Doubled strings and negative branes

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Recent Advances in T/U-dualities and Generalized Geometries
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VRIJE UNIVERSITEIT BRUSSEL

Fonds Wetenschappelijk Onderzoek Vlaanderen
Opening new horizons

Goal

- Explore some exotic dualities
- Use double field theory (DFT) and exceptional field theory (EFT) as a tool to do so.
- Questions: What solutions does DFT/EFT contain? What supergravities does DFT/EFT contain?

Solutions

Magnetic solutions:

NS5($t12345$)
$H_{ijk} = 3\partial_{[i}B_{jk]}$

Solutions

Magnetic solutions:

$$\begin{array}{ccc} \mathrm{NS5}(t12345) & \xrightarrow{T_x} & \mathrm{KKM}(t12345, x) \\ H_{ijk} = 3\partial_{[i}B_{jk]} & & F_{ij}{}^{x} = 2\partial_{[i}A_{j]}{}^{x} \end{array}$$

Solutions

Magnetic solutions:

$$\begin{array}{ccccc} \text{NS5}(t12345) & \xrightarrow{T_x} & \text{KKM}(t12345, x) & \xrightarrow{T_y} & 5_2^2(t12345, xy), \\ H_{ijk} = 3\partial_{[i} B_{jk]} & & F_{ij}{}^x = 2\partial_{[i} A_{j]}{}^x & & Q_i{}^{xy} = \partial_i \beta^{xy} \end{array}$$

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Electric solutions:

$$\mathrm{F1}\ (tz) \xrightarrow{T_z} \mathrm{pp}\ (t, \tilde{z})$$

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Electric solutions:

$$\begin{array}{ccccc} \text{F1 } (tz) & \xrightarrow{\quad T_z \quad} & \text{pp } (t, \tilde{z}) & \xrightarrow{\quad T_t \quad} & \text{neg F1 } (\tilde{t}\tilde{z}) \end{array}$$

Solutions

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Electric dual to exotic brane predicted in Bergshoeff, Ortin, Riccioni (2011). Solution in terms of bivector: Sakatani (2014). As T-dual: Welch (1994), Blair (2015), Park, Rey, Rim, Sakatani (2015).

String and wave

- Fundamental string solution:

$$
\begin{aligned}
ds^2 &= H^{-1}(-dt^2 + dz^2) + d\vec{x}_8^2 \,, \\
B &= (H^{-1} - 1) dt \wedge dz \,, \\
e^{-2\phi} &= H \,,
\end{aligned}
$$

  where

$$H = 1 + \frac{h}{r^6} \quad r \equiv \sqrt{\vec{x}_8^2}$$

- T-dual to pp-wave, $(t, z) \to (t, \tilde{z})$:

$$ds^2 = -H^{-1}dt^2 + H(d\tilde{z} + (H^{-1} - 1)dt)^2 + d\vec{x}_8^2 \,,$$

String and wave = doubled wave

- Embed in DFT
  - $(t, z) \to (t, z, \tilde{t}, \tilde{z})$.
  - $g_{ij}, B_{ij} \to \mathcal{H}_{MN} \in O(D, D)/O(1, D-1) \times O(1, D-1)$
  - Symmetries $\to$ generalised diffeomorphisms
  - Section condition: $\partial_i \tilde{\partial}^i = 0$
- F1 solution corresponds to

$$\mathcal{H} = \begin{pmatrix} g - Bg^{-1}B & Bg^{-1} \\ -g^{-1}B & g^{-1} \end{pmatrix} = \begin{pmatrix} H-2 & 0 & 0 & 1-H \\ 0 & 2-H & 1-H & 0 \\ 0 & 1-H & -H & 0 \\ 1-H & 0 & 0 & H \end{pmatrix}$$

- View as a wave with (generalised) momentum in $\tilde{z}$ direction: Berkeley, Berman, Rudolph (2014). Section choice: $\partial_z \neq 0$ ($\Rightarrow$ F1) or $\tilde{\partial}^z \neq 0$ ($\Rightarrow$ pp).
- Solves eom of $S = S_{DFT} + S_{DWS}$. Blair (2016)

String source: negative string

- Can I also choose $\partial_t \neq 0$ or $\tilde{\partial}^t \neq 0$?
- Timelike T-dual $(t, \tilde{z}) \to (\tilde{t}, \tilde{z})$:

$$\begin{aligned} ds^2 &= \tilde{H}^{-1}(-d\tilde{t}^2 + d\tilde{z}^2) + d\vec{x}_8^2 \\ B &= (\tilde{H}^{-1} - 1) d\tilde{t} \wedge d\tilde{z} \\ e^{-2\phi} &= |\tilde{H}| \,. \end{aligned}$$

  where $\tilde{H} = 2 - H = 1 - \frac{h}{|\vec{x}_8|^6}$.

