmatrix} \color{red}{J} + \color{red}{A} \\ \color{blue}{\bar{J}} - \color{blue}{\bar{A}} \end{pmatrix}$$

$$U^+ \approx 1 \qquad U^\pm = \frac{1}{2}(\phi^{-1} \pm \phi) \qquad \color{red}{A} = V_1 + B_1 \qquad \color{red}{J} = \partial_y + dy$$

$$U^- \approx \tfrac{1}{2} M \qquad \color{blue}{\bar{A}} = V_1 - B_1 \qquad \color{blue}{\bar{J}} = \partial_y - dy$$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

#

$B_{\mu y}$

$$\boxed{\begin{matrix} \phi^{-1}(\partial_y + B_1) \\ \phi\,(dy + V_1) \end{matrix}}$$

$g_{\mu y}$

$e^{\hat{a}}$

$$\sqrt{g_{yy}} = R = 1 + \tfrac{1}{2}v + \ldots \qquad v \sim \epsilon$$

$\langle M \rangle$

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix} \xrightarrow[LR]{} \begin{pmatrix} E^{L} \\ E^{R} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M \\ \frac{1}{2}M & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

$$U^+ \approx 1 \qquad U^\pm = \frac{1}{2}(\phi^{-1} \pm \phi) \qquad A = V_1 + B_1 \quad J = \partial_y + dy$$

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$\#$

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix} \qquad \boxed{\begin{matrix} \phi^{-1}(\partial_y + B_1) \\ \phi\,(dy + V_1) \end{matrix}} \quad g_{\mu y}$$

$e^{\hat{a}}$

$$\sqrt{g_{yy}} = R = 1 + \tfrac{1}{2} v + \dots \qquad v \sim \epsilon$$

$\langle M \rangle$

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix} \xrightarrow[LR]{} \begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M \\ \frac{1}{2}M & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

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So far, no enhancement of symmetry

$$E_A = \begin{pmatrix} e_a - \iota_{e_a} B \\ e^a \end{pmatrix}$$

$B_{\mu y}$

#

$$\phi^{-1}(\partial_y + B_1)$$

$$\phi\,(dy + V_1) \quad g_{\mu y}$$

$e^{\hat{a}}$

$$\sqrt{g_{yy}} = R = 1 + \tfrac{1}{2}v + \ldots \qquad v \sim \epsilon$$

$\langle M \rangle$

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_y + B_1 \\ dy + V_1 \end{pmatrix} \xrightarrow{LR} \begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} 1 & \tfrac{1}{2}M \\ \tfrac{1}{2}M & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

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$$U^- \approx \tfrac{1}{2}M \qquad \bar{A} = V_1 - B_1 \quad \bar{J} = \partial_y - dy$$

So far, no enhancement of symmetry , no double field theory

DFT

$$T\mathcal{M} \oplus T^*\mathcal{M} \longrightarrow T\mathcal{M}_{\text{N-1}} \oplus TS^1 \oplus T^*S^1 \oplus T^*\mathcal{M}_{\text{N-1}}$$

$$J = \partial_y + dy$$

$$\bar{J} = \partial_y - dy$$

DFT

$$T\mathcal{M} \oplus T^*\mathcal{M} \longrightarrow T\mathcal{M}_{\text{N-1}} \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_{\text{N-1}}$$

$$dy \approx \partial_{\tilde{y}}$$

$$J = \partial_y + dy$$

$$\bar{J} = \partial_y - dy$$

DFT

$$T\mathcal{M} \oplus T^*\mathcal{M} \longrightarrow T\mathcal{M}_{\text{N-1}} \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_{\text{N-1}}$$

$$dy \approx \partial_{\tilde{y}}$$

$$J = \partial_y + dy = \partial_y + \partial_{\tilde{y}} = \partial_{y^L}$$

$$\bar{J} = \partial_y - dy = \partial_y - \partial_{\tilde{y}} = \partial_{y^R}$$

DFT

$$T\mathcal{M} \oplus T^*\mathcal{M} \longrightarrow T\mathcal{M}_{\text{N-1}} \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_{\text{N-1}}$$

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Still, this is formal. No dependence on y or $\tilde{y}$

DFT

$$T\mathcal{M} \oplus T^*\mathcal{M} \longrightarrow T\mathcal{M}_{N-1} \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_{N-1}$$

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Still, this is formal. No dependence on y or $\tilde{y}$

Of course, we have not included momentum/winding modes $\sim e^{2iy}/e^{2i\tilde{y}}$

To include winding modes we need dependence on $S^1, \tilde{S}^1$

DFT & Enhancement of symmetry

$$TM \oplus T^*M \longrightarrow TM_{\text{N-1}} \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*M_{\text{N-1}}$$

$$dy \approx \partial_{\tilde{y}}$$

$$J = \partial_y + dy = \partial_y + \partial_{\tilde{y}} = \partial_{y^L}$$

$$\bar{J} = \partial_y - dy = \partial_y - \partial_{\tilde{y}} = \partial_{y^R}$$

Still, this is formal. No dependence on y or $\tilde{y}$

Of course, we have not included momentum/winding modes $\sim e^{2iy}/e^{2i\tilde{y}}$

To include winding modes we need dependence on $S^1, \tilde{S}^1$

To account for the enhancement of symmetry, we need to enlarge the generalized tangent space

Enhancement of symmetry

$$T\mathcal{M}_{\text{N-1}} \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_{\text{N-1}}$$

$$\begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M \\ \frac{1}{2}M & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

Enhancement of symmetry

$$T\mathcal{M}_{\text{N-1}} \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_{\text{N-1}}$$

$$\downarrow$$

$$T\mathcal{M}_{\text{N-1}} \oplus \underbrace{V_2 \oplus TS^1 \oplus T\tilde{S}^1 \oplus V_2^*}_{\text{O(3,3)}} \oplus T^*\mathcal{M}_{\text{N-1}}$$

$$\begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M \\ \frac{1}{2}M & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

