Yang-Baxter sigma-models & conformal twists

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AdS/CFT is a duality between type IIB superstrings on AdS_5 x S^5 and N=4 super Yang-Mills

rich integrable structure

can construct a flat connection Bena, Roiban, Polchinski (2003)

expansion of Lax pair results in finite set of conserved charges

less symmetric examples

recent progress in understanding integrable deformations of AdS geometries in a unified way

Yang-Baxter (YB) sigma-models

key to this approach is an r-matrix solution to classical Yang Baxter equation (CYBE)

set up correspondence between r-matrices and geometries

"gravity/CYBE correspondence"

touch of experimental physics

take your favourite r-matrix, plug into sigma-model, read off deformed geometry, repeat

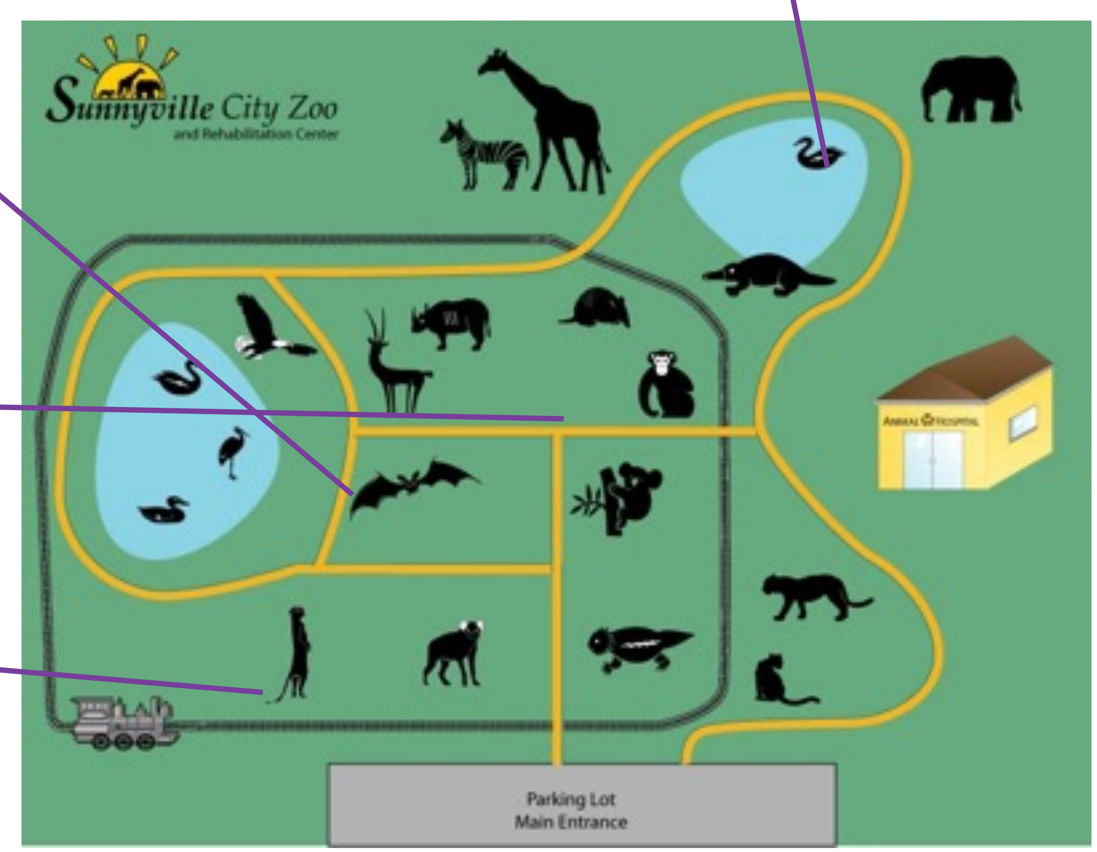
build up a zoo of integrable deformations

generalized supergravity

open strings

TsT transformations

non-Abelian T-duality



let us begin with Yang-Baxter prehistory

early TsT deformations of AdS_5 x S^5
inspired by NC deformations of N=4 sYM

Hashimoto, Itzhaki; Maldacena, Russo; Alishahiha, Oz, Sheikh-Jabbari (1999)

$$\mathrm{d}s^2 = \frac{(-\mathrm{d}t^2 + \mathrm{d}x_3^2 + \mathrm{d}z^2)}{z^2} + \frac{z^2}{(z^4+\eta^2)}(\mathrm{d}x_1^2 + \mathrm{d}x_2^2),$$

$$B = \frac{\eta}{z^4+\eta^2}\mathrm{d}x_1 \wedge \mathrm{d}x_2, \quad e^{2\Phi} = \frac{z^4}{(z^4+\eta^2)}.$$

$$[\hat{x}^\mu, \hat{x}^\nu] = i\Theta^{\mu\nu}, \quad \Theta^{12} = -\eta$$

in hindsight, we now understand

i) early T-duality shift T-duality (TsT)

integrable deformation Lunin, Maldacena; Frolov (2005)

ii) more generally, Yang-Baxter deformation

$$r = \frac{1}{2} P_1 \wedge P_2$$ Matsumoto, Yoshida (2014)

more milestones...