- This is a **negative F1**.
- Naked singularity at $\tilde{H} = 0$.
- N.b. generalised metric not singular. For $\tilde{H} < 0$, $\tilde{t}, \tilde{z}$ swap roles

Positive and negative branes

Dijkgraaf, Heidenreich, Jefferson, Vafa (2016):

- Brane solutions

$$ds^2 = H^\alpha ds^2_{worldvolume} + H^\beta ds^2_{transverse}$$

  involve harmonic functions of transverse coordinates

$$H = 1 + \frac{Q}{r^n} \qquad Q > 0 \sim \text{tension}$$

- Negative brane:

$$H = 1 - \frac{Q}{r^n}$$

- Naked singularity at $H = 0$.

Through the singularity

- Mutually SUSY branes can probe beyond singularity.
- Negative D0

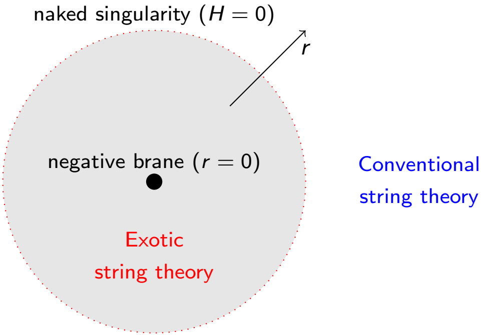
$$\begin{aligned} ds^2 &= -H^{-1/2}dt^2 + H^{1/2}ds_9^2 \\ e^{-2\Phi} &= H^{-3/2} \\ A_1 &= H^{-1}dt \end{aligned}$$

  lifts to **smooth** pp-wave M-theory background:

$$ds^2 = ds_9^2 + 2dtdy + Hdy^2$$

- For $H < 0$, we are reducing M-theory on **timelike** circle.
- $\Rightarrow$ for negative D0, theory beyond singularity has: signature (0,10), Euclidean strings.

Bubbles



- Negative brane surrounded by **bubble** within naked singularity
- Theory inside bubble: exotic string theory (signature flipped in worldvolume directions)
- What theory? Use dualities and work of Hull (1998).

A note on notation

- Type II variants: IIA/B$^{\pm\pm}$
- $\pm$ fundamental strings have Lorentzian/Euclidean worldsheets
- $\pm$ D1/D2 branes have Lorentzian/Euclidean worldvolumes
- M-theory variants: M$^{\pm}$
- $\pm$ M2 branes have Lorentzian/Euclidean worldvolumes
- Signature (t, s).
- Note here Lorentzian means: odd number of timelike directions, Euclidean means: even number of timelike directions.

Dualities and exotic theories

$$
\begin{tikzcd}
 & \begin{array}{c}\mathrm{IIA}^{++}\ (\mathrm{IIA}) \\ (1{,}9)\end{array} \\
\begin{array}{c}\mathrm{IIB}^{++}\ (\mathrm{IIB}) \\ (1{,}9)\end{array} \arrow[ur, leftrightarrow, "x"] &
\end{tikzcd}
$$