Enhancement of symmetry

$$T\mathcal{M}_{\text{N-1}} \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_{\text{N-1}}$$

$$\downarrow$$

$$T\mathcal{M}_{\text{N-1}} \oplus \underbrace{V_2 \oplus TS^1 \oplus T\tilde{S}^1 \oplus V_2^*}_{\text{O(3,3)}} \oplus T^*\mathcal{M}_{\text{N-1}}$$

$$\begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M \\ \frac{1}{2}M & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} E^a \\ E^a \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M^{ab} \\ \frac{1}{2}M^{ab} & 1 \end{pmatrix} \begin{pmatrix} J^b + A^b \\ \bar{J}^b - \bar{A}^b \end{pmatrix}$$

$M^{ab}(x)$

9 scalar fields

$A^a(x)$

$\bar{A}^a(x)$

6 vector fields

Enhancement of symmetry

$$T\mathcal{M}_{\text{N-1}} \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_{\text{N-1}}$$

$$\downarrow$$

$$T\mathcal{M}_{\text{N-1}} \oplus \underbrace{V_2 \oplus TS^1 \oplus T\tilde{S}^1 \oplus V_2^*}_{\text{O(3,3)}} \oplus T^*\mathcal{M}_{\text{N-1}}$$

$$\begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M \\ \frac{1}{2}M & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} E^a \\ E^a \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M^{ab} \\ \frac{1}{2}M^{ab} & 1 \end{pmatrix} \begin{pmatrix} J^b + A^b \\ \bar{J}^b - \bar{A}^b \end{pmatrix}$$

$M^{ab}(x)$

9 scalar fields

$A^a(x)$

$\bar{A}^a(x)$

6 vector fields

$J^a(y,\tilde{y})$

$\bar{J}^a(y,\tilde{y})$

Enhancement of symmetry

$$T\mathcal{M}_{\text{N-1}} \oplus TS^1 \oplus T\tilde{S}^1 \oplus T^*\mathcal{M}_{\text{N-1}}$$

$$\downarrow$$

$$T\mathcal{M}_{\text{N-1}} \oplus \underbrace{V_2 \oplus TS^1 \oplus T\tilde{S}^1 \oplus V_2^*}_{\text{O(3,3)}} \oplus T^*\mathcal{M}_{\text{N-1}}$$

$$\begin{pmatrix} E^L \\ E^R \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M \\ \frac{1}{2}M & 1 \end{pmatrix} \begin{pmatrix} J + A \\ \bar{J} - \bar{A} \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} E^a \\ E^a \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}M^{ab} \\ \frac{1}{2}M^{ab} & 1 \end{pmatrix} \begin{pmatrix} J^b + A^b \\ \bar{J}^b - \bar{A}^b \end{pmatrix}$$

$M^{ab}(x)$

9 scalar fields

$A^a(x)$

$\bar{A}^a(x)$

6 vector fields

Should satisfy SU(2)$_\text{L}$ algebra $J^a(y, \tilde{y})$

Should satisfy SU(2)$_\text{R}$ algebra $\bar{J}^a(y, \tilde{y})$

Effective action (for T^d)

Generalized Sherk-Schwarz compactification of DFT action

$$
\begin{aligned}
\mathcal{L} &= R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \mathcal{H}_{IJ} F^{I\mu\nu} F^{J}_{\mu\nu} + (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ} \\
&\quad - \frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)
\end{aligned}
$$

Effective action (for T^d)

Generalized Sherk-Schwarz compactification of DFT action

$$I = a, a$$

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \mathcal{H}_{IJ} F^{I\mu\nu} F^{J}_{\mu\nu} + (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ}$$

$$- \frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)$$

$$\begin{pmatrix} E_a \\ E^{L} \\ E^{R} \\ E^a \end{pmatrix} = \begin{pmatrix} e_a & \iota_{e_a} A & \iota_{e_a} \bar{A} & \iota_{e_a} B \\ 0 & 1 & \frac{1}{2} M & M \bar{A} \\ 0 & \frac{1}{2} M^t & 1 & M^t A \\ 0 & 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & \bar{J} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_A(x, y, \tilde{y}) = \qquad U_A{}^{A'}(x) \qquad E'_{A'}(y, \tilde{y})$$

Effective action (for T^d)

Generalized Sherk-Schwarz compactification of DFT action

$$I = \dot{a}, a$$

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \mathcal{H}_{IJ} F^{I\mu\nu} F^{J}_{\mu\nu} + (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ}$$

$$-\frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)$$

$$H = dB + F^I \wedge A_I$$

$$F^I = dA^I + \boxed{f^I{}_{JK}} A^J \wedge A^K$$

$$\boxed{[E'_J, E'_K] = f^I_{JK} E'_I}$$

$J, \bar{J}$

$$\begin{pmatrix} E_a \\ E^{L} \\ E^{R} \\ E^a \end{pmatrix} = \begin{pmatrix} e_a & \iota_{e_a} A & \iota_{e_a} \bar{A} & \iota_{e_a} B \\ 0 & 1 & \frac{1}{2} M & M \bar{A} \\ 0 & \frac{1}{2} M^t & 1 & M^t A \\ 0 & 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & \bar{J} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_A(x, y, \tilde{y}) = \qquad U_A{}^{A'}(x) \qquad E'_{A'}(y, \tilde{y})$$

Effective action (for T^d)

Generalized Sherk-Schwarz compactification of DFT action

$$I = \dot{a}, a$$

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \overset{\boxed{}^{2}}{\mathcal{H}_{IJ}} F^{I\mu\nu} F^{J}_{\mu\nu} + (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ}$$

$$-\frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)$$

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$J, \bar{J}$

$$\begin{pmatrix} E_a \\ E^{L} \\ E^{R} \\ E^a \end{pmatrix} = \begin{pmatrix} e_a & \iota_{e_a} A & \iota_{e_a} \bar{A} & \iota_{e_a} B \\ 0 & 1 & \frac{1}{2} M & M \bar{A} \\ 0 & \frac{1}{2} M^t & 1 & M^t A \\ 0 & 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & \bar{J} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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Effective action (for T^d)

Generalized Sherk-Schwarz compactification of DFT action

$$I = \dot{a}, a$$

$$\square^2 \approx \begin{pmatrix} 1 & M \\ M^t & 1 \end{pmatrix}$$

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} \mathcal{H}_{IJ} F^{I\mu\nu} F^J_{\mu\nu} + (D_\mu \mathcal{H})_{IJ} (D^\mu \mathcal{H})^{IJ}$$

$$- \frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)$$

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$$\boxed{[E'_J, E'_K] = f^I_{JK} E'_I}$$

