Scale invariance, T-duality and modified IIB supergravity

Integrable eta-deformations

Gleb Arutyunov

II Institute for Theoretical Physics

Hamburg University

Outline

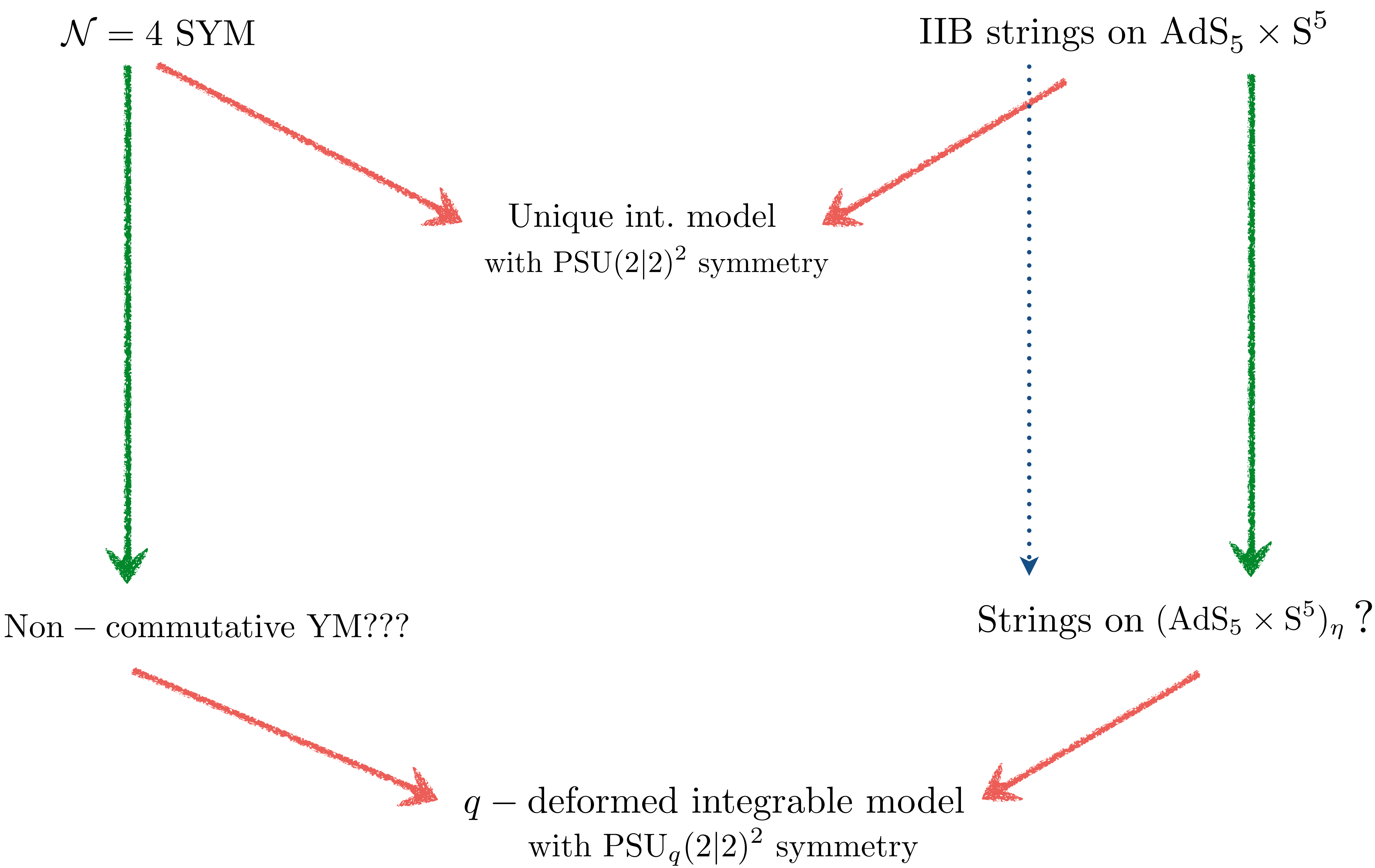
- *Deforming string sigma model*
- *Bosonic model*
- *Inclusion of fermions — RR fields*
- *Scale invariance - NSNS fields*
- *Modified type II equations*
- *Scale invariance - RR fields*
- *Conclusions and future problems*