YB sigma-models introduced for PCM - r-matrices satisfy mCYBE Klimcik (2002, 2008)

extension to symmetric cosets $AdS_5 \times S^5$

Delduc, Magro, Vicedo (2013) Kawaguchi, Matsumoto, Yoshida (2014)

generalized type IIB supergravity - DFT/EFT

Arutyunov, Borsato, Frolov, Hoare, Roiban, Tseytlin (2015)

efforts to understand transformation: non-commuting TsTs, non-Abelian T-duality

Borsato, Hoare, Matsumoto, Osten, Thompson, Tseytlin, van Tongeren, Wulff, Yoshida,...(2014-2017)

YB sigma-model

concreteness, AdS_5 coset

$$\mathcal{L} = \mathrm{Tr}\left[A\, P^{(2)} \circ \frac{1}{1 - 2\eta R_g \circ P^{(2)}} A\right]$$

$$R_g(X) \equiv g^{-1} R(g X g^{-1}) g,$$
$$A = -g^{-1} \mathrm{d}g, \quad g \in SO(4,2)$$

$$[R(X), R(Y)] - R([R(X), Y] + [X, R(Y)]) = -c^2 [X, Y]$$
$$X, Y \in \mathfrak{g}, \quad c \in \mathbb{C}$$

can write linear operator R as r-matrix

$$
\begin{aligned}
R(X) &= \mathrm{Tr}_2[r(1\otimes X)] = \sum_{i,j} r^{ij} b_i \mathrm{Tr}[b_j X], \\
r &= \frac{1}{2}\sum_{i,j} r^{ij} b_i \wedge b_j, \qquad b_i \in \mathfrak{so}(4,2)
\end{aligned}
$$

Abelian $\quad [b_i, b_j] = 0 \quad$ unimodular $\quad r^{ij}[b_i, b_j] = 0$

Borsato, Wulff (2016)

absence of conformal anomaly $\quad f_{ab}{}^{a} = 0$

Alvarez-Gaume et al.; Elitzur et al (1995) read off geometry

$$g = \exp[x^\mu P_\mu] \exp[(\log z) D]$$

$$g_{MN} = e^m_M e^n_N k_{(mn)}, \quad B_{MN} = e^m_M e^n_N k_{[nm]},$$
$$\mathrm{e}^\Phi = g_s (\det{}_5\, k)^{1/2}, \quad k_{mn} = k_{(mn)} + k_{[mn]}$$

$$k_m{}^n \equiv (\delta_m{}^n - 2\eta\lambda_m{}^n)^{-1},$$
$$\lambda_m{}^n \equiv \eta^{nl} \mathrm{Tr}[\mathbf{P}_l R_g(\mathbf{P}_m)]$$

key point: completely systematic way to generate integrable deformations

But what about the thread to NC YM?

Abelian, Jordanian ([h, e] = e) deformations - Drinfeld twists of symmetry algebra

conjectured AdS/CFT duality (unimodular)

Seiberg-Witten: low-energy limit of - Poincare

for (non-commuting) TsT this is natural

observed that NC parameter is the r-matrix

van Tongeren (2015, 2016)

We present a unified prescription starting from one key observation.

for all (homogeneous) closed string CYBE deformations of AdS_5, the open string metric is undeformed!

All information about the deformation is encoded in a noncommutaive (NC) parameter

YB community have been working out Drinfeld twists of full conformal algebra

“holographic noncommutativity" (z-dependence)

brute force proof that $\Theta^{MN} = -2\eta r^{MN}$

unimodularity: result of symmetry principle open string metric is undeformed

$$G_{MN} = \left(g - Bg^{-1}B\right)_{MN} ,$$

$$\Theta^{MN} = -\left((g+B)^{-1}B(g-B)^{-1}\right)^{MN} ,$$

Seiberg, Witten

$$G_s = g_s \mathrm{e}^{\Phi} \left(\frac{\det(g+B)}{\det g}\right)^{\frac{1}{2}} .$$

$$\mathrm{d}s^2_{\text{open}} = \frac{1}{z^2}(-\mathrm{d}x_0^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2 + \mathrm{d}z^2),$$

$$\Theta^{12} = -\eta, \quad G_s = g_s .$$

HI/MR solution

more generally (should also hold mCYBE)

$$G^{MN} = e_m^M e_n^N \eta^{mn}, \quad \Theta^{MN} = 2\eta e_m^M e_n^N \lambda^{mn}$$

let us consider Drinfeld twists of Poincare subalgebra

can formulate QFT on NC spacetime - Moyal star product

$$(f \star g)(x) = f(x) \mathrm{e}^{\frac{i}{2}\Theta^{\mu\nu}\overleftarrow{\partial_\mu}\overrightarrow{\partial_\nu}} g(x),$$
$$[f, g]_\star := i\Theta^{\mu\nu}\partial_\mu f \partial_\nu g + \mathcal{O}(\partial^3 f, \partial^3 g)$$