$J, \bar{J}$

$$\begin{pmatrix} E_a \\ E^L \\ E^R \\ E^a \end{pmatrix} = \begin{pmatrix} e_a & \iota_{e_a} A & \iota_{e_a} \bar{A} & \iota_{e_a} B \\ 0 & 1 & \frac{1}{2} M & M \bar{A} \\ 0 & \frac{1}{2} M^t & 1 & M^t A \\ 0 & 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & \bar{J} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_A(x, y, \tilde{y}) = U_A{}^{A'}(x) \quad E'_{A'}(y, \tilde{y})$$

Effective action (for T^d)

Generalized Sherk-Schwarz compactification of DFT action

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$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu} + \frac{1}{4} \bar{F}^{\dot{a}}_{\mu\nu} \bar{F}^{\dot{a}\mu\nu} + \frac{1}{4} M^{ab} F^{a}_{\mu\nu} \bar{F}^{b\mu\nu} + D_\mu M^{ab} D^\mu M^{ab}$$

$$- \frac{1}{12} f_{IJK} f_{LMN} \left(\mathcal{H}^{IL} \mathcal{H}^{JM} \mathcal{H}^{KN} - 3\, \mathcal{H}^{IL} \eta^{JM} \eta^{KN} + 2\, \eta^{IL} \eta^{JM} \eta^{KN} \right)$$

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$$\boxed{[E'_J, E'_K] = f^I_{JK} E'_I}$$

$J, \bar{J}$

$$\begin{pmatrix} E_a \\ E^L \\ E^R \\ E^a \end{pmatrix} = \begin{pmatrix} e_a & \iota_{e_a} A & \iota_{e_a} \bar{A} & \iota_{e_a} B \\ 0 & 1 & \frac{1}{2} M & M \bar{A} \\ 0 & \frac{1}{2} M^t & 1 & M^t A \\ 0 & 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & \bar{J} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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Effective action (for T^d)

Generalized Sherk-Schwarz compactification of DFT action

$$I = a, a$$

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu} + \frac{1}{4} \bar{F}^{a}_{\mu\nu} \bar{F}^{a\mu\nu} + \frac{1}{4} M^{ab} F^{a}_{\mu\nu} \bar{F}^{b\mu\nu} + D_\mu M^{ab} D^\mu M^{ab}$$

$$+ f_{abc} f_{abc} M^{aa} M^{bb} M^{cc}$$

$$H = dB + F^I \wedge A_I$$

$$F^I = dA^I + \boxed{f^I{}_{JK}} A^J \wedge A^K$$

$$\boxed{[\underset{J,\bar{J}}{E'_J}, E'_K] = f^I_{JK} E'_I}$$

$$\begin{pmatrix} E_a \\ E^L \\ E^R \\ E^a \end{pmatrix} = \begin{pmatrix} e_a & \iota_{e_a} A & \iota_{e_a} \bar{A} & \iota_{e_a} B \\ 0 & 1 & \frac{1}{2} M & M \bar{A} \\ 0 & \frac{1}{2} M^t & 1 & M^t A \\ 0 & 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & \bar{J} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_A(x, y, \tilde{y}) = \quad U_A{}^{A'}(x) \quad E'_{A'}(y, \tilde{y})$$

Effective action (for T^d)

Generalized Sherk-Schwarz compactification of DFT action

$$I = a, a$$

$$\mathcal{L} = R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu} + \frac{1}{4} \bar{F}^{a}_{\mu\nu} \bar{F}^{a\mu\nu} + \frac{1}{4} M^{ab} F^{a}_{\mu\nu} \bar{F}^{b\mu\nu} + D_\mu M^{ab} D^\mu M^{ab}$$

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Exactly string theory action!

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Generalized Sherk-Schwarz compactification of DFT action

$$I = \dot{a}, a$$

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$$+ f_{\dot{a}\dot{b}\dot{c}} f_{abc} M^{\dot{a}a} M^{\dot{b}b} M^{\dot{c}c}$$

Exactly string theory action!

Reproduces string theory masses of states at a point close to maximal enhancement point

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$$F^I = dA^I + \boxed{f^I{}_{JK}} A^J \wedge A^K$$

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$$M^{mn} = \underbrace{v^{mn}}_{\text{deviation from point of enhancement}} + M'^{mn}$$

$$\delta(g + B)_{mn}$$

G x G algebra

C-bracket

$$[V_1, V_2]_C = \frac{1}{2}(\mathcal{L}_{V_1} V_2 - \mathcal{L}_{V_2} V_1)$$

$$(\mathcal{L}_{V_1} V_2)^I = V_1^J \partial_J V_2^I + (\partial^I V_{1J} - \partial_J V_1^I) V_2^J$$

generalized Lie derivative

$$\boxed{[E'_J, E'_K]_C = f^I{}_{JK} E'_K}$$

J, $\bar{J}$

SU(2) x SU(2) algebra

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J, $\bar{J}$ $\quad$ ϵ^{abc}, ϵ^{abc}

SU(2) x SU(2) algebra

$$\boxed{V_2 + TS^1 + T\tilde{S}^1 + V_2^*}$$

C-bracket

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SU(2) x SU(2) algebra

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v_1^L, v_2^L $\qquad$ v_1^R, v_2^R

C-bracket

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generalized Lie derivative

The following J and $\bar{J}$ do the job

$$J = \begin{pmatrix} \cos 2y^L & \sin 2y^L & 0 \\ -\sin 2y^L & \cos 2y^L & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ \partial_{y^L} \end{pmatrix}$$

$$J = \begin{pmatrix} \cos 2y^R & \sin 2y^R & 0 \\ -\sin 2y^R & \cos 2y^R & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ \partial_{y^R} \end{pmatrix}$$

$$\boxed{[E'_J, E'_K]_C = f^I{}_{JK} E'_K}$$

$J, \bar{J}$ $\qquad$ $\epsilon^{abc}, \epsilon^{abc}$

SU(2) x SU(2) algebra

$$\boxed{V_2 + \boxed{TS^1 + TS^1} + V_2}$$

v_1^L, v_2^L $\qquad$ v_1^R, v_2^R

$v_\pm = v_1 \pm i\, v_2$ $\qquad$ $v_\pm = v_1 \pm i\, v_2$

C-bracket

$$[V_1, V_2]_C = \frac{1}{2}(\mathcal{L}_{V_1} V_2 - \mathcal{L}_{V_2} V_1)$$

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$$J = \begin{pmatrix} e^{2iy^R} & 0 & 0 \\ 0 & e^{-2iy^R} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_+ \\ v_- \\ \partial_{y^R} \end{pmatrix}$$

