Axial gravity, massless fermions and trace anomalies

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Introduction

- A symmetry of the classical action is a transformation of the fields that leaves the action invariant.
- Are these symmetries still valid in the quantum theory?

If not, the theory is anomalous!

- Two types of anomalies:
  1. Harmless
  2. Harmful - destroy consistency of QFT

Symmetries of classical action

- Classical action S is describing some matter field coupled to a curved background $g_{\mu\nu}$.
- Local diff - transformations: $\delta_\xi g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$

If classical action is invariant $\Rightarrow$ EMT is conserved

$$\delta_\xi S = 0 \qquad \Rightarrow \qquad \nabla^\mu T_{\mu\nu} = 0$$

- Energy-momentum tensor

$$T_{\mu\nu} = \frac{2}{\sqrt{g}} \frac{\delta S}{\delta g^{\mu\nu}}$$

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- Energy-momentum tensor

$$T_{\mu\nu} = \frac{2}{\sqrt{g}} \frac{\delta S}{\delta g^{\mu\nu}}$$

- Local Weyl transformations: $\delta_\omega g_{\mu\nu} = \omega(x) g_{\mu\nu}$

If classical action is invariant $\Rightarrow$ EMT is tracelsess

$$\delta_\omega S = 0 \qquad \Rightarrow \qquad T^\mu_\mu = 0$$

Ward identities

- Fundamental object is partition function:

$$Z[g] = \int D\bar{\psi} D\psi e^{iS}$$

- Effective action:

$$Z[g] = e^{iW[g]} \quad \Rightarrow \quad W[g] = -i \ln Z[g]$$

- Expectation (1-point function) of the EMT:

$$\langle\langle T_{\mu\nu}(x) \rangle\rangle = \frac{1}{Z[g]} \int D\bar{\psi} D\psi \, T_{\mu\nu}(x) e^{iS} = \frac{2}{\sqrt{g}} \frac{\delta W[g]}{\delta g^{\mu\nu}}$$

Ward identities

- If the quantum theory has the same symmetries as classical action

Ward identity for diff-invariance

$$\nabla^{\mu}\langle\langle T_{\mu\nu}(x)\rangle\rangle = 0$$

Ward identity for Weyl invariance

$$\langle\langle T^{\mu}_{\mu}(x)\rangle\rangle = 0$$

- If classical symmetry breaks after quantization $\Rightarrow$ anomalies

Diff and trace anomalies

- EMT conserved

Diff anomaly

$$\nabla^{\mu}\langle\langle T_{\mu\nu}(x)\rangle\rangle = 0$$

- Symmetry under local Weyl transformations is broken

Trace anomaly - P-even part

$$\langle\langle T^{\mu}_{\mu}(x)\rangle\rangle = a\,E + c\,W^2 \qquad \text{[Capper, Duff 1975]}$$

Euler density $E = R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2$
Weyl density $W^2 = R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2$

Diff and trace anomalies

- From WZ consistency conditions
  - There is no diff-anomalies

$$\nabla^{\mu}\langle\langle T_{\mu\nu}(x)\rangle\rangle = 0$$

  - General form of trace anomaly is

$$\langle\langle T^{\mu}_{\mu}(x)\rangle\rangle = a\,E + c\,W^2 + e\,P$$ [Bonora, Pasti, Tonin 1986]

Pontryagin density

$$P = \frac{1}{2}\,\varepsilon^{\mu\nu\rho\sigma}\,R_{\mu\nu}{}^{\alpha\beta}\,R_{\rho\sigma\alpha\beta}$$

Original calculation

[Bonora et al. 2014]

- Consider the action for left-handed Weyl fermion coupled to curved background in 4d.

$$S = \int d^4x \sqrt{|g|}\, i\overline{\psi_L}\gamma^\mu \left(\nabla_\mu + \frac{1}{2}\omega_\mu\right)\psi_L$$

