# Non-Abelian T-duality and Linear Quivers

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"Recent Advances in T/U-dualities and Generalized Geometries"
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# 1. Introduction & motivation: NATD in AdS/CFT

Non-Abelian T-duality (NATD) has proved to be very useful as a solution generating technique in AdS/CFT

Its realization in the CFT remains however quite unknown

Interestingly, some examples suggest that, contrary to its Abelian counterpart, NATD may change the CFT:

NATD of $AdS_5 \times S^5$ : Gaiotto & Maldacena geometry (dual to N=2 SCFTs (Gaiotto theories))

NATD of $AdS_5 \times T^{1,1}$ : Bah, Beem, Bobev, Wecht geometry (dual to N=1 SCFTs (Sicilian quivers))

See however Jesús Montero's talk

Indeed, contrary to its Abelian counterpart, NATD has not been proven to be a symmetry of string theory

Applying NATD to an AdS/CFT pair, a new AdS background is generated which may have associated a different CFT dual, which, moreover, may only exist in the strong coupling regime

New way to describe new CFTs through AdS/CFT

In all examples the CFTs seem to originate from M5-branes

The focus of this talk will be on the interplay between NATD and AdS/CFT, *taking the CFT side*

Based on Y.L., Carlos Núñez, 1603.04440

And also:
- Y.L., Niall Macpherson, Jesús Montero, Carlos Núñez, 1609.09061
- Y.L., Carlos Núñez, Salomón Zacarías, 1703.00417
- Georgios Itsios, Y.L., Jesús Montero, Carlos Núñez, 1705.09661

# Outline:

1. Introduction and motivation: NATD in AdS/CFT

2. Basics of NATD: i) NATD vs Abelian T-duality
   ii) NATD as a solution generating technique

3. The Sfetsos-Thompson $AdS_5 \times S^2$ background

   3.1. Short review about Gaiotto-Maldacena geometries

   3.2. ST as a GM geometry

4. The $AdS_4 \times S^2 \times S^2$ example

5. Conclusions

# 2. Basics of NATD: i) NATD vs Abelian T-duality

Using the string sigma-model Rocek and Verlinde proved that Abelian T-duality is a symmetry to all orders in $g_s$ and $\alpha'$

The extension to arbitrary wordsheets determines the global properties of the dual variable:

$$\theta \in [0, 2\pi] \quad \xrightarrow{\text{T}} \quad \tilde{\theta} \in [0, 2\pi]$$

In the non-Abelian case neither proof works

Variables living in a group manifold are substituted by variables living in its Lie algebra

$$g \in SU(2) \quad \xrightarrow{\text{NAT}} \quad \chi \in \mathbb{R}^3$$

In the absence of global information the new variables remain non-compact

In more detail:

Rocek and Verlinde's formulation of Abelian T-duality for ST in a curved background (Buscher'88) :

$$\boxed{S=\frac{1}{4\pi\alpha'}\int\Big(g_{\mu\nu}\,dX^\mu\wedge *dX^\nu+B_{\mu\nu}\,dX^\mu\wedge dX^\nu\Big)+\frac{1}{4\pi}\int R^{(2)}\phi}$$

i) Identify an Abelian isometry: $X^\mu=\{\theta,X^\alpha\}$ such that

$\theta\rightarrow\theta+\epsilon$ and $\partial_\theta(\text{backgrounds})=0$

ii) Gauge the isometry: $d\theta\rightarrow D\theta=d\theta+A$

$A$ non-dynamical gauge field / $\delta A=-d\epsilon$

iii) Add a Lagrange multiplier term: $\tilde{\theta}\, dA$ , such that

$$\int \mathcal{D}\tilde{\theta} \;\rightarrow\; dA = 0 \;\Rightarrow\; A \text{ exact}$$

(in a topologically trivial worldsheet)

+ fix the gauge: $A = 0 \rightarrow$ Original theory

iv) Integrate the gauge field

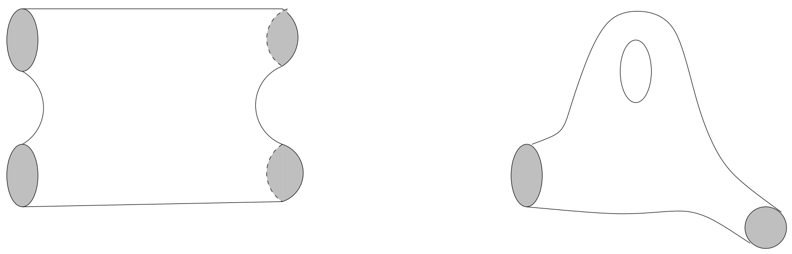
+ fix the gauge: $\theta = 0 \quad \rightarrow$ Dual sigma model:

$\{\theta, X^\alpha\} \rightarrow \{\tilde{\theta}, X^\alpha\}$ and

$(\tilde{g}, \tilde{B}_2, \tilde{\phi})$ given by Buscher's formulae

- Conformal invariance? Original and dual theories can be obtained from the gauged Lagrangian either gauging a vectorial or an axial combination of chiral currents

- Arbitrary worldsheets? (symmetry of string perturbation theory):



(a) (b)

Large gauge transformations: $$\oint_\gamma d\epsilon = 2\pi n\,;\;\; n \in \mathbb{Z}$$

To fix them:

Multivalued Lagrange multiplier: $$\oint_\gamma d\tilde{\theta} = 2\pi m\,;\;\; m \in \mathbb{Z}$$
such that

$$\int [\text{exact}] \;\rightarrow\; dA = 0 \qquad \textbf{+} \qquad \int [\text{harmonic}] \Rightarrow A \text{ exact}$$

This fixes the periodicity of the dual variable

# Non-Abelian T-duality

(De la Ossa, Quevedo'93)

Non-Abelian continuous isometry: $X^m \to g^m_n X^n \,, g \in G$

i) Gauge it: $dX^m \to DX^m = dX^m + A^m_n X^n$

$A \in$ Lie algebra of $G$ $\qquad A \to g(A+d)g^{-1}$

ii) Add a Lagrange multiplier term: $\mathrm{Tr}(\chi F)$

$F = dA - A \wedge A$

$\chi \in$ Lie Algebra of $G$, $\quad \chi \to g\chi g^{-1}$, such that

$\int \mathcal{D}\chi \;\to\; F = 0 \;\Rightarrow\; A$ exact

(in a topologically trivial worldsheet)

+ fix the gauge: $A = 0 \;\Rightarrow$ Original theory

iii) Integrate the gauge field + fix the gauge $\rightarrow$ Dual theory

However:

- Non-involutive
- Higher genus generalization? Set to zero $W_\gamma = P\, e^{\oint_\gamma A}$
- Global properties?

  For SU(2): $\chi \in \mathbb{R}^3$: Global completion of $\mathbb{R}^3$ ?
- Conformal invariance not proved in general (only vectorial gauging)

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True symmetry in String Theory?

Still useful as a solution generating technique

# ii) NATD as a solution generating technique

Need to know how the RR fields transform

Sfetsos and Thompson (2010) extended Hassan's derivation in the Abelian case:
Implement the relative twist between left and right movers in the bispinor formed by the RR fields

# 3. The ST $AdS_5 \times S^2$ background

$$\begin{tikzcd}
AdS_5 \times S^5 \arrow[d, leftrightarrow] \arrow[r, leftrightarrow, "\text{NATD}"] & \text{N=2} \quad AdS_5 \times S^2 \in \text{GM} \arrow[d, leftrightarrow] \\
\text{N=4 SYM} & \text{N=2 SCFT}
\end{tikzcd}$$

(Sfetsos, Thompson'10)

- Gaiotto-Maldacena geometries encode the information about the dual CFT
- Useful example to study the CFT realization of NATD

• Take the $AdS_5 \times S^5$ background

$$ds^2 = ds^2_{AdS_5} + L^2\Big(d\alpha^2 + \sin^2\alpha d\beta^2 + \cos^2\alpha ds^2(S^3)\Big)$$

$$F_5 = 8L^4 \sin\alpha \cos^3\alpha d\alpha \wedge d\beta \wedge \mathrm{Vol}(S^3)$$ + Hodge dual

• Dualize it w.r.t. one of the $SU(2)$ symmetries

In spherical coordinates adapted to the remaining $SU(2)$:

$$ds^2 = ds^2_{AdS_5} + L^2\Big(d\alpha^2 + \sin^2\alpha d\beta^2\Big) + \frac{d\rho^2}{L^2\cos^2\alpha} + \frac{L^2\cos^2\alpha\rho^2}{\rho^2 + L^4\cos^4\alpha} ds^2(S^2)$$

$$B_2 = \frac{\rho^3}{\rho^2 + L^4\cos^4\alpha}\mathrm{Vol}(S^2)\,, \qquad e^{-2\phi} = L^2\cos^2\alpha(L^4\cos^4\alpha + \rho^2)$$

$$F_2 = L^4 \sin\alpha\cos^3\alpha d\alpha \wedge d\beta, \qquad F_4 = B_2 \wedge F_2$$