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Straightforward generalization to SU(2)^d x SU(2)^d

$$\boxed{[E'_J, E'_K]_C = f^I{}_{JK} E'_K}$$

$J, \bar{J}$ $\quad$ $\epsilon^{abc}, \epsilon^{abc}$

What about other enhancement groups?

T^2

$\color{red}{SU(2)^2} \times \color{blue}{SU(2)^2}$

$\color{red}{SU(3)} \times \color{blue}{SU(3)}$

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T^2

$SU(2)^2 \times SU(2)^2$

$SU(3) \times SU(3)$ 3 positive roots : 2 simple, 1 non-simple

$$[J^\alpha, J^\beta] = J^{\alpha+\beta}$$

does not arise from any obvious extension of the previous construction

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$$\tilde{\mathcal{L}}_{E_I} E_J = \mathcal{L}_{E_I} E_J + \Omega_{IJ}{}^K E_K$$

Cocycle tensor

$$\underset{\alpha\beta\gamma}{\Omega_{IJK}} = \begin{cases} (-1)^{\alpha * \beta}\ \delta_{\alpha+\beta+\gamma} & \text{if two roots are positive} \\ -(-1)^{\alpha * \beta}\ \delta_{\alpha+\beta+\gamma} & \text{if two roots are negative} \end{cases}$$

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This reproduces $\boxed{[E'_J, E'_K]_{\tilde{C}} = f^I{}_{JK} E'_K}$ for any group

Effective action found is good close to enhancement point

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Can we find a description "good" for all moduli space ?

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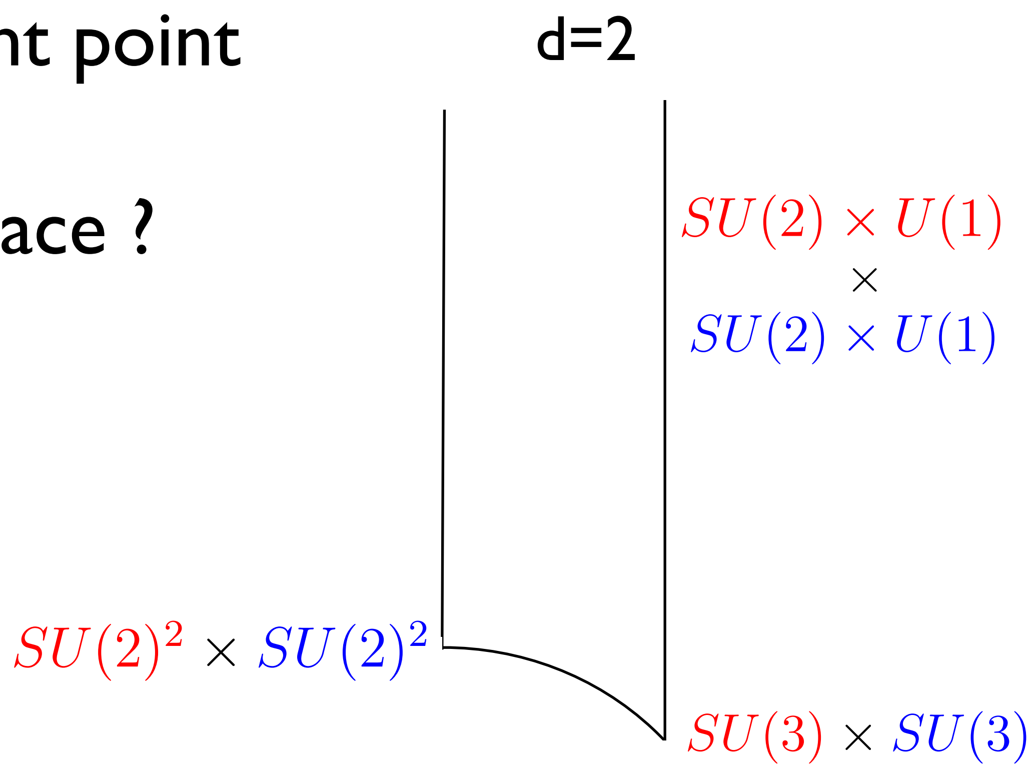

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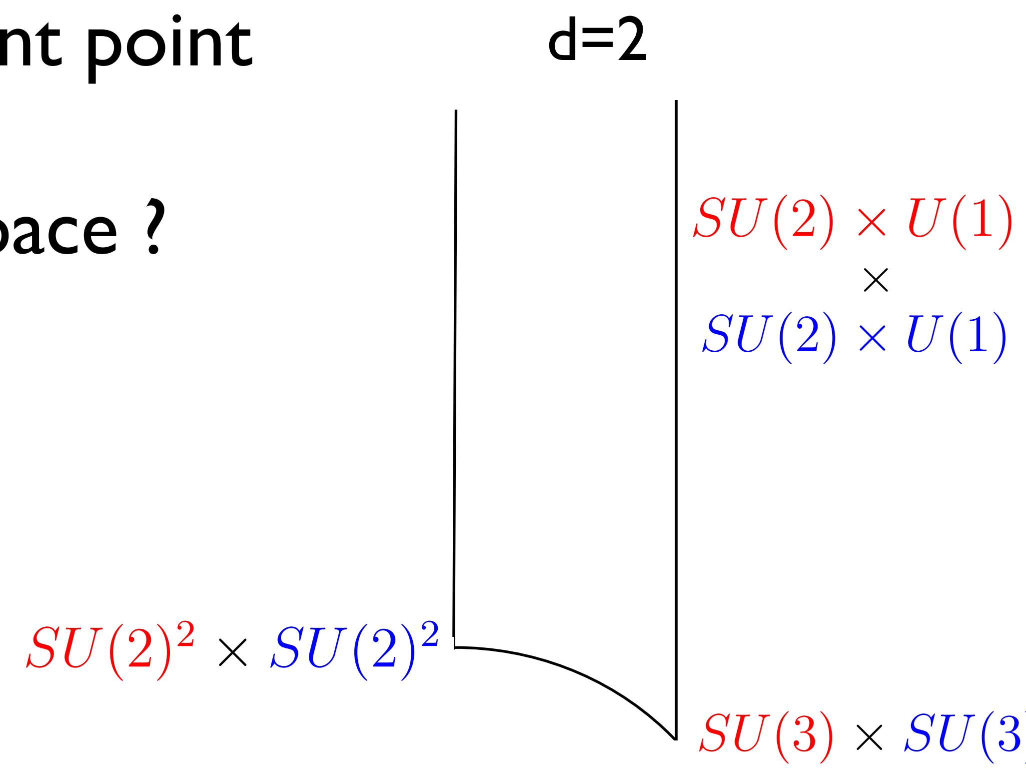
Can we find a description "good" for all moduli space ?