With Riccardo Borsato, Sergey Frolov, Ben Hoare, Radu Roiban and Arkady Tseytlin

Deforming the primary model

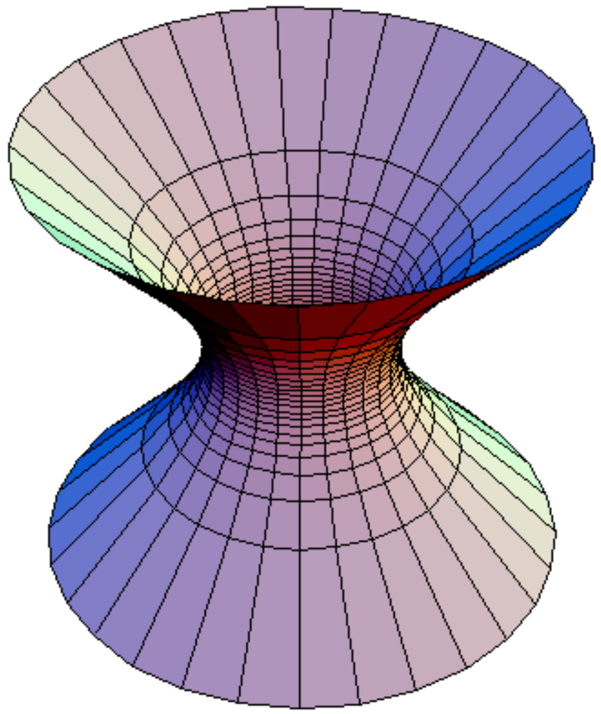
Klimcik '08

Delduc, Magro, Vicedo '13

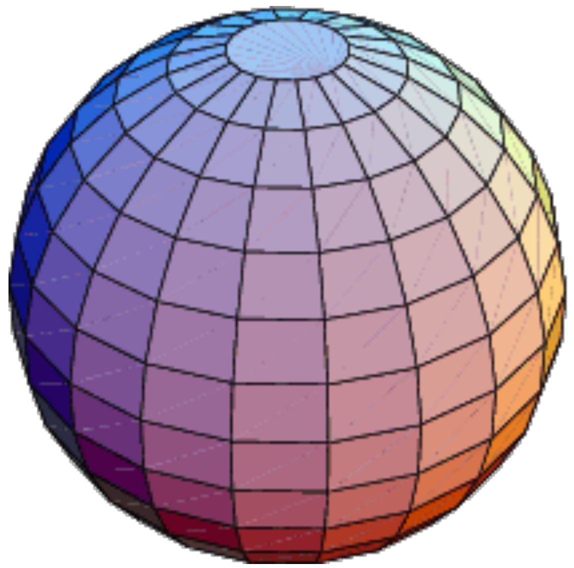


Undeformed theory: symmetries and Lagrangian

Coset $\mathrm{PSU}(2,2|4)/\mathrm{SO}(4,1)\times\mathrm{SO}(5)$



$\mathrm{SU}(2,2)/\mathrm{SO}(4,1)\sim\mathrm{AdS}_5$



$\mathrm{SU}(4)/\mathrm{SO}(5)\sim\mathrm{S}^5$

coset element $\mathfrak{g}(\tau,\sigma)$
Current $A_\alpha\equiv-\mathfrak{g}^{-1}\partial_\alpha\mathfrak{g}\in\mathfrak{su}(2,2|4)$
Realisation of $\mathfrak{su}(2,2|4)$ in terms of 8×8 matrices

Undeformed theory: symmetries and Lagrangian

$\mathbf{Z}_4$-grading: $\mathfrak{f} = \mathfrak{su}(2,2|4) = \mathfrak{f}^{(0)} + \mathfrak{f}^{(1)} + \mathfrak{f}^{(2)} + \mathfrak{f}^{(3)}$

$$\Omega(\mathfrak{f}^{(k)}) = i^k \mathfrak{f}^{(k)}$$

Projectors $P^{(k)}$, $\qquad d \equiv P^{(1)} + 2P^{(2)} - P^{(3)}$

$$\mathcal{L} = -\frac{g}{4}(\gamma^{\alpha\beta} - \epsilon^{\alpha\beta})\, \mathrm{str}\Big(A_\alpha \cdot d(A_\beta)\Big)$$

This expression reproduces the expected Green – Schwarz Lagrangian for IIB superstring in $\mathrm{AdS}_5 \times \mathrm{S}^5$

Metsaev, Tseytlin '98

Deforming the theory

$\eta \in [0,1[$

$$\mathcal{L}_\eta = -\frac{g}{4}(1+\eta^2)(\gamma^{\alpha\beta} - \epsilon^{\alpha\beta})\,\mathrm{str}\Big(A_\alpha d \cdot \mathcal{O}^{-1}(A_\beta)\Big)$$

$$d \equiv P^{(1)} + \frac{2}{1-\eta^2} P^{(2)} - P^{(3)}$$

$$\mathcal{O} \equiv \mathbf{1} - \eta\, R_{\mathfrak{g}} \circ d$$

$$R_{\mathfrak{g}}(\cdot) \equiv \mathrm{Adj}_{\mathfrak{g}^{-1}} \circ R \circ \mathrm{Adj}_{\mathfrak{g}}(\cdot) = \mathfrak{g}^{-1} R(\mathfrak{g}(\cdot)\mathfrak{g}^{-1})\mathfrak{g}$$

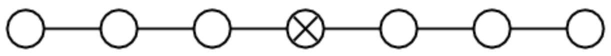
modified classical Yang-Baxter equation (mcYBe)

$$[RM, RN] - R([RM,N] + [M,RN]) = [M,N]$$

Delduc, Magro, Vicedo '13

Generic properties if the deformed models

- *Classically integrable*
- κ *-symmetry*
- *Hidden symmetry* $\mathrm{PSU}_q(2,2|4)$
- *At least 8 non-trivial R's, corresponding to Dynkin diagrams*

For the rest of the talk we choose R which corresponds to the standard Dynkin diagram 

$$R(M_{jk}) = \begin{cases} -i\ M_{jk}, & j<k \\ 0, & j=k \\ +i\ M_{jk}, & j>k \end{cases}$$

Other interesting choices are possible!

Bosonic sigma-model

$$S=-\frac{\tilde g}{2}\int d\tau d\sigma\left(\gamma^{\alpha\beta}\partial_\alpha X^M\partial_\beta X^N G_{MN}-\epsilon^{\alpha\beta}\partial_\alpha X^M\partial_\beta X^N B_{MN}\right)$$

$$\varkappa=\frac{2\eta}{1-\eta^2}$$

$$\tilde g\equiv g\sqrt{1+\varkappa^2}$$

$$f_\pm(x)=1\pm x^2$$

$$ds^2=-\frac{f_+(\rho)}{f_-(\varkappa\rho)}dt^2+\frac{1}{f_+(\rho)f_-(\varkappa\rho)}d\rho^2+\rho^2 d\Theta_3^\rho$$
$$+\frac{f_-(r)}{f_+(\varkappa r)}d\phi^2+\frac{1}{f_-(r)f_+(\varkappa r)}dr^2+r^2 d\Theta_3^r$$

$$d\Theta_3^\rho\equiv\frac{1}{1+\varkappa^2\rho^4\sin^2\zeta}(d\zeta^2+\cos^2\zeta d\psi_1^2)+\sin^2\zeta d\psi_2^2$$
$$d\Theta_3^r\equiv\frac{1}{1+\varkappa^2 r^4\sin^2\xi}(d\xi^2+\cos^2\xi d\chi_1^2)+\sin^2\xi d\chi_2^2$$

$$B=\varkappa\Big(\frac{\rho^4\sin 2\zeta}{1+\varkappa^2\rho^4\sin^2\zeta}d\psi_1\wedge d\zeta-\frac{r^4\sin 2\xi}{1+\varkappa^2 r^4\sin^2\xi}d\chi_1\wedge d\xi\Big)$$