- New Gaiotto-Maldacena geometry
- What about $\rho$ ?
  - Background perfectly smooth for all $\rho \in \mathbb{R}^+$
  - No global properties inferred from the NATD
  - How do we interpret the running of $\rho$ to infinity in the CFT?
- Singular at $\alpha = \pi/2$ where the original $S^3$ shrinks (due to the presence of NS5-branes)

  This is the tip of a cone with $S^2$ boundary $\rightarrow$

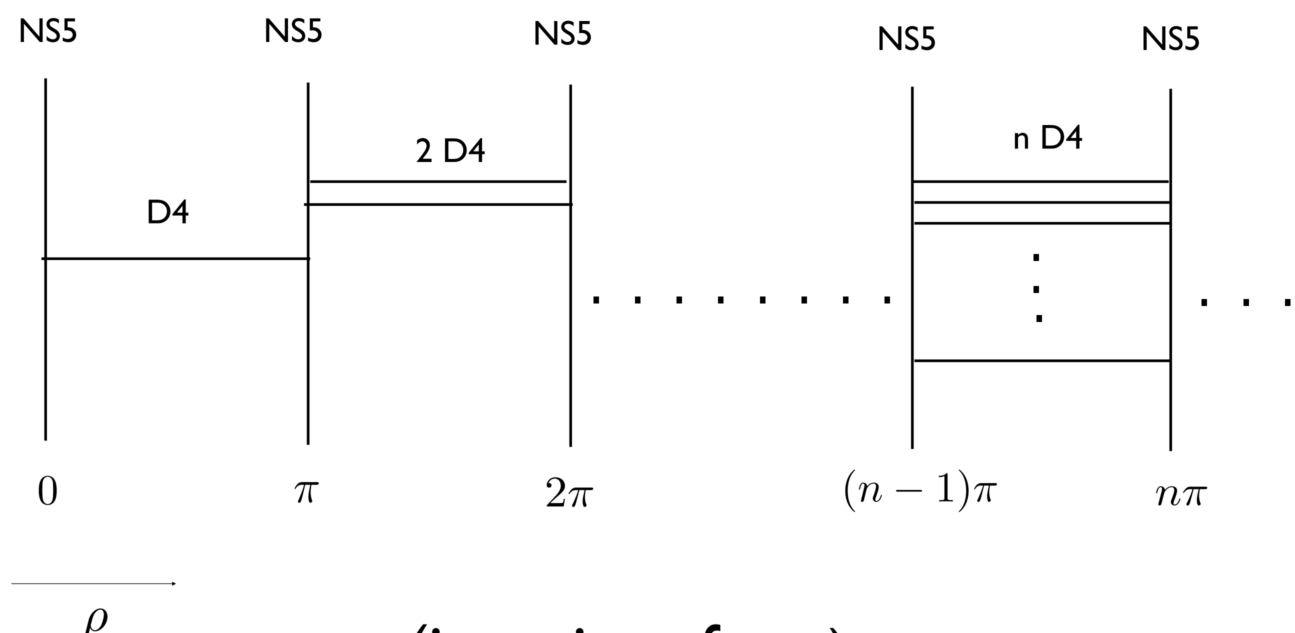
  Large gauge transformations $B_2 \rightarrow B_2 - n\pi \mathrm{Vol}(S^2)$
  for $\rho \in [(n-1)\pi, n\pi]$

This modifies the Page charges such that $N_4 = nN_6$ in each $[(n-1)\pi, n\pi]$ interval

We have also $N_5$ charge, such that every time we cross a $\pi$ interval one unit of NS5 charge is created

This is compatible with a D4/NS5 brane set-up:

D4: $\mathbb{R}^{1,3}, \rho$
NS5: $\mathbb{R}^{1,3}, \alpha, \beta$




(in units of $N_6$ )