☞ We can, but $SU(2) \times SU(2) \not\subset SU(3)$



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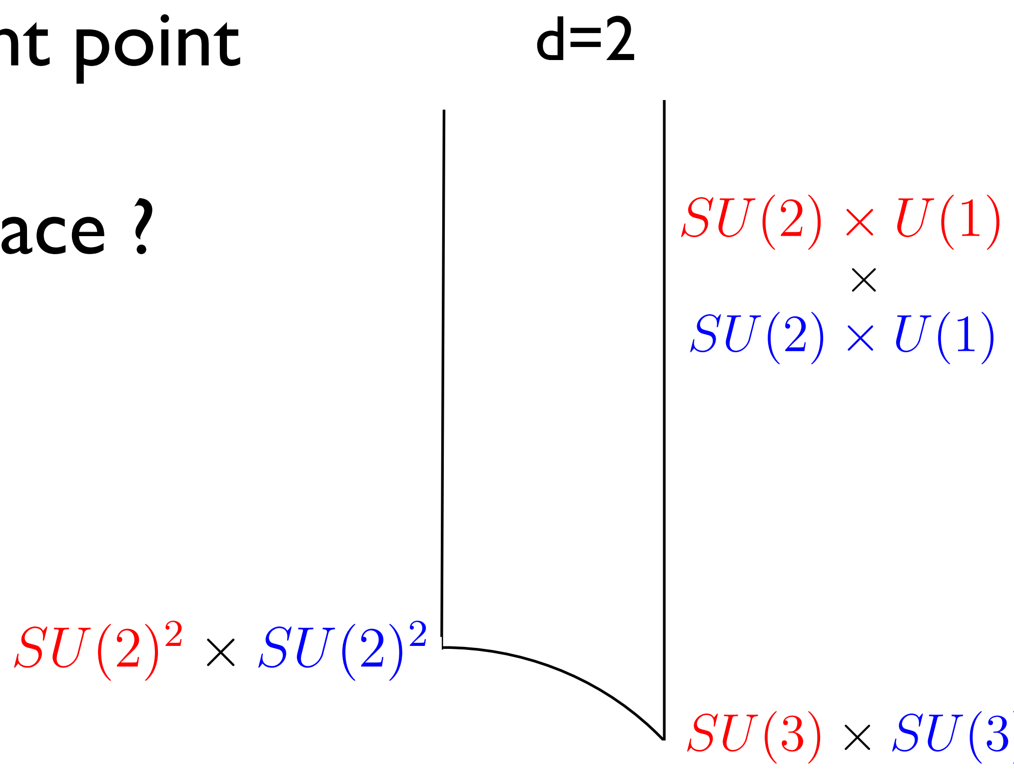

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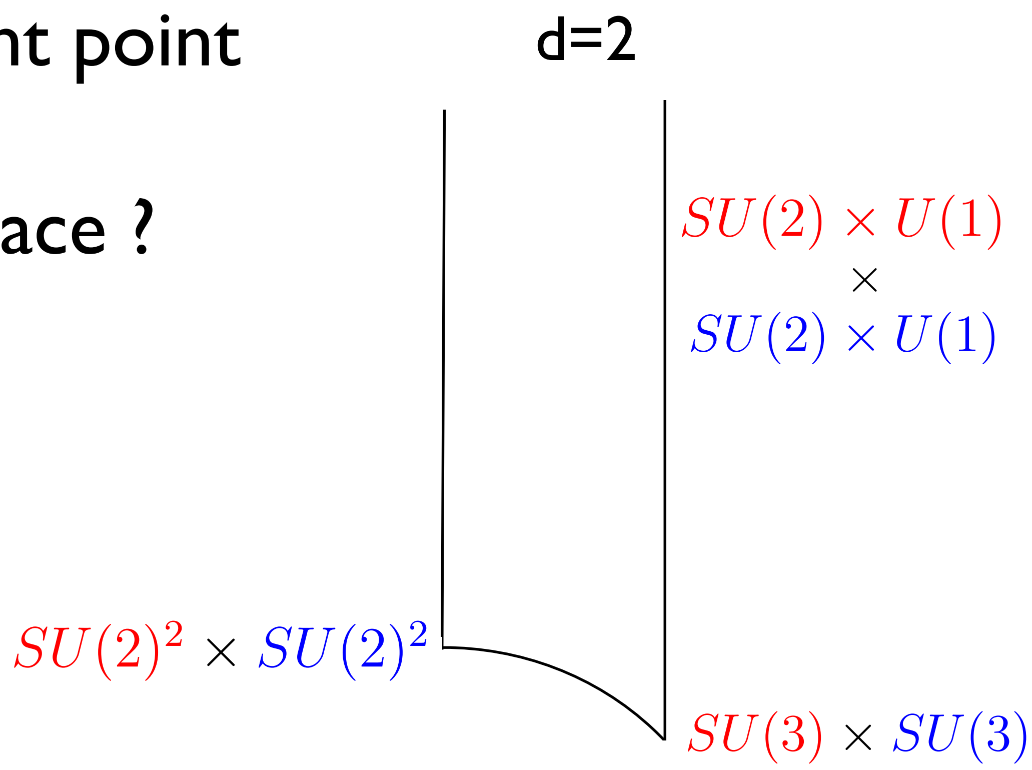