- The limit $\varkappa\to 0\Longrightarrow \mathrm{AdS}_5\times\mathrm{S}^5$
- The metric and B-field are invariant under shifts of $t,\phi,\psi_1,\psi_2,\chi_1,\chi_2$
- The ranges $0\le\rho\le\frac{1}{\varkappa}\,,\quad 0\le r\le 1$
- The deformed AdS is singular at $\rho=1/\varkappa$

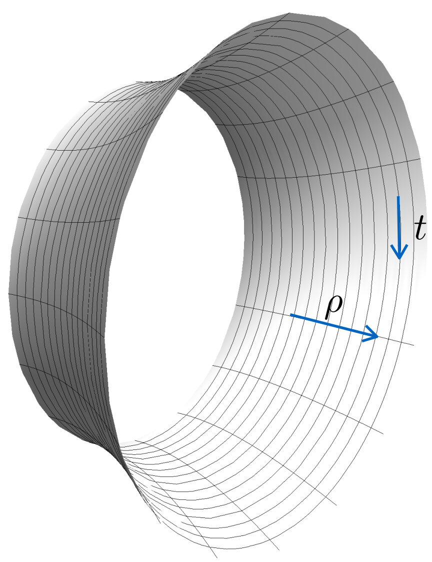
Borsato, Frolov and G.A. '13

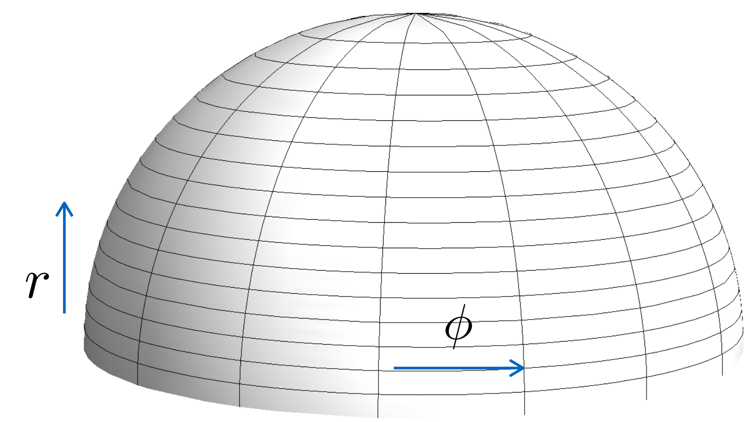
Geometry

$$-\frac{1+\rho^2}{1-\varkappa^2\rho^2}\mathrm{d}t^2+\frac{\mathrm{d}\rho^2}{(1+\rho^2)(1-\varkappa^2\rho^2)}$$

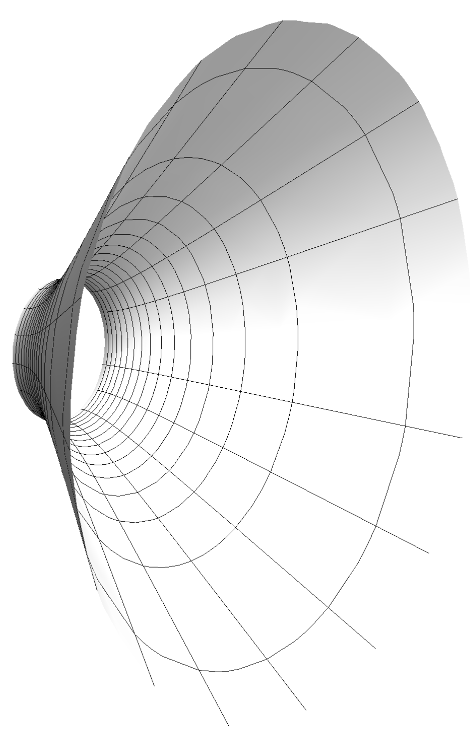
$$\frac{1-r^2}{1+\varkappa^2 r^2}\mathrm{d}\phi^2+\frac{\mathrm{d}r^2}{(1-r^2)(1+\varkappa^2 r^2)}$$

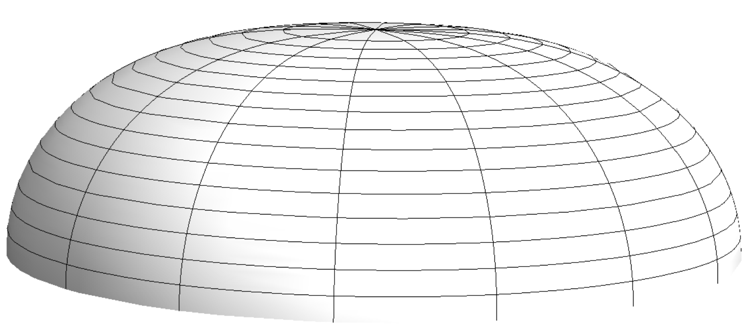
$\varkappa = 0$





$\varkappa = 1$





RR fields

Borsato, Frolov and G.A. '15

Green-Schwarz Lagrangian at quadratic order in fermions

$$\mathcal{L}_{\theta^2} = -\frac{g}{2}\left(\gamma^{\alpha\beta}\delta^{IJ} + \epsilon^{\alpha\beta}\sigma_3^{IJ}\right) i\bar{\theta}_I e_\alpha^m \gamma_m D_\beta^{JK}\theta_K$$

NSNS

$$D_\alpha^{IJ} = \delta^{IJ}\left(\partial_\alpha - \frac{1}{4}\omega_\beta^{mn}\gamma_{mn}\right) + \frac{1}{8}\sigma_3^{IJ} e_\alpha^m H_{mnp}\gamma^{np}$$
$$-\frac{1}{8}e^\phi\left(\epsilon^{IJ}\gamma^p F_p^{(1)} + \frac{1}{3!}\sigma_1^{IJ}\gamma^{pqr}F_{pqr}^{(3)} + \frac{1}{2\cdot 5!}\epsilon^{IJ}\gamma^{pqrst}F_{pqrst}^{(5)}\right)e_\alpha^m\gamma_m$$

RR

Extract $e^\Phi F^{(n)}$, plug in SUGRA equations for $F^{(n)}$ $\Longrightarrow$ Ist order differential equation for Φ!