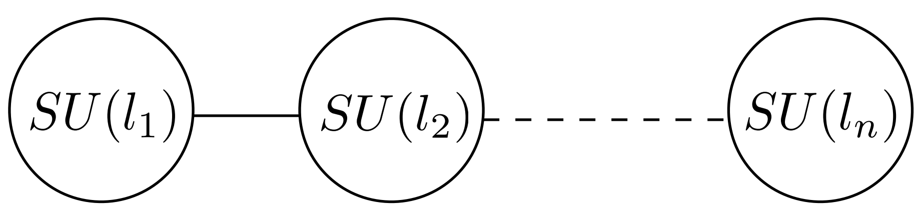
These D4/NS5 brane set-ups realize 4d $\mathcal{N}=2$ field theories with gauge groups connected by bifundamentals (Witten'97)

Having the D4 finite extension in the $\rho$ direction, the field theory living in them is 4d at low energies, with effective gauge coupling:

$$\frac{1}{g_4^2} \sim \rho_{n+1} - \rho_n$$

For $l_n$ D4-branes in $[\rho_n, \rho_{n+1}]$ the gauge group is $SU(l_n)$ and there are $(l_{n-1}, l_n)$ and $(l_n, l_{n+1})$ hypermultiplets.

The field theory is then described by a quiver

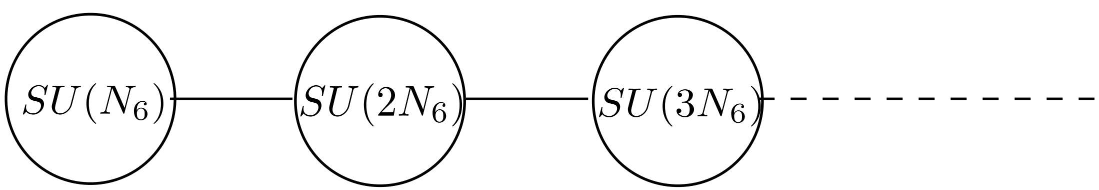


The beta-function for each $SU(l_n)$ gauge theory vanishes if

$$2l_n = l_{n+1} + l_{n-1}$$

This condition is satisfied by our brane configuration, which has $l_n = nN_6$

It corresponds to an infinite linear quiver:




This is in agreement with Gaiotto-Maldacena

# 3.1 Short review of GM geometries

Generic backgrounds dual to 4d N=2 SCFTs.

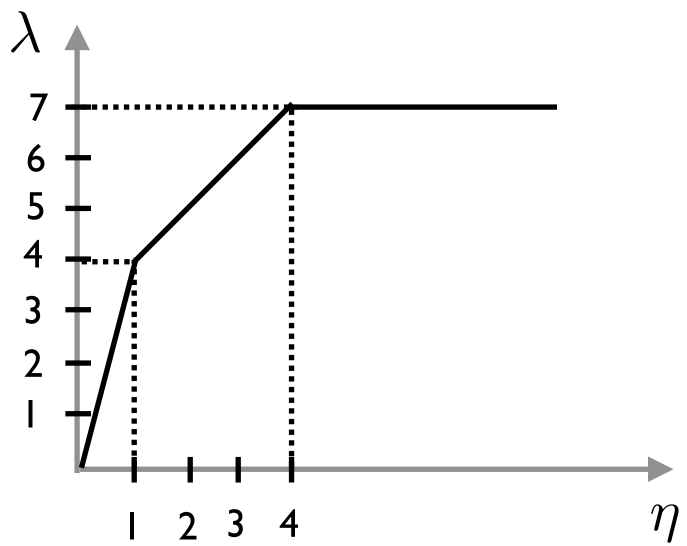
Described in terms of a function $V(\sigma, \eta)$ solving a Laplace eq. with a given charge density $\lambda(\eta)$ at $\sigma = 0$

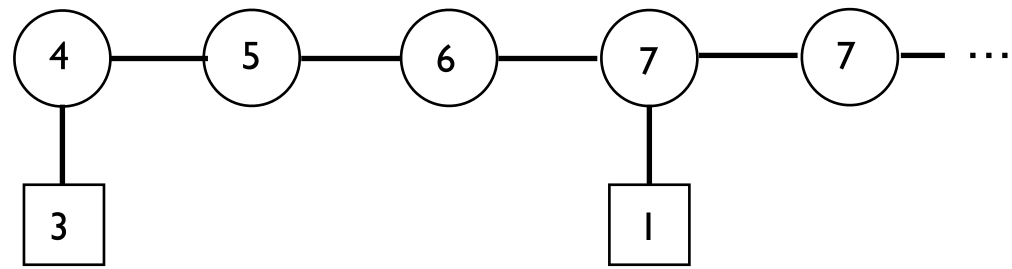
$$\partial_\sigma[\sigma \partial_\sigma V] + \sigma \partial_\eta^2 V = 0, \qquad \lambda(\eta) = \sigma \partial_\sigma V(\sigma, \eta)|_{\sigma=0}$$