Moyal bracket not Lorentz invariant - puzzle

Chaichian, Kulish, Nishijima, Tureanu (2004)

co-product of Hopf-Poincare algebra deformed

simplest twist of Poincare algebra

$$\mathcal{F} = e^{-2i\eta r} = e^{\frac{i}{2}\Theta^{\mu\nu} P_\mu \wedge P_\nu}$$

other Poincare twists

HI/MR twist is simplest - constant NC parameter

Poincare twists (P, M)- linear, quadratic x-dependence

$$r = \frac{1}{2} M_{01} \wedge M_{23}$$

Lukierski, Woronowicz (2005)

$$\Theta^{02} = -\,2\sinh\frac{\eta}{2}\cdot x^1 x^3, \qquad \Theta^{03} = 2\sinh\frac{\eta}{2}\cdot x^1 x^2,$$

$$\Theta^{12} = -\,2\sinh\frac{\eta}{2}\cdot x^0 x^3, \qquad \Theta^{13} = 2\sinh\frac{\eta}{2}\cdot x^0 x^2.$$

agrees with our proposal at leading order more generally, (new) conformal twists

e. g.

$$r_1 = \frac{1}{2} D \wedge K_1,$$
$$r_2 = \frac{1}{2}(P_0 - P_3) \wedge (D + M_{03}),$$

$$\Theta^{1\mu} = \eta x^\mu (x_\nu x^\nu + z^2), \quad \Theta^{1z} = \eta z (x_\nu x^\nu + z^2),$$
$$\Theta^{-+} = -4\eta x^+, \quad \Theta^{-i} = -2\eta x^i, \quad \Theta^{-z} = -2\eta z,$$
$$x^\pm = x^0 \pm x^3$$

for AdS_5 can be proved $\Theta^{MN} = -2\eta r^{MN}$

same result from conformal twist

$$\mathrm{d}s^2 = -\mathrm{d}x^+\mathrm{d}x^- + (\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2$$

$$P_0 - P_3 = -2\partial_-, \quad D + M_{03} = -2x^+\partial_+ - x^1\partial_1 - x^2\partial_2$$

$$\begin{aligned} f(x) \star g(x) &= m \circ \mathcal{F}(f(x) \otimes g(x)) = m \circ e^{-2i\eta r_2}(f(x) \otimes g(x)) \\ &= m \circ e^{-i\eta(P_0 - P_3)\wedge(D + M_{03})}(f(x) \otimes g(x)) \end{aligned}$$

$$\begin{aligned} [x^\mu, x^\nu]_\star &= x^\mu \star x^\nu - x^\nu \star x^\mu \\ &= -i\eta(x^+\eta^{\mu+}\eta^{\nu-} - x^1\eta^{\mu+}\eta^{\nu 1} - x^2\eta^{\mu+}\eta^{\nu 1} - \mu \leftrightarrow \nu) \end{aligned}$$

if r-matrices are generally non-Abelian

led to generalized supergravity

Arutyunov, Frolov, Hoare, Roiban, Tseytlin (2015)

$$R_{MN} - \frac{1}{4} H_{MKL} H_N{}^{KL} - T_{MN} + D_M X_N + D_N X_M = 0\,,$$
$$\frac{1}{2} D^K H_{KMN} + \frac{1}{2} F^K F_{KMN} + \frac{1}{12} F_{MNKLP} F^{KLP} = X^K H_{KMN} + D_M X_N - D_N X_M\,,$$
$$R - \frac{1}{12} H^2 + 4 D_M X^M - 4 X_M X^M = 0$$

$$X_M \equiv I_M + Z_M\,,$$
$$Z_M = \partial_M \Phi - B_{MN} I^N$$

I is Killing - supergravity recovered when I vanishes

efforts to recover from DFT/EFT

Baguet, Magro, Samtleben; Sakatani, Uehara, Yoshida

symmetry principle

Λ-symmetry $\quad B \to B + \mathrm{d}\Lambda \quad$ supergravity invariant

$$S_{\mathrm{DBI}} = \int e^{-\Phi} \sqrt{\det(g + B + \mathrm{d}A)} \quad B \to B + \mathrm{d}\Lambda, \quad A \to A - \Lambda$$