- Perturbative calculation around flat background

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

with redefinition of the field $\psi \to |g|^{\frac{1}{4}}\psi$

- The action up to h^2

$$S \approx \int d^4x \left[\frac{i}{2}(\delta^\mu_a - \frac{1}{2}h^\mu_a)\overline{\psi_L}\gamma^a \overset{\leftrightarrow}{\partial}_\mu \psi_L + \frac{1}{16}\epsilon^{\mu abc}\,\partial_\mu h_{a\lambda}\, h^\lambda_b\, \bar{\psi}_L\gamma_c\gamma_5\psi_L\right]$$

Original calculation

- The calculation of trace anomaly is based on Feynmann diagrams and dimensional regularization
- Vertices

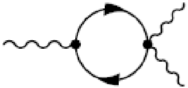
$$V_{ffh} : -\frac{i}{8}\left[(p+p')_\mu \gamma_\nu + (p+p')_\nu \gamma_\mu\right] P_L$$

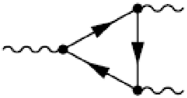
$$V^{\epsilon}_{ffhh} : \frac{1}{64} t_{\mu\nu\mu'\nu'\kappa\lambda}(k-k')^\lambda \gamma^\kappa P_L$$

where $P_L = \frac{1+\gamma_5}{2}$ and

$$t_{\mu\nu\mu'\nu'\kappa\lambda} = \eta_{\mu\mu'}\epsilon_{\nu\nu'\kappa\lambda} + \eta_{\nu\nu'}\epsilon_{\mu\mu'\kappa\lambda} + \eta_{\mu\nu'}\epsilon_{\nu\mu'\kappa\lambda} + \eta_{\nu\mu'}\epsilon_{\mu\nu'\kappa\lambda}$$

- Diagrams





Original calculation

- Parity-odd part of trace of EMT at h^2 order

$$\langle\langle T^{\mu}_{\mu}(x)\rangle\rangle^{(2)}_{\mathrm{P}} = \frac{1}{2}\int d^4x_1 d^4x_2\, \mathcal{T}^{\mu_1\nu_1\mu_2\nu_2}(x,x_1,x_2)\, h_{\mu_1\nu_1}(x_1)\, h_{\mu_2\nu_2}(x_2)$$

where

$$\mathcal{T}^{\mu_1\nu_1\mu_2\nu_2}(x,x_1,x_2) = -\eta_{\mu\nu}\left\langle T^{\mu\nu}_{(0)}(x)\, T^{\mu_1\nu_1}_{(0)}(x_1)\, T^{\mu_2\nu_2}_{(0)}(x_2)\right\rangle_{\mathrm{P-odd}}$$

- $T^{\mu\nu}_{(0)}$ is the energy momentum tensor in the flat space

$$T^{\mu\nu}_{(0)} = -\frac{i}{4}\left(\overline{\psi_L}\gamma^{\mu}\overset{\leftrightarrow}{\partial^{\nu}}\psi_L + (\mu\leftrightarrow\nu)\right)$$

Original calculation

- The explicit calculation gives [Bonora et al. 2014]

$$\langle\langle T^{\mu}_{\mu}\rangle\rangle^{(2)}_{\mathrm{P}} = \frac{i}{768\pi^2}\epsilon^{\mu\nu\lambda\rho}\left(\partial_\mu\partial_\sigma h^{\tau}_{\nu}\,\partial_\lambda\partial_\tau h^{\sigma}_{\rho} - \partial_\mu\partial_\sigma h^{\tau}_{\nu}\,\partial_\lambda\partial^\sigma h_{\tau\rho}\right)$$

- Covariantize:

$$\mathcal{A}_0 \equiv \langle\langle T^{\mu}_{\mu}\rangle\rangle = \frac{i}{768\pi^2}P \quad \Rightarrow \quad e_L = \frac{i}{768\pi^2}$$

- For RH fermion $e_R = -\frac{i}{768\pi^2}$

- Calculate in 2 ways:

  1. Repeat calculation for P-odd part of the trace anomaly for Weyl fermions coupled to curved background in 4d in a more pedantic way
  2. Introduce axial gravity - use Dirac fermions coupled to metric-axial-tensor (MAT) gravity