Regularity and quantization of charges impose strong constraints on the allowed form of $\lambda(\eta)$, which encodes the information of the dual CFT:

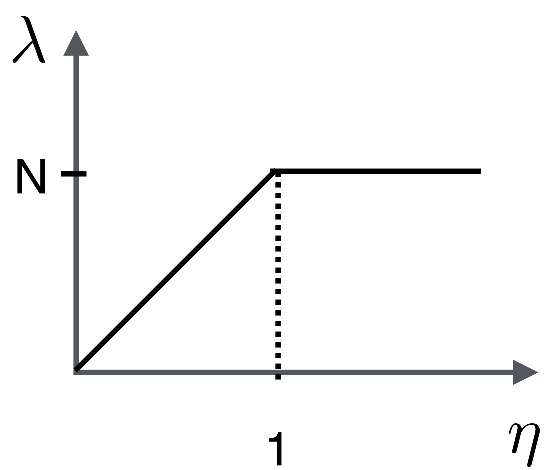
- A $SU(n_i)$ gauge group is associated to each integer value of $\eta = \eta_i$ , with $n_i$ given by $\lambda(\eta_i) = n_i$
- A kink in the line profile corresponds to extra $k_i$ fundamentals attached to the gauge group at the node $n_i$

## For example:

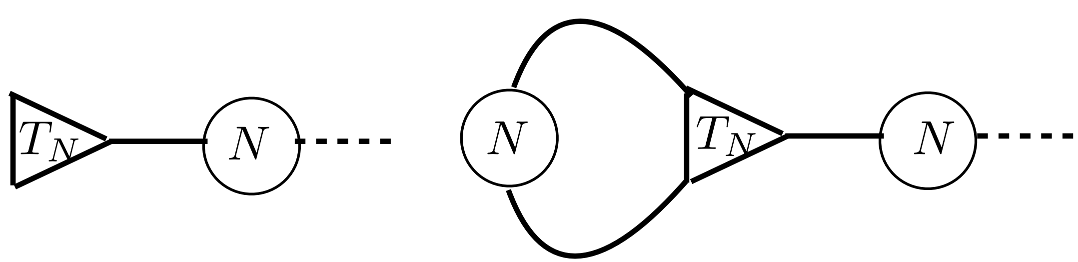



## The Maldacena-Nunez solution:




$$\lambda_{MN}(\eta) = \frac{N}{2}(|\eta + 1| - |\eta - 1|)$$



Interesting for our work:

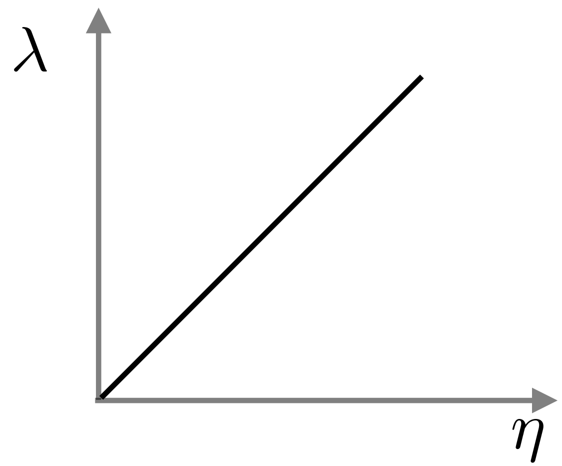
Following Reid-Edwards and Stefanski'10 (see also Aharony, Berdichevsky, Berkooz'12) , the MN solution can be taken as a building block for N=2 IIA solutions: Any allowed profile of the line charge density can be viewed as a sum of suitably re-scaled and shifted $\lambda_{MN}$ profiles

We can use this to *complete* the NATD solution

# 3.2. The NATD as a GM geometry

GM geometry with $\lambda(\eta) = \eta \, , \; \eta \sim \rho \, , \, \sigma = \sin\alpha$