$$\frac{1}{G_s} \sqrt{\det G} = \frac{e^{-\Phi}}{g_s} \sqrt{\det(g + B)}$$

$$0 = \delta_\Lambda S_{\mathrm{tot}} = \delta_\Lambda S_{\mathrm{sugra}} + \delta_\Lambda S_{\mathrm{DBI}} = \int \sqrt{G} \nabla_M^{(G)} \Theta^{MN} \Lambda_N$$

generalized supergravity $\quad \nabla_M \Theta^{MN} = I^N$

an example

$$r = \frac{1}{2} K_1 \wedge D$$

$$\begin{aligned}
\mathrm{d}s^2 &= \frac{z^2[\mathrm{d}t^2 + \mathrm{d}{x_1}^2 + \mathrm{d}z^2] + \eta^2(x_1^2 + t^2 + z^2)^2(\mathrm{d}t - tz^{-1}\mathrm{d}z)^2}{z^4 + \eta^2(t^2 + z^2)(x_1^2 + t^2 + z^2)^2} \\
&+ \frac{t^2(-\mathrm{d}\phi^2 + \cosh^2\phi\,\mathrm{d}\theta^2)}{z^2} + \mathrm{d}s^2(S^5)\,, \\
B &= \frac{\eta(x_1^2 + t^2 + z^2)}{z^4 + \eta^2(t^2 + z^2)(x_1^2 + t^2 + z^2)^2}\mathrm{d}x_1 \wedge (t\mathrm{d}t + z\mathrm{d}z)\,, \\
\Phi &= \frac{1}{2}\log\left[\frac{z^4}{z^4 + \eta^2(t^2 + z^2)(x_1^2 + t^2 + z^2)^2}\right]
\end{aligned}$$

$$\begin{aligned}
x_0 &= t\sinh\phi, \\
x_2 &= t\cosh\phi\cos\theta, \\
x_3 &= t\cosh\phi\sin\theta
\end{aligned}$$

an example

special conformal symmetry

$$I = \eta \left[(t^2 - x_1^2 + z^2)\partial_{x_1} - 2x_1(t\partial_t + z\partial_z) \right]$$

$$\Theta^{tx_1} = \eta t(t^2 + x_1^2 + z^2), \quad \Theta^{zx_1} = \eta z(t^2 + x_1^2 + z^2)$$

$$\nabla_M \Theta^{Mx_1} = \eta(t^2 - x_1^2 + z^2) = I^{x_1}$$

$$\nabla_M \Theta^{Mt} = -2\eta x_1 t = I^t\,,$$

$$\nabla_M \Theta^{Mz} = -2\eta x_1 z = I^z$$

picture works for YB deformations based on mCYBE

take the ABF solution

$$g_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} = -\frac{(1+\rho^2)\mathrm{d}t^2}{1-\kappa^2\rho^2} + \frac{\mathrm{d}\rho^2}{(1+\rho^2)(1-\kappa^2\rho^2)} + \frac{\rho^2\mathrm{d}\zeta^2}{1+\kappa^2\rho^4\sin^2\zeta}$$
$$+ \frac{\rho^2\cos^2\zeta\mathrm{d}\psi_1^2}{1+\kappa^2\rho^4\sin^2\zeta} + \rho^2\sin^2\zeta\mathrm{d}\psi_2^2,$$
$$B = -\frac{\kappa\rho^4\sin(2\zeta)}{2(1+\kappa^2\rho^4\sin^2\zeta)}\mathrm{d}\zeta\wedge\mathrm{d}\psi_1 - \frac{\kappa\rho}{1-\kappa^2\mathrm{d}\rho}\mathrm{d}t\wedge\mathrm{d}\rho$$

$$\Theta^{\zeta\psi_1} = \kappa\tan\zeta, \qquad \Theta^{t\rho} = -\kappa\rho,$$
$$I^t = 4\kappa, \quad I^{\psi_1} = 2\kappa$$

what is the connection to DFT/EFT?

recent interesting paper - 1705.07116 Sakamoto, Sakatani, Yoshida

beta-twists from generalized diffeomorphisms

what they did not tell you (to maintain mystery?)

$$\beta = \Theta$$

$$f^{a}{}_{ab} = Q_{a}{}^{ab} = 0 \Leftrightarrow \nabla_a \Theta^{ab} = 0$$

$$R^{abc} = 3\Theta^{d[a}\partial_d \Theta^{bc]}$$

is there a nice DFT equivalent for Killing vector I?

homogeneous YB deformations of AdS and conformal twists have same algebraic structure

open string metric is undeformed

NC parameter is the r-matrix

NC parameter encodes information about YB deformation, e. g. whether a supergravity solution, or not

same picture holds for r-matrix solutions to mCYBE

conformal twists of CFT Araujo, OC, Sheikh-Jabbari

integrability of NC deformations of N=4 sYM

Beisert, Roiban (2005)