Green-Schwarz Lagrangian at quadratic order in fermions

Reaching the canonical form of the GS Lagrangian

$$\theta \to U\theta, \qquad X^M \to X^M + \bar{\theta} f^M \theta$$
U, f^M : 32 × 32 matrices, functions of X

$$S_{\text{bos}}(X) \to S_{\text{bos}}(X) + \delta S_{\text{bos}}(X, \bar{\theta} f \theta)$$

$$\bar{\theta}_I M_{IJ} \theta_J \to \bar{\theta}_I \left(\bar{U} M U\right)_{IJ} \theta_J$$

Guiding principle: find U and f^M to get a canonical term with ∂

$$-\frac{\tilde{g}}{2}\left(\gamma^{\alpha\beta}\delta^{IJ} + \epsilon^{\alpha\beta}\sigma_3^{IJ}\right) \, i\bar{\theta}_I \, \tilde{e}_\alpha^m \gamma_m \partial_\beta \theta_J$$

$\tilde{e}_M^m$ is the deformed vielbein

We found such a field redefinition! The terms with ω and $H_{\mu\nu\rho}$ came out correctly!

Results for the RR-sector (ABF background)

$$e^{\phi}F_1 = -4\varkappa^2\sqrt{1+\varkappa^2}\; c_F^{-1}\; \rho^3 \sin\zeta, \qquad e^{\phi}F_6 \;= -4\varkappa^2\sqrt{1+\varkappa^2}\; c_F^{-1}\; r^3 \sin\xi,$$

$$\begin{aligned}
e^{\phi}F_{014} &= +4\varkappa\sqrt{1+\varkappa^2}\; c_F^{-1}\; \rho^2 \sin\zeta, & e^{\phi}F_{123} &= -4\varkappa\sqrt{1+\varkappa^2}\; c_F^{-1}\; \rho,\\
e^{\phi}F_{569} &= -4\varkappa\sqrt{1+\varkappa^2}\; c_F^{-1}\; r^2 \sin\xi, & e^{\phi}F_{678} &= +4\varkappa\sqrt{1+\varkappa^2}\; c_F^{-1}\; r,\\
e^{\phi}F_{046} &= -4\varkappa^3\sqrt{1+\varkappa^2}\; c_F^{-1}\; \rho r^3 \sin\xi, & e^{\phi}F_{236} &= +4\varkappa^3\sqrt{1+\varkappa^2}\; c_F^{-1}\; \rho^2 r^3 \sin\zeta \sin\xi,\\
e^{\phi}F_{159} &= -4\varkappa^3\sqrt{1+\varkappa^2}\; c_F^{-1}\; \rho^3 r \sin\zeta, & e^{\phi}F_{178} &= -4\varkappa^3\sqrt{1+\varkappa^2}\; c_F^{-1}\; \rho^3 r^2 \sin\zeta \sin\xi,
\end{aligned}$$

$$\begin{aligned}
e^{\phi}F_{01234} &= +4\sqrt{1+\varkappa^2}\; c_F^{-1}, & e^{\phi}F_{02346} &= +4\varkappa^4\sqrt{1+\varkappa^2}\; c_F^{-1}\rho^3 r^3 \sin\zeta \sin\xi,\\
e^{\phi}F_{01459} &= +4\varkappa^2\sqrt{1+\varkappa^2}\; c_F^{-1}\rho^2 r \sin\zeta, & e^{\phi}F_{01478} &= +4\varkappa^2\sqrt{1+\varkappa^2}\; c_F^{-1}\rho^2 r^2 \sin\zeta \sin\xi,\\
e^{\phi}F_{04569} &= -4\varkappa^2\sqrt{1+\varkappa^2}\; c_F^{-1}\rho r^2 \sin\xi, & e^{\phi}F_{04678} &= +4\varkappa^2\sqrt{1+\varkappa^2}\; c_F^{-1}\rho r.
\end{aligned}$$

$$c_F = \sqrt{1-\varkappa^2\rho^2}\sqrt{1+\varkappa^2\rho^4\sin^2\zeta}\sqrt{1+\varkappa^2 r^2}\sqrt{1+\varkappa^2 r^4\sin^2\xi}.$$

SUGRA eoms are not satisfied! ✗

Maldacena-Russo background is reproduced! ✓

Maldacena-Russo background

$$\varkappa \to 0\,, \quad \rho \to \rho/\sqrt{\varkappa}\,, \quad t \to \sqrt{\varkappa}t\,, \quad \zeta \to \zeta_0 + \sqrt{\varkappa}\zeta$$

$$\psi_1 \to \sqrt{\varkappa}\,\psi_1/\cos\zeta_0\,, \quad \psi_2 \to \sqrt{\varkappa}\,\psi_2/\sin\zeta_0$$

$$ds^2 = \rho^2(-dt^2 + d\psi_2^2) + \frac{\rho^2(d\zeta^2 + d\psi_1^2)}{1+\rho^4\sin^2\zeta_0} + \frac{d\rho^2}{\rho^2}\,,$$

$$B_a = \frac{\rho^4\sin\zeta_0}{1+\rho^4\sin^2\zeta_0}d\psi_1\wedge d\zeta$$

[Matsumoto, Yoshida '14] [Frolov, unpublished '14]

$\log c_F^{-1} \Longrightarrow$ $$\phi = \phi_0 - \frac{1}{2}\log(1+\rho^4\sin^2\zeta_0)$$

$F_{014} \Longrightarrow$ $$F_3 = 4\rho^3\sin\zeta_0 e^{-\phi_0}\,dt\wedge d\psi_2\wedge d\rho$$

$F_{01234} \Longrightarrow$ $$F_5 = 2e^{-\phi_0}\left(\frac{2\rho^3}{1+\rho^4\sin^2\zeta_0}dt\wedge d\psi_2\wedge d\psi_1\wedge d\zeta\wedge d\rho + r^3\sin 2\xi d\phi\wedge d\phi_2\wedge d\phi_1\wedge d\xi\wedge dr\right)$$

[Maldacena, Russo '99]