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- Shown that from $SU(2) \times SU(3) \times SU(2) \times SU(3)$

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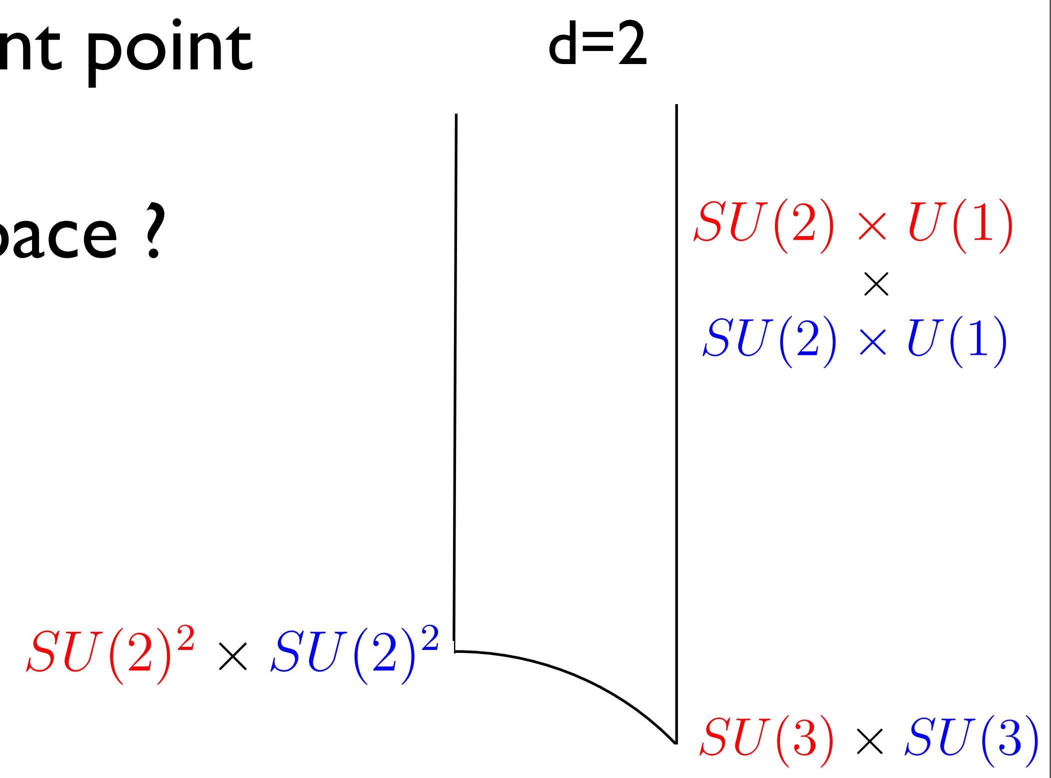

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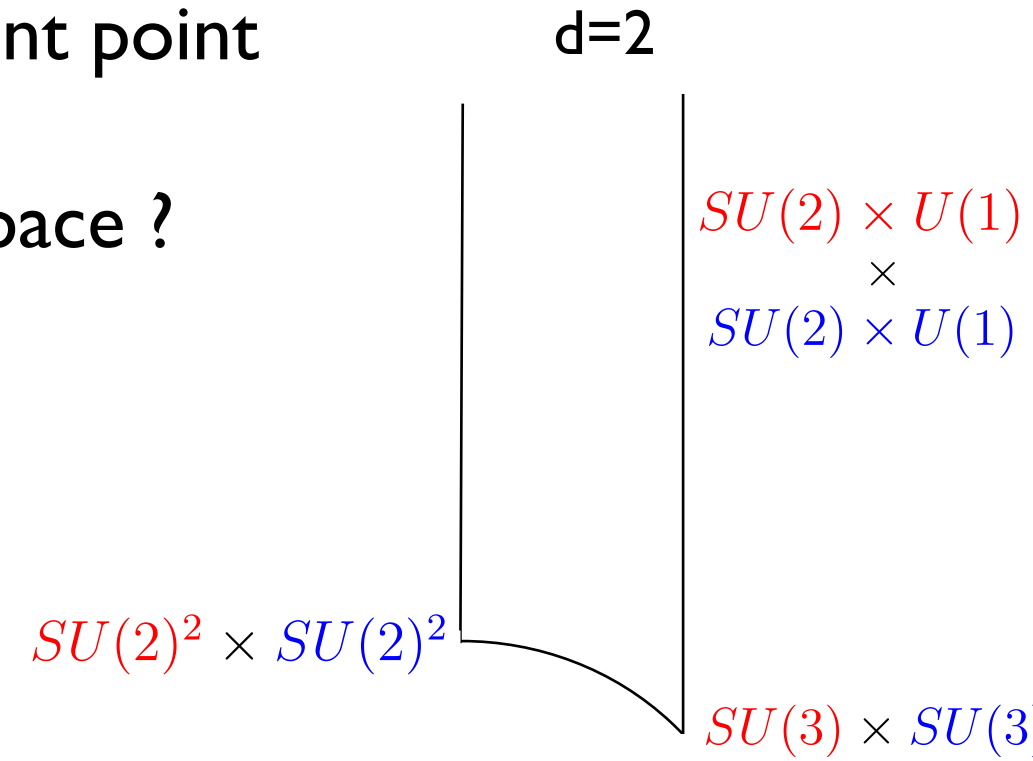
- Shown that from $SU(2) \times SU(3) \times SU(2) \times SU(3) \rightarrow SU(3) \times \mathbf{U(1)} \times SU(3) \times \mathbf{U(1)}$

Effective action found is good close to enhancement point



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$\rightarrow SU(2) \times SU(2) \times \mathbf{U}(\mathbf{1}) \times SU(2) \times SU(2) \times \mathbf{U}(\mathbf{1})$

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Can we find a description "good" for all moduli space ?