1. Calculation with Weyl fermions

- No redefinition of the field
- There are additional vertices, up to h^2 order:

$$
\begin{aligned}
V'_{ffh} &: \quad \frac{i}{4}\eta_{\mu\nu}(\not{p}+\not{p}')\, P_L \\
V'_{ffhh} &: \quad \frac{3i}{64}\Big[\,\big((p+p')_{\mu}\gamma_{\mu'}\eta_{\nu\nu'}+(p+p')_{\mu}\gamma_{\nu'}\eta_{\nu\mu'}+\{\mu\leftrightarrow\nu\}\big) \\
&\qquad +\big((p+p')_{\mu'}\gamma_{\mu}\eta_{\nu\nu'}+(p+p')_{\mu'}\gamma_{\nu}\eta_{\mu\nu'}+\{\mu'\leftrightarrow\nu'\}\big)\Big]\, P_L \\
V''_{ffhh} &: \quad -\frac{i}{16}\Big[\eta_{\mu\nu}\,\big((p+p')_{\mu'}\gamma_{\nu'}+(p+p')_{\nu'}\gamma_{\mu'}\big) \\
&\qquad +\eta_{\mu'\nu'}\,\big((p+p')_{\mu}\gamma_{\nu}+(p+p')_{\nu}\gamma_{\mu}\big)\Big]\, P_L \\
V'''_{ffhh} &: \quad \frac{i}{8}(\not{p}+\not{p}')(\eta_{\mu\nu}\eta_{\mu'\nu'}-\eta_{\mu\nu'}\eta_{\mu'\nu}-\eta_{\mu\mu'}\eta_{\nu\nu'})\, P_L
\end{aligned}
$$

1. Calculation with Weyl fermions

- Flat-space energy momentum $T^{\mu\nu}_{(0)}(x)$ contains an additional term

$$T^{\mu\nu}_{(0)} = -\frac{i}{4}\left(\overline{\psi_L}\gamma^\mu \overset{\leftrightarrow}{\partial^\nu}\psi_L + (\mu \leftrightarrow \nu)\right) + \frac{i}{2}\eta^{\mu\nu}\,\overline{\psi_L}\gamma^\lambda \overset{\leftrightarrow}{\partial}_\lambda \psi_L$$

- The explicit calculation gives

$$\langle\langle T^\mu_\mu \rangle\rangle^{(2)}_{\mathrm{P}} = -\frac{3i}{768\pi^2}\epsilon^{\mu\nu\lambda\rho}\left(\partial_\mu\partial_\sigma h^\tau_\nu\,\partial_\lambda\partial_\tau h^\sigma_\rho - \partial_\mu\partial_\sigma h^\tau_\nu\,\partial_\lambda\partial^\sigma h_{\tau\rho}\right)$$

- Covariantize the result

$$\langle\langle T^\mu_\mu \rangle\rangle = -\frac{3i}{768\pi^2}P$$

- Not the result we expect!

1. Calculation with Weyl fermions

- Check conservation of EMT, it is not zero

$$\nabla^{\mu}\langle\langle T_{\mu\nu}(x)\rangle\rangle \neq 0$$

- Introduce counterterm

$$\mathcal{C} = -\frac{1}{2}\int \omega\, h_{\mu}^{\mu}\, \mathcal{A}_0,$$

which cancels diff-anomaly and trace anomaly becomes

$$\langle\langle T_{\mu}^{\mu}\rangle\rangle = \frac{i}{768\pi^2} P$$

2. MAT gravity

- Ispired by Bardeen's method for chiral gauge anomalies
- Use Dirac field and avoid subtleties with Weyl fermions
- In addition to ordinary gravity introduce axial metric $\Rightarrow$ metric-axial-tensor (MAT) gravity

$$G_{\mu\nu} = g_{\mu\nu} + \gamma_5 \, f_{\mu\nu}$$

2. MAT gravity

- Vielbein:

$$E^a_\mu = e^a_\mu + \gamma_5 c^a_\mu, \qquad \hat{E}^\mu_a = \hat{e}^\mu_a + \gamma_5 \hat{c}^\mu_a$$

- Connection:

$$\Gamma^\lambda_{\mu\nu} = \Gamma^{(1)\lambda}_{\mu\nu} + \gamma_5 \Gamma^{(2)\lambda}_{\mu\nu}$$