Kappa-symmetry

The deformed model has kappa-symmetry

32-dim spinors $K_{\alpha I}$ to parameterise kappa-symmetry transf.
$\Theta_I = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \theta_I$ and Γ_m are 32×32 Gamma matrices

The same field redefinitions bring the kappa-symmetry transformations to the standard form:

$$\delta_\kappa X^M = -\frac{i}{2}\, \bar{\Theta}_I \delta^{IJ} \Gamma^M \delta_\kappa \Theta_J + \mathcal{O}(\Theta^3),$$
$$\delta_\kappa \Theta_I = -\frac{1}{4}(\delta^{IJ}\gamma^{\alpha\beta} - \sigma_3^{IJ}\epsilon^{\alpha\beta})\Gamma_\beta K_{\alpha J} + \mathcal{O}(\Theta^2),$$

RR-couplings can be read off from the variation of the world-sheet metric and they are the same as found before!

$$\delta_\kappa \gamma^{\alpha\beta} = 2i\, \Pi^{IJ\,\alpha\alpha'} \Pi^{JK\,\beta\beta'}\, \bar{K}_{I\alpha'} D^{KL}_{\beta'} \Theta_L + \mathcal{O}(\Theta^4)$$
$$\Pi^{IJ\,\alpha\alpha'} \equiv \frac{\delta^{IJ}\gamma^{\alpha\alpha'} + \sigma_3^{IJ}\epsilon^{\alpha\alpha'}}{2}$$

Contradiction with [Bergshoeff, Sezgin, Townsend, '86] and [Grisaru, Howe, Mezincescu, Nilsson, Townsend '86]?

kappa-symmetry implies supergravity constraints (eoms?)

Differential constraints can start showing up only from variations of quartic action in fermions!

HT background in the T-dualized model

$$\hat{ds}^2 = -\frac{1-\varkappa^2\rho^2}{1+\rho^2}d\hat{t}^2 + \frac{d\rho^2}{(1+\rho^2)(1-\varkappa^2\rho^2)} + \frac{d\hat{\psi}_1^2}{\rho^2\cos^2\zeta} + (\rho d\zeta + \varkappa\rho\tan\zeta\, d\hat{\psi}_1)^2 + \frac{d\hat{\psi}_2^2}{\rho^2\sin^2\zeta}$$
$$+ \frac{1+\varkappa^2 r^2}{1-r^2}d\hat{\varphi}^2 + \frac{dr^2}{(1-r^2)(1+\varkappa^2 r^2)} + \frac{d\hat{\phi}_1^2}{r^2\cos^2\xi} + (r d\xi - \varkappa r\tan\xi\, d\hat{\phi}_1)^2 + \frac{d\hat{\phi}_2^2}{r^2\sin^2\xi}$$

$$\hat{B} = 0\,, \qquad \hat{\mathcal{F}}_1 = 0 = \hat{\mathcal{F}}_3$$

$$\hat{\mathcal{F}}_5 = \frac{4i\sqrt{1+\varkappa^2}}{\sqrt{1+\rho^2}\sqrt{1-r^2}}\Big[\big(d\hat{t} + \frac{\varkappa\rho d\rho}{1-\varkappa^2\rho^2}\big)\wedge\frac{d\hat{\psi}_2}{\rho\sin\zeta}\wedge\frac{d\hat{\psi}_1}{\rho\cos\zeta}\wedge(r d\xi - \varkappa r\tan\xi\, d\hat{\phi}_1)\wedge\big(\frac{dr}{1+\varkappa^2 r^2} + \varkappa r d\hat{\varphi}\big)$$
$$-\big(d\hat{\varphi} - \frac{\varkappa r dr}{1+\varkappa^2 r^2}\big)\wedge\frac{d\hat{\phi}_2}{r\sin\xi}\wedge\frac{d\hat{\phi}_1}{r\cos\xi}\wedge(\rho d\zeta + \varkappa\rho\tan\zeta\, d\hat{\psi}_1)\wedge\big(\frac{d\rho}{1-\varkappa^2 r^2} + \varkappa\rho d\hat{t}\big)\Big]$$

$$\hat{\mathcal{F}} \equiv e^{\phi}\hat{F}$$

T-duality

$$\mathcal{F}_{\rm ABF}$$

$$\hat{\phi} = \phi_0 - 4\varkappa(\hat{t}+\hat{\varphi}) - 2\varkappa(\hat{\psi}_1 - \hat{\phi}_1) + \log\frac{(1-\varkappa^2\rho^2)^2(1+\varkappa^2 r^2)^2}{\rho^2 r^2\sqrt{1+\rho^2}\sqrt{1-r^2}\sin 2\zeta\sin 2\xi}$$

- An exact solution of IIB sugra (metric is non-diagonal)
- Flux $\hat{\mathcal{F}}_5$ is imaginary
- Dilaton is linear in isometric coordinates
- Formally T-dualizing back is not possible!

[Hoare, Tseytlin '15]

Scale invariance of the eta-deformed model

NSNS-sector

[Frolov, Hoare, Roiban Tseytlin and G.A. ’15]

Scale vs Weyl Invariance for the bosonic sigma model

Scale invariance: $\delta g_{\mu\nu} = \epsilon g_{\mu\nu}$ $\int T^\mu_\mu = 0$ or locally $T^\mu_\mu = \partial_\mu K^\mu$

Weyl invariance: $\delta g_{\mu\nu} = \epsilon(x) g_{\mu\nu}$ $T^\mu_\mu = 0$

Scale invariance conditions for the bosonic sigma model:

$$\beta^G_{mn} \equiv R_{mn} - \frac{1}{4} H_{mkl} H_n{}^{kl} = -D_m X_n - D_n X_m$$

$$\beta^B_{mn} \equiv \frac{1}{2} D^k H_{kmn} + \partial_m Y_n - \partial_n Y_m$$

Weyl invariance conditions for the bosonic sigma model:

$$X_m = \partial_m \phi \,, \qquad Y_m = 0$$

ϕ is a dilation

$$\partial_m \beta^\phi = 0 \,, \qquad \beta^\phi \equiv R - \frac{1}{12} H^2_{mnk} + 4 D^2 \phi - 4 \partial^m \phi \partial_m \phi$$