Dualities and exotic theories

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\begin{array}{c}\mathrm{IIB}^{++}\ (\mathrm{IIB})\\ (1,9)\end{array} \arrow[dr, leftrightarrow, "t"'] & & \begin{array}{c}\mathrm{IIB}^{+-}\ (\mathrm{IIB}^{*})\\ (1,9)\end{array} \arrow[dl, leftrightarrow, "x"] \\
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 & \begin{array}{c}\mathrm{IIA}^{++}\ (\mathrm{IIA}) \\ (1{,}9)\end{array} \arrow[dr, leftrightarrow, "t"] & & & \begin{array}{c}\mathrm{IIA}^{-+}\ (\mathrm{IIA}_E) \\ (0{,}10)\end{array} & \cdots \\
\begin{array}{c}\mathrm{IIB}^{++}\ (\mathrm{IIB}) \\ (1{,}9)\end{array} \arrow[ur, leftrightarrow, "x"] \arrow[dr, leftrightarrow, "t"'] & & \begin{array}{c}\mathrm{IIB}^{+-}\ (\mathrm{IIB}^*) \\ (1{,}9)\end{array} \arrow[r, leftrightarrow, dashed, "S"] & \begin{array}{c}\mathrm{IIB}^{-+}\ (\mathrm{IIB}') \\ (1{,}9)\end{array} \arrow[ur, leftrightarrow, "t" near start, "x" near end] \arrow[dr, leftrightarrow, "x" near start, "t" near end] & & \begin{array}{c}\mathrm{IIB}^{--} \\ (3{,}7)\end{array} \\
 & \begin{array}{c}\mathrm{IIA}^{+-}\ (\mathrm{IIA}^*) \\ (1{,}9)\end{array} \arrow[ur, leftrightarrow, "x"'] & & & \begin{array}{c}\mathrm{IIA}^{--} \\ (2{,}8)\end{array} \arrow[ur, leftrightarrow, "x" near start, "t" near end] &
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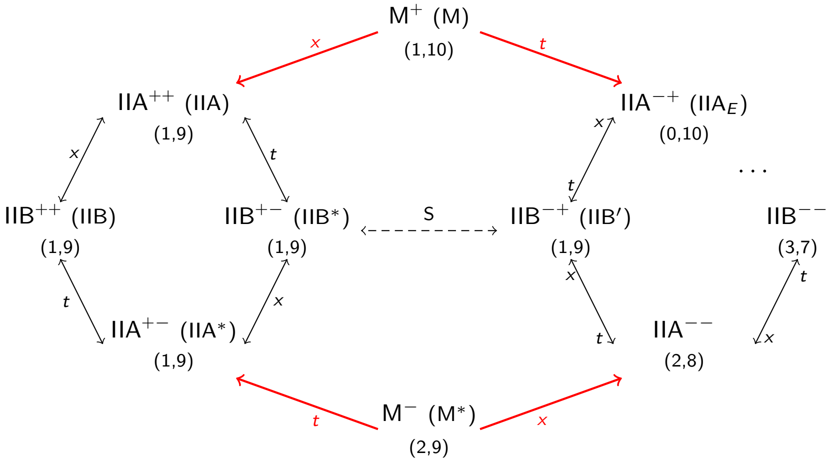
Dualities and exotic theories



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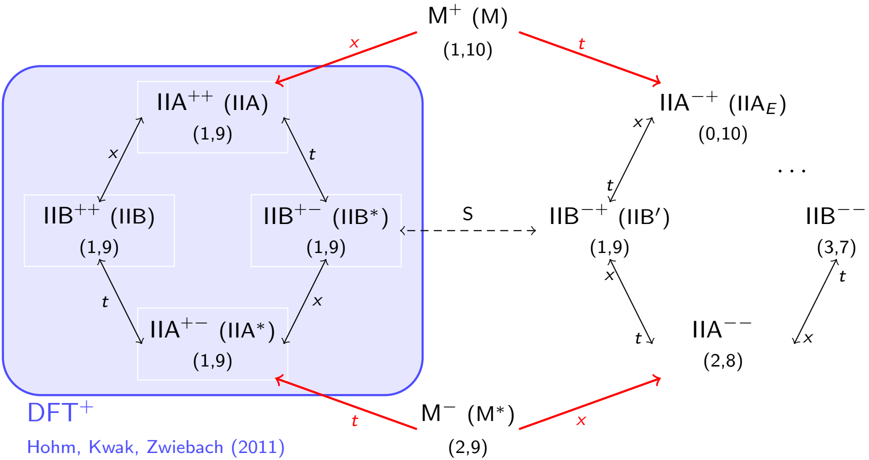