- Riemann:

$$\mathcal{R}_{\mu\nu\lambda}{}^\rho = \mathcal{R}^{(1)}_{\mu\nu\lambda}{}^\rho + \gamma_5 \mathcal{R}^{(2)}_{\mu\nu\lambda}{}^\rho$$

- The MAT spin connection is introduced in analogy

$$\Omega^{ab}_\mu = E^a_\nu \left(\partial_\mu \hat{E}^{\nu b} + \hat{E}^{\sigma b} \Gamma^\nu_{\sigma\mu} \right) = \Omega^{(1)ab}_\mu + \gamma_5 \Omega^{(2)ab}_\mu$$

2. MAT gravity

- The action

$$S = \int d^4x \, i\overline{\psi}\sqrt{|\bar{G}|}\gamma^a \hat{E}_a^\mu \left(\partial_\mu + \frac{1}{2}\Omega_\mu\right)\psi \equiv \int d^4x \overline{\psi}\sqrt{|\bar{G}|}\mathcal{O}\psi$$

is invariant under diffeomorphisms with parameter $\Xi^\mu = \xi^\mu + \gamma_5 \zeta^\mu$

$$\delta_\Xi G_{\mu\nu} = \mathcal{D}_\mu \Xi_\nu + \mathcal{D}_\nu \Xi_\mu$$

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is invariant under diffeomorphisms with parameter $\Xi^\mu = \xi^\mu + \gamma_5\zeta^\mu$

$$\delta_\Xi G_{\mu\nu} = \mathcal{D}_\mu \Xi_\nu + \mathcal{D}_\nu \Xi_\mu$$

- There are two independent classically conserved EMT

$$T^{\mu\nu} = 2\overline{\psi}\frac{\overleftarrow{\delta}\ \mathcal{O}}{\delta G_{\mu\nu}}\psi$$

$$T_5^{\mu\nu} = 2\overline{\psi}\frac{\overleftarrow{\delta}\ \mathcal{O}}{\delta G_{\mu\nu}}\gamma_5\psi$$

2. MAT gravity

- There are two types of Weyl transformations: the usual

$$\delta_\omega G_{\mu\nu} = 2\omega G_{\mu\nu}$$

and the axial one

$$\delta_\eta G_{\mu\nu} = 2\gamma_5 \eta G_{\mu\nu}$$

- Two trace conditions

$$T^{\mu\nu} g_{\mu\nu} + T_5^{\mu\nu} f_{\mu\nu} = 0,$$

$$T^{\mu\nu} f_{\mu\nu} + T_5^{\mu\nu} g_{\mu\nu} = 0,$$

2. MAT gravity

- Expand the action

$$S = \int d^4x \, i\overline{\psi}\sqrt{|\bar{G}|}\gamma^a \hat{E}_a^\mu \left(\partial_\mu + \frac{1}{2}\Omega_\mu\right)\psi$$

- We use redefinition $\psi \to |\bar{G}|^{\frac{1}{4}}\psi$ and expand around flat background

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad f_{\mu\nu} = k_{\mu\nu}$$

2. MAT gravity

- Expand the action

$$S = \int d^4x \, i\overline{\psi}\sqrt{|\bar{G}|}\gamma^a \hat{E}_a^\mu \left(\partial_\mu + \frac{1}{2}\Omega_\mu\right)\psi$$

- We use redefinition $\psi \to |\bar{G}|^{\frac{1}{4}}\psi$ and expand around flat background

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad f_{\mu\nu} = k_{\mu\nu}$$

- Vertices: The 2-fermion-1-graviton vertices:

$$\begin{aligned} V_{ffh} &: \quad -\frac{i}{8}\left[(p+p')_\mu\gamma_\nu + (p+p')_\nu\gamma_\mu\right] \\ V_{ffk} &: \quad -\frac{i}{8}\left[(p+p')_\mu\gamma_\nu + (p+p')_\nu\gamma_\mu\right]\gamma_5 \end{aligned}$$