Central charge identity (Curci-Paffuti identity)

!

$$T = \beta^G_{mn} \partial_a x^m \partial^a x^n + \beta^B_{mn} \epsilon^{ab} \partial_a x^m \partial_b x^n, \qquad T = \nabla^a N_a, \qquad N_a = 2(X_m \partial_a x^m + \epsilon_a{}^b Y_m \partial_b x^m)$$

This cannot be cancelled by a local counterterm unless $X_m = \partial_m \phi, \; Y_m = 0$

Scale invariance conditions for the GS superstring

1)

$$\mathcal{T}_{mn} \equiv \frac{1}{2}\mathcal{F}_m\mathcal{F}_n + \frac{1}{4}\mathcal{F}_{mpq}\mathcal{F}_n{}^{pq} + \frac{1}{4\times 4!}\mathcal{F}_{mpqrs}\mathcal{F}_n{}^{pqrs} - \frac{1}{2}G_{mn}\left(\frac{1}{2}\mathcal{F}_k\mathcal{F}^k + \frac{1}{12}\mathcal{F}_{kpq}\mathcal{F}^{kpq}\right)$$

$$\beta^G_{mn} \equiv R_{mn} - \frac{1}{4}H_{mkl}H_n{}^{kl} - \mathcal{T}_{mn} = -D_m X_n - D_n X_m$$

$$\beta^B_{mn} \equiv \frac{1}{2}D^k H_{kmn} + \mathcal{K}_{mn} = X^k H_{kmn} + \partial_m Y_n - \partial_n Y_m$$

$$\mathcal{K}_{mn} \equiv \frac{1}{2}\mathcal{F}^k\mathcal{F}_{kmn} + \frac{1}{12}\mathcal{F}_{mnklp}\mathcal{F}^{\,klp}$$

$$\mathcal{F}_m \equiv e^{\phi}F_m\,, \quad \mathcal{F}_{mnk} \equiv e^{\phi}F_{mnk}\,, \quad \mathcal{F}_{mnklp} \equiv e^{\phi}F_{mnklp}$$

For $X_m = \partial_m\phi\,,\ Y_m = 0$ these equations follow from IIB supergravity action

2) For RR-fields UV finiteness of the 2d sigma model should imply 2nd order differential equations:

$$\beta^{\mathcal{F}}_{k_1\ldots k_s} \equiv \frac{1}{2}D^2\mathcal{F}_{k_1\ldots k_s} + \ldots = X^m\partial_m\mathcal{F}_{k_1\ldots k_s} + \sum_i \mathcal{F}_{k_1\ldots m\ldots k_s}\partial_{k_i}X^m$$

X & Y of the eta-deformed model

$$
\begin{aligned}
X \equiv X_m dx^m = {} & c_0 \frac{1+\rho^2}{1-\varkappa^2\rho^2} dt + c_1 \rho^2 \sin^2\zeta \, d\psi_2 + c_2 \frac{\rho^2\cos^2\zeta}{1+\varkappa^2\rho^4\sin^2\zeta} d\psi_1 \\
& + c_3 \frac{1-r^2}{1+\varkappa^2 r^2} d\varphi + c_4 r^2 \sin^2\xi \, d\phi_2 + c_5 \frac{r^2\cos^2\xi}{1+\varkappa^2 r^4 \sin^2\xi} d\phi_1 \\
& + \frac{\varkappa^2\rho^4\sin 2\zeta}{2(1+\varkappa^2\rho^4\sin^2\zeta)} d\zeta + \frac{1}{\rho}\left(1 - \frac{3}{1-\varkappa^2\rho^2} + \frac{2}{1+\varkappa^2\rho^4\sin^2\zeta}\right) d\rho \\
& + \frac{\varkappa^2 r^4 \sin 2\xi}{2(1+\varkappa^2 r^4 \sin^2\xi)} d\xi + \frac{1}{r}\left(1 - \frac{3}{1+\varkappa^2 r^2} + \frac{2}{1+\varkappa^2 r^4 \sin^2\xi}\right) dr
\end{aligned}
$$

Isometry

$$
X_m = I_m + Z_m \, , \qquad D_m I_n + D_n I_m = 0 \, , \qquad D^m I_m = 0
$$

$$
\begin{aligned}
Y \equiv Y_m dx^m = {} & 4\varkappa \frac{1+\rho^2}{1-\varkappa^2\rho^2} dt + 2\varkappa \frac{\rho^2\cos^2\zeta}{1+\varkappa^2\rho^4\sin^2\zeta} d\psi_1 \\
& + 4\varkappa \frac{1-r^2}{1+\varkappa^2 r^2} d\varphi - 2\varkappa \frac{r^2\cos^2\xi}{1+\varkappa^2 r^4\sin^2\xi} d\phi_1 \\
& + \frac{\varkappa^2\rho^4\sin 2\zeta}{2(1+\varkappa^2\rho^4\sin^2\zeta)} d\zeta + \frac{1}{\rho}\left(1 - \frac{3}{1-\varkappa^2\rho^2} + \frac{2(\varkappa^{-1}c_2 - 1)}{1+\varkappa^2\rho^4\sin^2\zeta}\right) d\rho \\
& + \frac{\varkappa^2 r^4\sin 2\xi}{2(1+\varkappa^2 r^4\sin^2\xi)} d\xi + \frac{1}{r}\left(1 - \frac{3}{1+\varkappa^2 r^2} - \frac{2(\varkappa^{-1}c_5 + 1)}{1+\varkappa^2 r^4\sin^2\xi}\right) dr
\end{aligned}
$$

For $\quad c_0 = c_3 = 4\varkappa \, , \qquad c_1 = c_4 = 0 \, , \qquad c_2 = -c_5 = 2\varkappa$

$$
Y_m = X_m
$$

Surprisingly, for these coefficients $\quad \bar\beta^X \equiv R - \frac{1}{12} H^2_{mnk} + 4 D_k X^k - 4 X_k X^k = 0 \qquad \partial_m \bar\beta^X = 0$