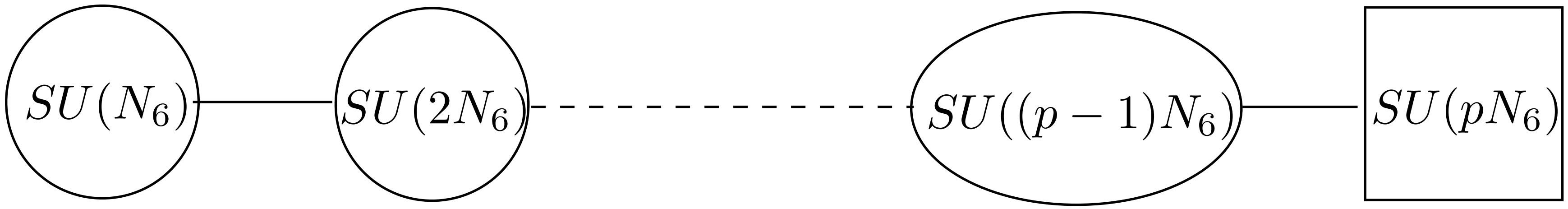
$\lambda(\eta) = \eta \Rightarrow$ Infinite linear quiver, consistent with the brane set-up:




Next, we will complete the quiver and, using holography, *complete* the geometry (both for large $\rho$ and at the singularity)

Example in which the field theory *informs* the geometry

A natural way to complete the quiver is by adding fundamentals:




This completion reproduces correctly the value of the holographic central charge:

From the geometry:

$$c_{NATD} \sim V_{int} \sim \int_0^{\eta_*} f(\eta)d\eta = \frac{N_6^2 N_5^3}{12}$$

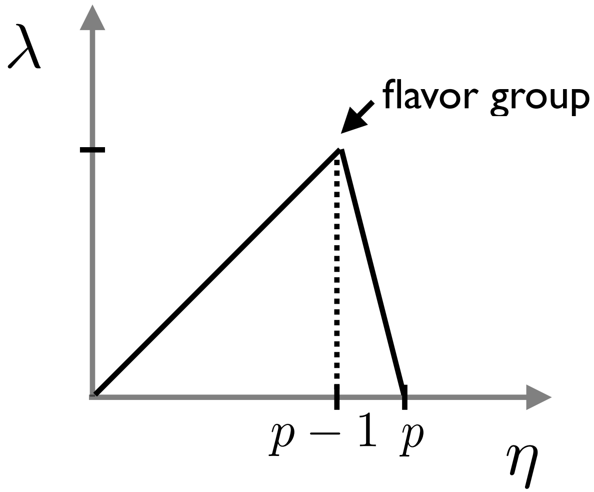
(Klebanov, Kutasov, Murugan'08)

In the field theory we can use: $c = \frac{1}{12}(2n_v + n_h)$ (Shapere, Tachikawa'08)

This gives

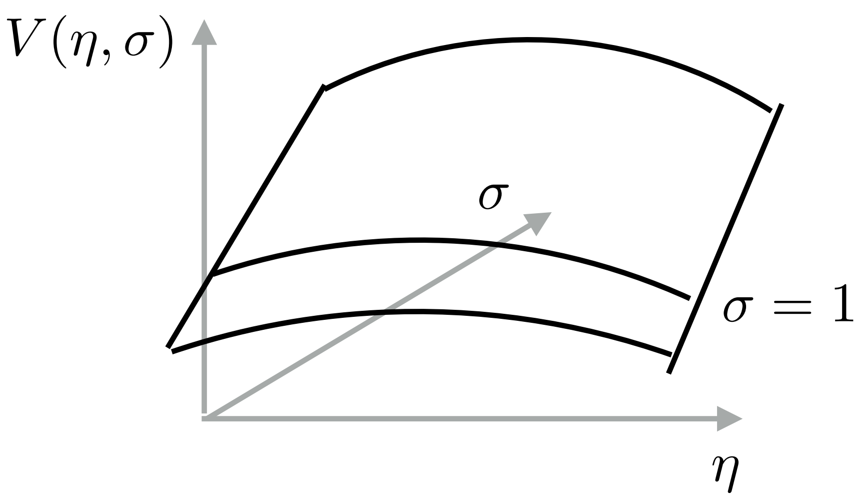
$$c = \frac{N_6^2 p^3}{12}\left[1 - \frac{1}{p} - \frac{2}{p^2 N_6^2} + \frac{2}{N_6^2 p^3}\right] \approx \frac{N_6^2 p^3}{12}$$

## In the geometry, the completed quiver corresponds to




$$\frac{\lambda(\eta)}{N_6} = \begin{cases} \eta & 0 \le \eta \le p-1 \\ (1-p)\eta + (p^2-p) & (p-1) \le \eta \le p \end{cases}$$

This charge density can be obtained as a superposition of MN solutions:




This superposition completes the NATD solution, and removes the singularity

The singularity can be interpreted as a result of cutting the space at $\sigma = 1$

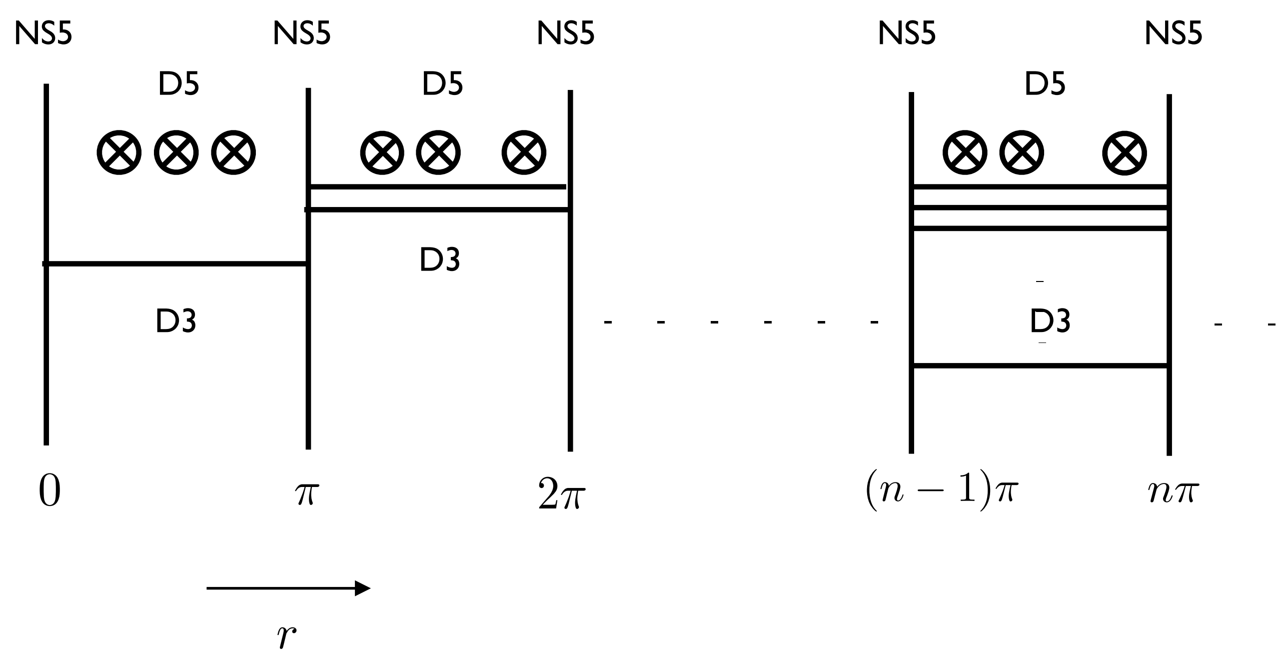
Can we find other examples where the NATD solution belongs to a classification with known field theory dual, to check these ideas?

# 4. The $AdS_4 \times S^2 \times S^2$ example

Non-Abelian T-duality on a reduction to IIA of $AdS_4 \times S^7/\mathbb{Z}_k$

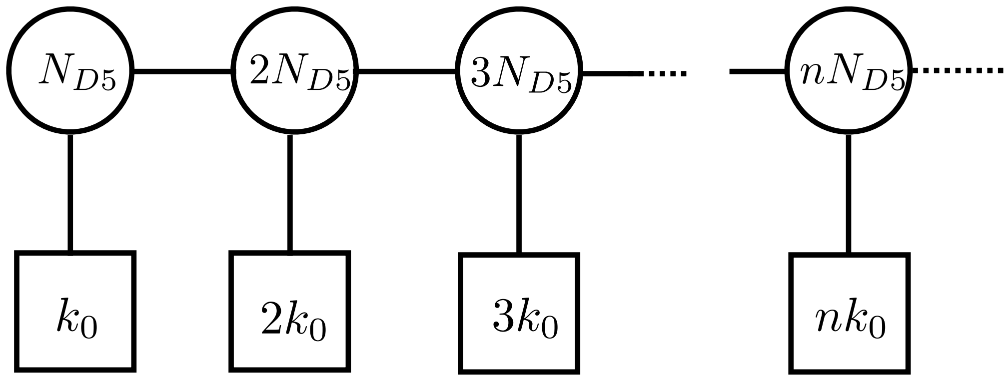
$\rightarrow$ IIB $AdS_4 \times S^2 \times S^2$ background, N=4 SUSY, in the classification of D'Hoker, Estes and Gutperle'07