2. MAT gravity

- There are six 2-fermion-2-graviton vertices:

$$
\begin{aligned}
V_{ffhh} \quad &: \quad \frac{3i}{64}\Big[\big((p+p')_{\mu}\gamma_{\mu'}\eta_{\nu\nu'} + (p+p')_{\mu}\gamma_{\nu'}\eta_{\nu\mu'} + \{\mu \leftrightarrow \nu\}\big) \\
&\quad + \big((p+p')_{\mu'}\gamma_{\mu}\eta_{\nu\nu'} + (p+p')_{\mu'}\gamma_{\nu}\eta_{\mu\nu'} + \{\mu' \leftrightarrow \nu'\}\big)\Big] \\
V_{ffkk} \quad &: \quad \frac{3i}{64}\Big[\big((p+p')_{\mu}\gamma_{\mu'}\eta_{\nu\nu'} + (p+p')_{\mu}\gamma_{\nu'}\eta_{\nu\mu'} + \{\mu \leftrightarrow \nu\}\big) \\
&\quad + \big((p+p')_{\mu'}\gamma_{\mu}\eta_{\nu\nu'} + (p+p')_{\mu'}\gamma_{\nu}\eta_{\mu\nu'} + \{\mu' \leftrightarrow \nu'\}\big)\Big] \\
V_{ffhk} \quad &: \quad \frac{3i}{64}\Big[\big((p+p')_{\mu}\gamma_{\mu'}\eta_{\nu\nu'} + (p+p')_{\mu}\gamma_{\nu'}\eta_{\nu\mu'} + \{\mu \leftrightarrow \nu\}\big) \\
&\quad + \big((p+p')_{\mu'}\gamma_{\mu}\eta_{\nu\nu'} + (p+p')_{\mu'}\gamma_{\nu}\eta_{\mu\nu'} + \{\mu' \leftrightarrow \nu'\}\big)\Big]\gamma_5 \\
V^{\epsilon}_{ffhh} \quad &: \quad \frac{1}{64}\, t_{\mu\nu\mu'\nu'\kappa\lambda}\,(k-k')^{\lambda}\gamma^{\kappa}\,\gamma_5 \\
V^{\epsilon}_{ffkk} \quad &: \quad \frac{1}{64}\, t_{\mu\nu\mu'\nu'\kappa\lambda}\,(k-k')^{\lambda}\gamma^{\kappa}\,\gamma_5 \\
V^{\epsilon}_{ffhk} \quad &: \quad \frac{1}{64}\, t_{\mu\nu\mu'\nu'\kappa\lambda}\,(k-k')^{\lambda}\gamma^{\kappa}
\end{aligned}
$$

2. MAT gravity

- Flat space EMT

$$T^{\mu\nu} \equiv T^{\mu\nu}_{(0,0)} = -\frac{i}{4}\left(\overline{\psi}\gamma^{\mu}\overleftrightarrow{\partial}^{\nu}\psi + \mu \leftrightarrow \nu\right),$$

and

$$T_5^{\mu\nu} \equiv T^{\mu\nu}_{5(0,0)} = \frac{i}{4}\left(\overline{\psi}\gamma_5\gamma^{\mu}\overleftrightarrow{\partial}^{\nu}\psi + \mu \leftrightarrow \nu\right)$$

- The quantum Ward identities for the Weyl and axial Weyl symmetry

$$\mathcal{T}(x) \equiv \langle\langle T^{\mu\nu}\rangle\rangle g_{\mu\nu} + \langle\langle T_5^{\mu\nu}\rangle\rangle f_{\mu\nu} = 0$$

and

$$\mathcal{T}_5(x) \equiv \langle\langle T^{\mu\nu}\rangle\rangle f_{\mu\nu} + \langle\langle T_5^{\mu\nu}\rangle\rangle g_{\mu\nu} = 0$$