ABF analog of the dilation

$$\partial_m Z_n - \partial_n Z_m + I^k H_{kmn} = 0$$

Zero mode

$$(\mathcal{L}_I B)_{mn} = I^k \partial_k B_{mn} + B_{kn} \partial_m I^k - B_{km} \partial_n I^k$$

Solution:

$$Z_m = \partial_m \phi + B_{km} I^k$$

$$X_m = Y_m = I_m + Z_m = \partial_m \phi + (G_{km} + B_{km}) I^k$$

$$\phi = \frac{1}{2} \log \frac{(1-\kappa^2\rho^2)^3(1+\kappa^2 r^2)^3}{(1+\kappa^2\rho^4 \sin^2\zeta)(1+\kappa^2 r^4 \sin^2\xi)}$$

Precisely the isometric part of the HT dilation $\hat{\phi}$ under the standard T-duality **!**

Modified type II equations
RR-sector

[Frolov, Hoare, Roiban Tseytlin and G.A. ’15]

Standard IIB sugra equations

Introduce $Z = Z_m dx^m = d\phi$

$\mathcal{F} = e^{\phi} F$

1) $D^m \mathcal{F}_m - Z^m \mathcal{F}_m - \frac{1}{6} H^{mnp} \mathcal{F}_{mnp} = 0 \ ,$ $\quad d\mathcal{F}_1 - Z \wedge \mathcal{F}_1 = 0 \ ,$

2) $D^p \mathcal{F}_{pmn} - Z^p \mathcal{F}_{pmn} - \frac{1}{6} H^{pqr} \mathcal{F}_{mnpqr} = 0 \ ,$ $\quad d\mathcal{F}_3 - Z \wedge \mathcal{F}_3 + H_3 \wedge \mathcal{F}_1 = 0 \ ,$

3) $D^r \mathcal{F}_{rmnpq} - Z^r \mathcal{F}_{mnpq} + \frac{1}{36} \varepsilon_{mnpqrstuvw} H^{rst} \mathcal{F}^{uvw} = 0 \ ,$ $\quad d\mathcal{F}_5 - Z \wedge \mathcal{F}_5 + H_3 \wedge \mathcal{F}_3 = 0 \ .$

Dynamical equations ↑ (left column) Bianchi identities ↑ (right column)

$$\star \mathcal{F}_5 = \mathcal{F}_5$$

↑ Self-duality of the five- form

I-modified sugra equations

1)

$$D^m \mathcal{F}_m - Z^m \mathcal{F}_m - \tfrac{1}{6} H^{mnp} \mathcal{F}_{mnp} = 0 \,, \qquad I^m \mathcal{F}_m = 0 \,,$$
$$(d\mathcal{F}_1 - Z \wedge \mathcal{F}_1)_{mn} - I^p \mathcal{F}_{mnp} = 0 \,.$$

2)

$$D^p \mathcal{F}_{pmn} - Z^p \mathcal{F}_{pmn} - \tfrac{1}{6} H^{pqr} \mathcal{F}_{mnpqr} - (I \wedge \mathcal{F}_1)_{mn} = 0 \,,$$
$$(d\mathcal{F}_3 - Z \wedge \mathcal{F}_3 + H_3 \wedge \mathcal{F}_1)_{mnpq} - I^r \mathcal{F}_{mnpqr} = 0 \,.$$

3)

$$D^r \mathcal{F}_{rmnpq} - Z^r \mathcal{F}_{rmnpq} + \tfrac{1}{36} \varepsilon_{mnpqrstuvw} H^{rst} \mathcal{F}^{uvw} - (I \wedge \mathcal{F}_3)_{mnpq} = 0 \,,$$
$$(d\mathcal{F}_5 - Z \wedge \mathcal{F}_5 + H_3 \wedge \mathcal{F}_3)_{mnpqrs} + \tfrac{1}{6} \varepsilon_{mnpqrstuvw} I^t \mathcal{F}^{uvw} = 0 \,.$$

! Surprisingly, there exists combinations of the above equations that depends on Z and I through the combination $Z = X + I$ only:

$$D^m \mathcal{F}_m - X^m \mathcal{F}_m - \tfrac{1}{6} H^{mnp} \mathcal{F}_{mnp} = 0 \,,$$
$$D^p \mathcal{F}_{pmn} - X^p \mathcal{F}_{pmn} - \tfrac{1}{6} H^{pqr} \mathcal{F}_{mnpqr} + (d\mathcal{F}_1 - X \wedge \mathcal{F}_1)_{mn} = 0 \,,$$
$$D^r \mathcal{F}_{rmnpq} - X^r \mathcal{F}_{rmnpq} + \tfrac{1}{36} \varepsilon_{mnpqrstuvw} H^{rst} \mathcal{F}^{uvw} + (d\mathcal{F}_3 - X \wedge \mathcal{F}_3 + H_3 \wedge \mathcal{F}_1)_{mnpq} = 0$$

These equations are sufficient to derive 2nd order equations which should express scale invariance conditions

Equations for RR-fields as scale invariance conditions

Equations for RR fields as scale invariance conditions

Requirements:

(i) vanish on the supergravity equations with $X = d\phi,\ Y = 0$

(ii) depend on Z and I through $X = Z + I$

(iii) depend on X through Lie derivatives

Compact form of the *I*-modified equations

$$d\mathcal{F}_{2n+1} - Z \wedge \mathcal{F}_{2n+1} + H_3 \wedge \mathcal{F}_{2n-1} - \star(I \wedge \star\mathcal{F}_{2n+3}) = 0 \ , \qquad n = -1, 0, \ldots \ ,$$
$$d \star \mathcal{F}_{2n+1} - Z \wedge \star\mathcal{F}_{2n+1} - H_3 \wedge \star\mathcal{F}_{2n+3} + \star(I \wedge \mathcal{F}_{2n-1}) = 0 \ , \qquad n = 0, 1, \ldots \ .$$