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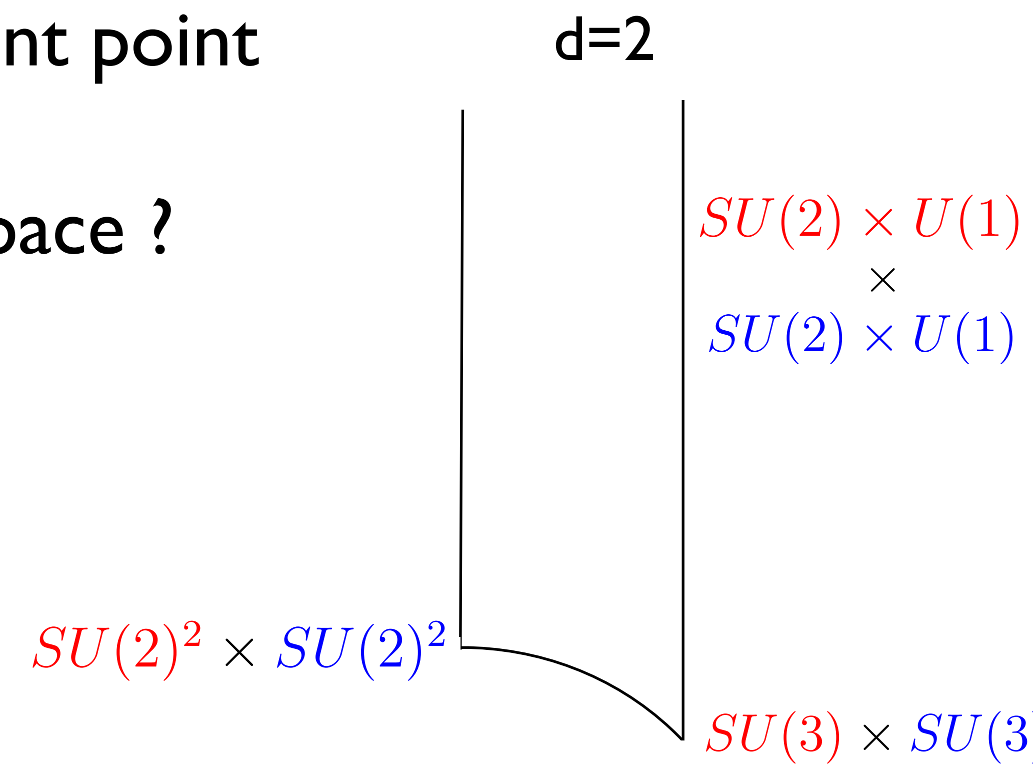
☞ We need a larger group coming from an enhancement in T^3



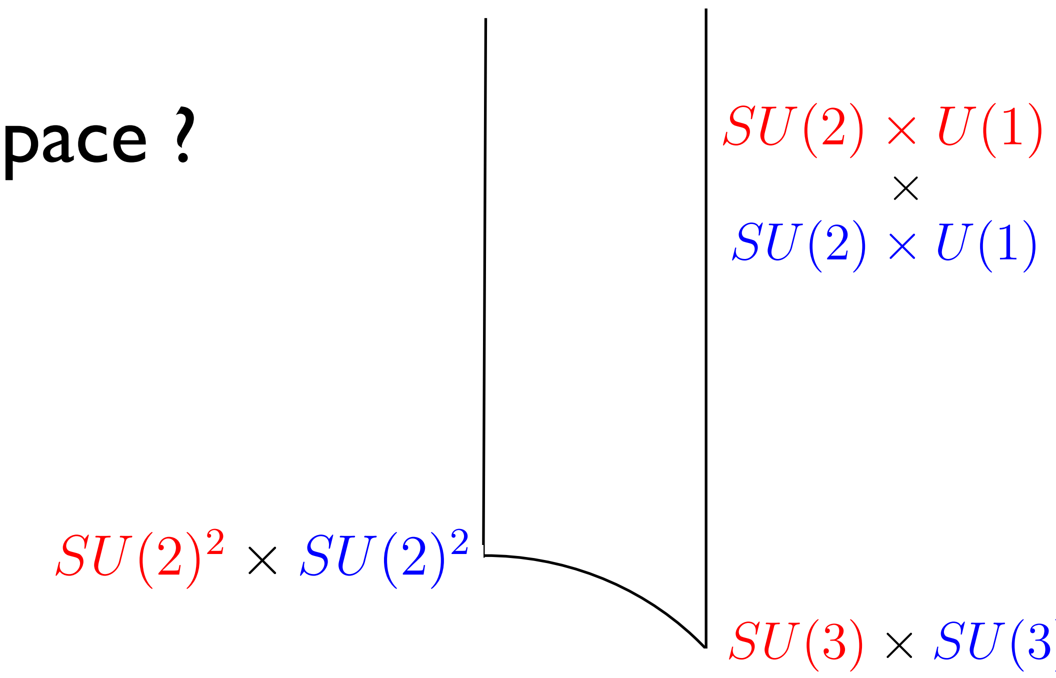
• Shown that from $SU(2)\times SU(3)\times SU(2)\times SU(3)$

$$\begin{aligned}
&\to SU(3)\times \mathbf{U(1)} \times SU(3)\times \mathbf{U(1)}\\
&\to SU(2)\times SU(2)\times \mathbf{U(1)} \times SU(2)\times SU(2)\times \mathbf{U(1)}\\
&\to SU(2)\times U(1)\times \mathbf{U(1)} \times SU(2)\times U(1)\times \mathbf{U(1)}
\end{aligned}$$