2. MAT gravity

- It turns out that only the 3-point correlators contribute
- Up to second order in the graviton field (lowest order)

$$\langle\langle T^{\mu}_{\mu}\rangle\rangle = -\frac{i}{384\pi^2}\epsilon^{\mu\nu\lambda\rho}\left(\partial_\mu\partial_\sigma h^\tau_\nu\,\partial_\lambda\partial_\tau k^\sigma_\rho - \partial_\mu\partial_\sigma h^\tau_\nu\,\partial_\lambda\partial^\sigma k_{\tau\rho}\right)$$

$$\begin{aligned}\langle\langle \mathcal{T}_5{}^{\mu}{}_{\mu}\rangle\rangle = &-\frac{i}{768\pi^2}\epsilon^{\mu\nu\lambda\rho}\left(\partial_\mu\partial_\sigma h^\tau_\nu\,\partial_\lambda\partial_\tau h^\sigma_\rho - \partial_\mu\partial_\sigma h^\tau_\nu\,\partial_\lambda\partial^\sigma h_{\tau\rho}\right)\\ &-\frac{i}{768\pi^2}\epsilon^{\mu\nu\lambda\rho}\left(\partial_\mu\partial_\sigma k^\tau_\nu\,\partial_\lambda\partial_\tau k^\sigma_\rho - \partial_\mu\partial_\sigma k^\tau_\nu\,\partial_\lambda\partial^\sigma h_{\tau\rho}\right)\end{aligned}$$

2. MAT gravity

- Covariantize the results:

$$\langle\!\langle T_\mu^\mu(x) \rangle\!\rangle = \frac{i}{768\pi^2}\epsilon^{\mu\nu\lambda\rho}\mathcal{R}^{(1)\sigma\tau}_{\mu\nu}\mathcal{R}^{(2)}_{\lambda\rho\sigma\tau}$$

$$\langle\!\langle T_{5\mu}{}^\mu(x) \rangle\!\rangle = \frac{i}{1536\pi^2}\,\epsilon^{\mu\nu\lambda\rho}\left(\mathcal{R}^{(1)\sigma\tau}_{\mu\nu}\mathcal{R}^{(1)}_{\lambda\rho\sigma\tau} + \mathcal{R}^{(2)\sigma\tau}_{\mu\nu}\mathcal{R}^{(2)}_{\lambda\rho\sigma\tau}\right)$$

2. MAT gravity

- Collapsing limit (for left-handed Weyl fermion)

$$h_{\mu\nu} \to \frac{h_{\mu\nu}}{2}, \quad k_{\mu\nu} \to \frac{h_{\mu\nu}}{2}$$

- The anomaly becomes

$$\langle\langle T_\mu^\mu \rangle\rangle = \frac{i}{768\pi^2} P$$

2. MAT gravity

- Collapsing limit (for right-handed Weyl fermion)

$$h_{\mu\nu} \to \frac{h_{\mu\nu}}{2}, \quad k_{\mu\nu} \to -\frac{h_{\mu\nu}}{2}$$

- The anomaly becomes

$$\langle\!\langle T^{\mu}_{\mu} \rangle\!\rangle = -\frac{i}{768\pi^2} P$$

2. MAT gravity

- Collapsing limit

$$h_{\mu\nu} \to h_{\mu\nu}, \quad k_{\mu\nu} \to 0$$

- The action reduces to the Dirac action (or to Majorana action if ψ satisfies reality condition)
- The anomaly, as expected, vanishes

$$\langle\langle T^{\mu}_{\mu} \rangle\rangle = 0$$

- There is an anomaly in the axial sector (analog of Kimura-Delbourgo-Salam anomaly for the axial current)

$$\langle\langle T^{\mu}_{5\mu} \rangle\rangle = \frac{i}{768\pi^2} P$$

Conclusion

- Recalculated the parity odd trace anomaly in two ways:
  - With Weyl fermions
  - MAT gravity with Dirac fermions

P-odd part is given by Pontryagin density in 4d

- The usual opposing argument: "In 4d massless Weyl and Majorana fermion are indistinguishable."
  - Classically, there is one-to-one correspondence between massless Weyl and Majorana fermion
  - We do not expect Pontryagin anomaly for Majorana fermion
  - However, the path integral measure is different for Majorana and Weyl fermion!

Conclusion

- Strange imaginary coefficient $e = \pm\frac{i}{768\pi^2}$
  - Could break unitarity
  - Theories with chiral unbalance not consistent
  - Only Dirac and Majorana fermions!
- Outlook:
  - Schwinger-DeWitt method
  - Calculation with different regularizations