$n = -1 :$ $\quad \star(I \wedge \star\mathcal{F}_1) = I^m \mathcal{F}_m = 0$

Act on the first equation with $\star d \star$ and on the second with $d\star$

Equations for RR fields as scale invariance conditions

1)

$$\underline{D^2\mathcal{F}_m - R_{mn}\mathcal{F}^n + \tfrac{1}{4}(R - \tfrac{3}{4}H^2)\mathcal{F}_m}$$
$$+ \tfrac{1}{2}H^{pnk}H_{mpn}\mathcal{F}_k - \tfrac{1}{6}D_m H^{pnk}\mathcal{F}_{pnk} - \tfrac{1}{2}H^{pnk}D_p\mathcal{F}_{nkm}$$
$$= 2(X^p D_p \mathcal{F}_m + D_m X^p \mathcal{F}_p) + \beta^G_{mn}\mathcal{F}^n - \tfrac{1}{2}\beta^B_{nk}\mathcal{F}^{nk}{}_m \ .$$

2)

$$\underline{D^2\mathcal{F}_{nkm} - R_{a[n}\mathcal{F}^a{}_{km]} + R_{ab[nk}\mathcal{F}^{ab}{}_{m]} + \tfrac{1}{4}(R - \tfrac{3}{4}H^2)\mathcal{F}_{nkm}}$$
$$+ \tfrac{1}{2}H^{abc}H_{ab[n}\mathcal{F}_{km]c} - \tfrac{1}{2}H^{abc}H_{a[nk}\mathcal{F}_{m]bc}$$
$$+ D^a H_{a[nk}\mathcal{F}_{m]} + H_{a[nk}D^a\mathcal{F}_{m]} - \mathcal{F}_a D^a H_{nkm}$$
$$- \tfrac{1}{6}D_{[n}H^{abc}\mathcal{F}_{km]abc} - \tfrac{1}{2}H^{abc}D_a\mathcal{F}_{bcnkm}$$
$$= 2(X^a D_a \mathcal{F}_{nkm} + D_{[n}X^a\mathcal{F}_{km]a}) + \beta^G_{a[n}\mathcal{F}^a{}_{km]} + \beta^B_{[nk}\mathcal{F}_{m]} - \tfrac{1}{2}\beta^B_{ab}\mathcal{F}^{ab}{}_{nkm}$$

3)

$$\underline{D^2\mathcal{F}_{ijklm} - R_{a[i}\mathcal{F}^a{}_{jklm]} + R_{ab[ij}\mathcal{F}^{ab}{}_{klm]} + \tfrac{1}{4}(R - \tfrac{3}{4}H^2)\mathcal{F}_{ijklm}}$$
$$+ \tfrac{1}{2}H^{abc}H_{ab[i}\mathcal{F}_{jklm]c} - \tfrac{1}{2}H^{abc}H_{a[ij}\mathcal{F}_{klm]bc}$$
$$+ D^a H_{a[ij}\mathcal{F}_{klm]} + H_{a[ij}D^a\mathcal{F}_{klm]} - \mathcal{F}_{a[ij}D^a H_{klm]}$$
$$+ \tfrac{1}{12}\varepsilon_{ijklmbdef}(D_a H^{abc}\mathcal{F}^{def} + H^{abc}D_a\mathcal{F}^{def} - \mathcal{F}^{abc}D_a H^{def}) =$$
$$= 2(X^a D_a \mathcal{F}_{ijklm} + D_{[i}X^a\mathcal{F}_{jklm]a}) + \beta^G_{a[i}\mathcal{F}^a{}_{jklm]} + \beta^B_{[ij}\mathcal{F}_{klm]} + \tfrac{1}{12}\varepsilon_{ijklmabcde}(\beta^B)^{ab}\mathcal{F}^{cde}$$

All three equations contain the expected Hodge-de Rham operator, vector X generates reparametrization and bosonic beta-functions appeared

T-duality as a origin of the *I*-deformation (NSNS case)

$$\hat{ds}^2 = e^{2\hat{a}(x)}[d\hat{y} + \hat{A}_\mu(x)dx^\mu]^2 + g_{\mu\nu}(x)dx^\mu dx^\nu \;, \qquad \hat{\phi} = -c\,\hat{y} + f(x)$$

$\hat{R}_{mn} + 2\hat{D}_m\hat{D}_n\hat{\phi} = 0\,, \quad \bar{\beta}^{\hat{\phi}} = 0$

$\partial_{\hat{y}}\hat{\phi} = -c \neq 0$

T-dualtity

$$ds^2 = e^{2a(x)}dy^2 + g_{\mu\nu}(x)dx^\mu dx^\nu \;, \qquad B = \hat{A}_\mu(x)\,dy \wedge dx^\mu \;, \qquad a = -\hat{a}$$

The condition $\hat{R}_{mn} + 2\hat{D}_m\hat{D}_n\hat{\phi} = 0$ expressed in terms of new fields reads as

$$R_{mn} + D_m X_n + D_n X_m = 0$$

With

$$X_m dx^m \equiv I_m dx^m + Z_m dx^m = c\,e^{-2a}\,dy + \left[\partial_\mu(\hat{\phi} - \hat{a}) + c\,\hat{A}_\mu\right] dx^\mu$$

Analogously, $\bar{\beta}^X \equiv R - \frac{1}{12}H^2_{mnk} + 4D^m X_m - 4X^m X_m = 0$

Conclusions

and

future problems

- Does *I*-modified equations follow from kappa-symmetry. Yes!

[Wulff and Tseytlin '16]

- What about other deformations, e.g. corresponding to the solutions of the CYBE. Yes!

[Yoshida et al. '16], [Hoare and van Tongeren '16]

- What is the relation to the mirror model ?

[van Tongeren and G.A. '14], [Pachol and van Tongeren '16]

- Eta-deformed model is scale invariant but not Weyl invariant. *Classical kappa-symmetry implies scale invariance only!* Could this model still be used to define a critical string?

Eta-deformed model is related to the lambda-model by Poisson-Lie duality and analytic continuation. The latter model propagates in the IIB background and therefore is Weyl invariant. Extra fields? Non-locality?

[Yoshida et al. '17]

- Is there any way to obtain *I*-modified gravity equations from some Lagrangian?

- What is *I*-modification for background fermionic fields?

- *I*-modification destroys local supersymmetry. If there is still any local (hidden) symmetry?

- What is a dual (non-commutative) gauge theory?


PBZ
PARTNER BANKA