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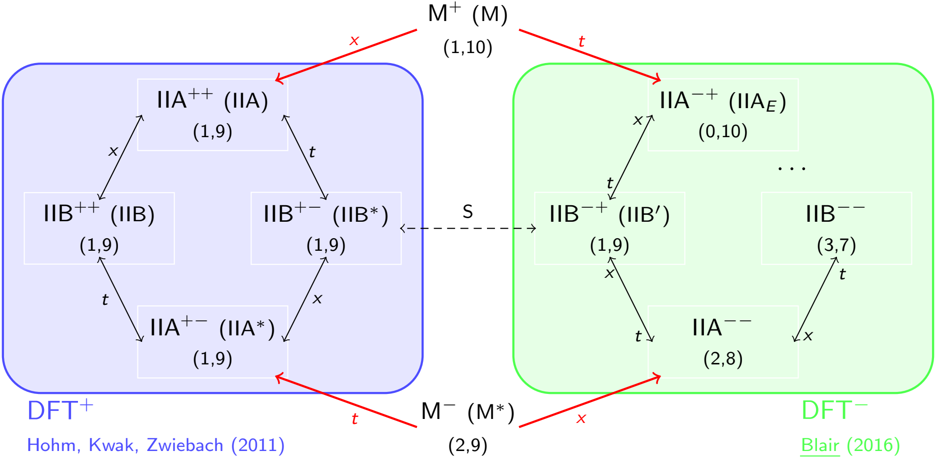

Worldsheet for Euclidean strings

- Euclidean string worldsheet action:

$$S = \frac{T}{2} \int d^2\sigma \left(|\gamma|^{1/2} \gamma^{\alpha\beta} \partial_\alpha X^i \partial_\beta X^j g_{ij} + \epsilon^{\alpha\beta} B_{ij} \partial_\alpha X^i \partial_\beta X^j \right)$$

- Parametrise Euclidean worldsheet metric

$$\gamma_{\alpha\beta} = \Omega \begin{pmatrix} u^2 + \tilde{u}^2 & \tilde{u} \\ \tilde{u} & 1 \end{pmatrix}$$

(conformal gauge: $\tilde{u} = 0$, $u = 1$).

- Write $\mathcal{L} = \partial_1 X^i P_i - \mathrm{Ham}(X, P)$, $P_i = \partial \mathcal{L} / \partial(\partial_1 X^i)$:

$$S = \int d^2\sigma \left(\partial_1 X^i P_i - \frac{Tu}{2} \mathcal{H}_{MN} Z^M Z^N - \frac{T\tilde{u}}{2} \eta_{MN} Z^M Z^N \right)$$

$$Z^M = \begin{pmatrix} \partial_2 X^i \\ T^{-1} P_i \end{pmatrix} \quad \eta_{MN} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Doubled worldsheet for Euclidean strings

- Define duals $\partial_2 \tilde{X}_i = T^{-1} P_i$. Then

$$S_{EDWS} = T \int d^2\sigma \Big(\frac{1}{2} \partial_1 X^M \eta_{MN} \partial_2 X^N - \frac{u}{2} \mathcal{H}_{MN} \partial_2 X^M \partial_2 X^N - \frac{\tilde{u}}{2} \eta_{MN} \partial_2 X^M \partial_2 X^N \Big)$$

- The generalised metric is

$$\mathcal{H}_{MN} = \begin{pmatrix} -g_{ij} - B_{ik} g^{kl} B_{lj} & B_{ik} g^{kj} \\ -g^{ik} B_{kj} & g^{ij} \end{pmatrix}$$

- Note minus sign in top left!

Doubled worldsheet for Euclidean strings

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Generalised metric for Euclidean strings

- Not an element of $O(D,D)$: if $\mathcal{H}^{MN} \equiv \eta^{MP}\eta^{NQ}\mathcal{H}_{PQ}$ then $\mathcal{H}^{MN}\mathcal{H}_{NP} = -\delta^M_P$. (So $(\eta^{-1}\mathcal{H})^2 = -1$).

- However $\pm i\mathcal{H} \in O(D,D;\mathbb{C})/O(D;\mathbb{C}) \times O(D;\mathbb{C})$.

- Buscher changes signature:

$$\mathcal{H}_{MN} = \begin{pmatrix} -g & 0 \\ 0 & g^{-1} \end{pmatrix} \to \begin{pmatrix} g^{-1} & 0 \\ 0 & -g \end{pmatrix} \equiv \begin{pmatrix} -\tilde{g} & 0 \\ 0 & \tilde{g}^{-1} \end{pmatrix},$$

  and the dual metric is $\tilde{g} = -g^{-1}$

- Vielbein subtleties: $\tilde{e} = \pm i e^{-1} \to$ work with complexified coset, restrict to real forms.