Analysis of charges: (D3,NS5,D5) brane set-up:




Gaiotto and Witten'08: 3d N=4 $T_{\rho}^{\hat{\rho}}(N)$ theories

$T^{\hat{\rho}}_{\rho}(N)$ field theories flow to CFTs in the infrared if the partitions satisfy certain conditions, that are satisfied by our brane set-up




The holographic duals of these CFTs are known (Assel, Bachas, Estes and Gomis'11)

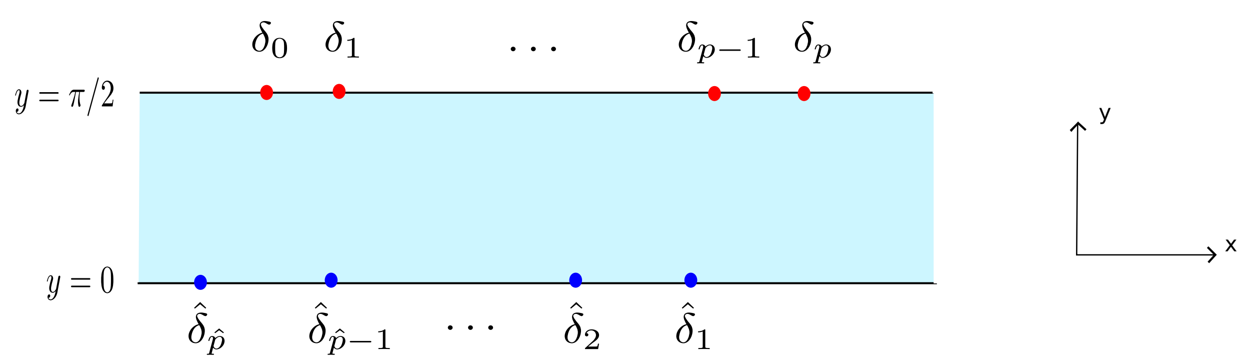
They belong to the general class of $AdS_4 \times S^2 \times S^2$ geometries in D'Hoker, Estes and Gutperle'07

These are fibrations of $AdS_4 \times S^2 \times S^2$ over a Riemann surface that can be completely determined from two harmonic functions $h_1(z, \bar{z}), h_2(z, \bar{z})$

Assel, Bachas, Estes and Gomis'11 showed how to determine these functions from the (D3, NS5, D5) brane set-ups associated to $T_\rho^{\hat\rho}(N)$ theories:

$$h_1 = -\frac{1}{4}\sum_{a=1}^{p} N_5^a \log\tanh\left(\frac{i\frac{\pi}{2}+\delta_a - z}{2}\right) + cc$$

$$h_2 = -\frac{1}{4}\sum_{b=1}^{\hat p} \hat N_5^b \log\tanh\left(\frac{z-\hat\delta_b}{2}\right) + cc$$




The positions of the D5 and NS5 branes are determined, in turn, from the linking numbers of the configuration:

$$\hat\delta_b - \delta_a = \log\tan\left(\frac{\pi}{2}\frac{l_a \hat l_b}{N}\right)$$

The $h_1, h_2$ functions computed from our *completed* brane set-up agree with those associated to the non-Abelian T-dual geometry in the region $x, y \sim 0$ , far from the location of the branes

The non-Abelian T-dual arises as a result of zooming-in in a particular region of the *completed* solution

This completion smoothes out the singularities and defines the geometry globally

# 5. Conclusions

- NATD geometries dual to infinite linear quivers

$\rightarrow$ Different CFTs after NATD

D3 $\rightarrow$ (D4,NS5) (D2,D6) $\rightarrow$ (D3,NS5,D5)

- Quivers completed, and thereof the geometries, to define the CFTs

- NATD as a zooming-in in a patch of the completed geometry

In fact, Penrose limit of superstar solution (see C. Nuñez's talk)

- General pattern?

$AdS_5 \times T^{1,1}$ $\rightarrow$ (D4,NS5,NS5') brane set-up

(see J. Montero's talk)

Dual CFTs originating from M5-branes

$AdS_6 \times S^4:$ (D4,D8) system $\rightarrow$ (D5,NS5,D7)

$AdS_6 \times S^2$ IIB solutions recently classified by D'Hoker, Gutperle, Karch and Uhlemann'16

THANKS!