decompactify

Effective action found is good close to enhancement point

d=2



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$$\to SU(3) \times \mathbf{U(1)} \times SU(3) \times \mathbf{U(1)}$$
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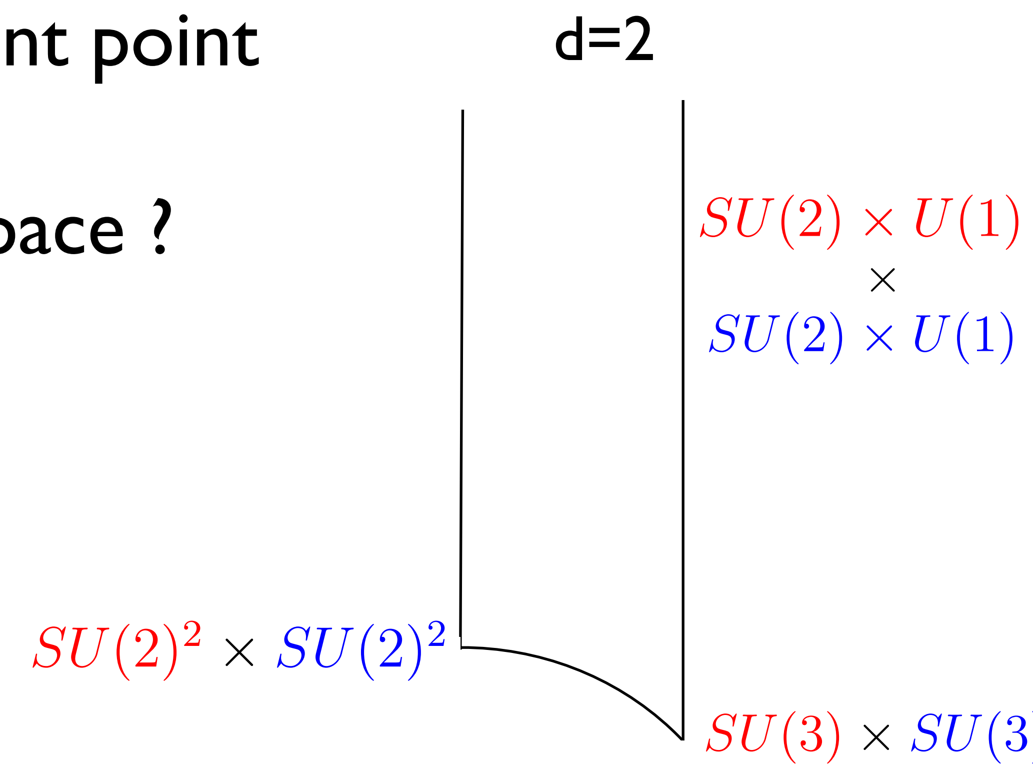
$$\to SU(2) \times U(1) \times \mathbf{U(1)} \times SU(2) \times U(1) \times \mathbf{U(1)}$$

decompactify

☞ To describe all moduli space of T^3, need $SU(4) \times SU(2) \times SU(4) \times SU(2)$ coming from an enhancement in T^4

Effective action found is good close to enhancement point



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- Shown that from $SU(2)\times SU(3)\times SU(2)\times SU(3)$

$$\to SU(3)\times \mathbf{U(1)}\times SU(3)\times \mathbf{U(1)}$$
$$\to SU(2)\times SU(2)\times \mathbf{U(1)}\times SU(2)\times SU(2)\times \mathbf{U(1)}$$
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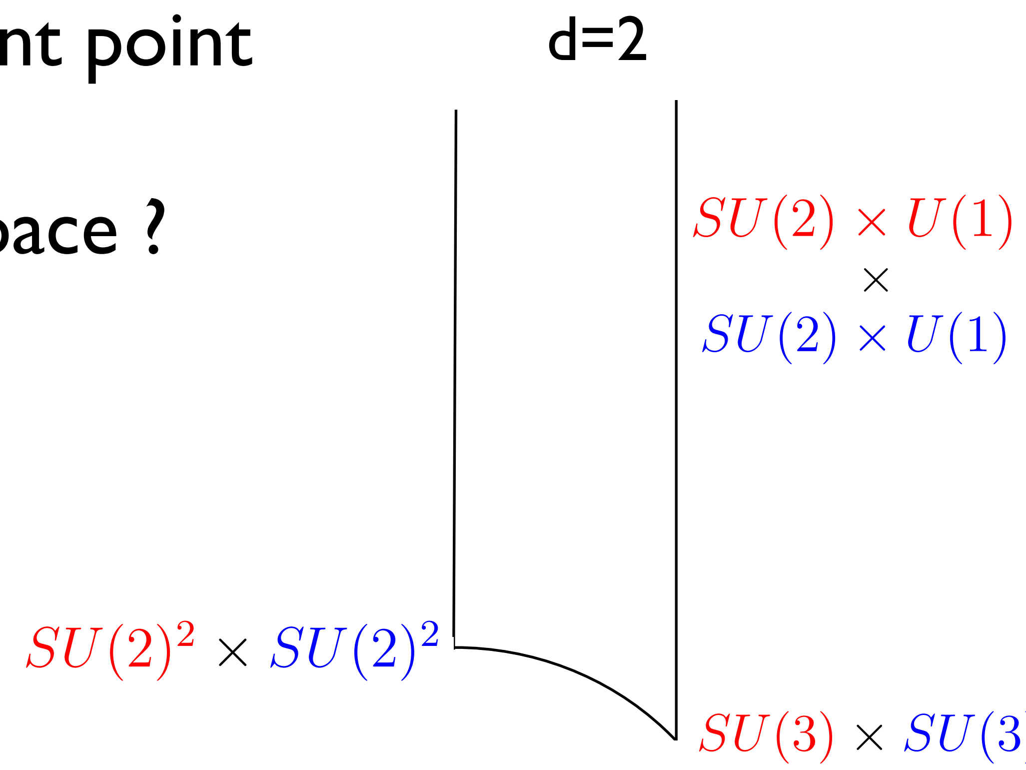
decompactify

☞ To describe all moduli space of T^3, need $SU(4)\times SU(2)\times SU(4)\times SU(2)$ coming from an enhancement in T^4

☞ To describe all moduli space of T^4, need to consider enhancement groups on T^7

Effective action found is good close to enhancement point



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- Shown that from $SU(2)\times SU(3)\times SU(2)\times SU(3)$

$$\to SU(3)\times \mathbf{U(1)}\times SU(3)\times \mathbf{U(1)}$$
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decompactify

☞ To describe all moduli space of T^3, need $SU(4)\times SU(2)\times SU(4)\times SU(2)$ coming from an enhancement in T^4

☞ To describe all moduli space of T^4, need to consider enhancement groups on T^7

☞ But action not a good low energy action

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- We can describe all moduli space. But...
  - Systematics...?
  - Is that truncation of any use?