$$g = e(+1)e \to \tilde{g} = \tilde{e}(+1)\tilde{e} = \tilde{e}_{\mathbb{R}}(-1)\tilde{e}_{\mathbb{R}}$$

  c.f. Bergshoeff, Hartong, Ploegh, Rosseel, Van den Bleeken (2007)

Action for DFT$^-$

- Construct action using invariance under generalised diffeomorphisms. Generalised Ricci scalar

$$\mathcal{R}^- = 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M d\,\partial_N d + 4\partial_M\mathcal{H}^{MN}\,\partial_N d$$
$$- \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\,\partial_N\mathcal{H}_{KL} + \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL}$$

- Reduces to

$$\mathcal{R}^- \xrightarrow{\tilde{\partial}^i=0} R + \frac{1}{12}H^2 - 4(\nabla\phi)^2 + 4\nabla^2\phi$$

- "Wrong sign" for H^2 term: as expected in IIA/B$^{-\pm}$.

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Return to the negative string

- F1 solution in IIA/B^{++}

  $\downarrow$ T-duality on z

- pp solution in IIB/A^{++}

  $\downarrow$ T-duality on t

- neg F1 solution in IIA/B^{+-}
- Theory inside bubble is IIA/B^{++}. Different section?

Full DFT$^+$

- For IIA/B$^{+\pm}$: include RR fields as in Hohm, Kwak, Zwiebach (2011). $O(D,D) \to$ (S)pin(D,D).

- Spin representative of generalised metric $\mathbb{S}$. Explicitly parametrise $\mathbb{S} = \pm S_{\mathcal{H}(g,B)}$ corresponding to $\mathcal{H} = \mathcal{H}(g,B)$.

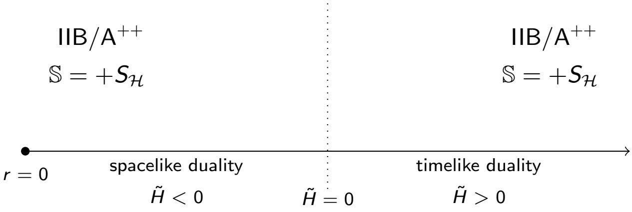
- Buscher duality in i direction $\mathcal{H} \to \mathcal{H}'$,

$$S'_{\mathcal{H}'} = \mathrm{sign}(g_{ii}) S_{\mathcal{H}'}$$

Return to the negative string again

- Consider pp-wave $ds^2 = (H-2)dt^2 + 2(1-H)dtd\tilde{z} + Hd\tilde{z}^2$.
- Isometry $k = \frac{\partial}{\partial t}$, $k^2 = g_{tt} = (H-2) = -\tilde{H}$.
- T-duality in isometry direction is timelike for $\tilde{H} > 0$ and spacelike for $\tilde{H} < 0$.

pp-wave:



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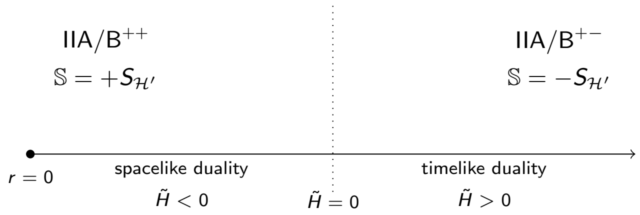
negative string



Fundamental field $\mathbb{S}$. Choice of parametrisation discontinuous. Same section, different interpretations.

Bivector parametrisation

- Alternative parametrisation: $(g_{ij}, B_{ij}) \to (\tilde{g}^{ij}, \beta^{ij})$.
- For negative string
$$d\tilde{s}^2 = H^{-1}\left(d\tilde{t}^2 + d\tilde{z}^2\right) + ds_8^2$$
$$\beta^{\tilde{t}\tilde{z}} = H^{-1} - 1$$
- Important for timelike dualities, see Malek (2013). Exotic theory with signature change, wrong sign kinetic terms REPLACED by theory with bivector. Cause: generalised Lorentz gauge fixing.
- DFT$^-$ seems different ($O(D, D; \mathbb{C})$).
- What is the general picture?

Final questions

- Full generalisation to EFT: covering all of the duality web.
- Black holes: horizons ($g_{tt} = 0$) dual to singularities ($\tilde{g}_{tt} = \frac{1}{g_{tt}} = \infty$). Partially discussed in Arvanitakis and Blair (2016).
- What is the ultimate status of exotic theories (including DFT?) within string theory?

Thanks for listening!