noncommutative gravity with self-dual variables

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outline

- introduction
- the palatini-holst action
- noncommutative gravity with a self-dual connection
- the commutative limit
- perturbative expansion in the deformation parameter
- further extensions of the model
- conclusions / discussion

marco de cesare, mairi sakellariadou, patrizia vitale , arXiv:1806.04666

introduction

spacetime noncommutativity maybe a key feature characterising geometric structure of spacetime at planck scale
(radical departure from standard description of ST as riemannian manifold)

aim: *to build a dynamical theory of gravity consistent with the noncommutative structure we assume and which can recover GR in the appropriate regime*

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spacetime noncommutativity requires the introduction of new gravitational d.o.f.

$\Rightarrow$

- $\exists$ correction terms relevant close to the noncommutativity scale, leading to higher-derivative interactions
- the commutative limit is a modified theory of gravity (bimetric theory featuring both torsion and non-metricity)

the palatini-holst action

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palatini-holst action in the commutative case

$$\boxed{S[e,\omega] = -\frac{1}{16\pi G} P_{IJKL} \int e^I \wedge e^J \wedge F^{KL}(\omega)}$$

capital latin letters denote internal lorentz symmetries

tetrad one-forms

field strength: gauge curvature of the spin connection one-form ω

$$F^{IJ}(\omega) = \mathrm{d}\omega^{IJ} + \omega^{IK} \wedge \omega_K^{\ J}$$

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tensor in the internal space

$$P_{IJKL} = \frac{1}{2}\varepsilon_{IJKL} + \frac{1}{\beta}\delta_{IJKL}$$

$$F^{IJ}(\omega) = \mathrm{d}\omega^{IJ} + \omega^{IK} \wedge \omega_K{}^J$$

$\delta^{IJ}{}_{KL}$ *identity on the space of rank-2 antisymmetric tensors in internal space*

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in the torsion-free case, the dynamics is not affected by the particular value of the barbero-immirzi parameter, which can be a priori an arbitrary complex number

the palatini action is recovered if one sets $1/\beta = 0$, with β the **barbero-immirzi** complex-valued parameter *palatini-holst action in the commutative case*

$$\boxed{S[\boldsymbol{e},\boldsymbol{\omega}] = -\frac{i}{32\pi G}\int \mathrm{Tr}\left[\boldsymbol{e}\wedge\boldsymbol{e}\wedge\left(*_{\mathrm{H}}\boldsymbol{F}+\frac{1}{\beta}\boldsymbol{F}\right)\right]}$$

capital latin letters denote internal lorentz symmetries

trace over the lie algebra

$$*_{\mathrm{H}}\, F^{IJ} = \frac{1}{2}\varepsilon^{IJ}{}_{KL}F^{KL}$$

$$*_{\mathrm{H}}\, \boldsymbol{F} = -i\boldsymbol{F}\gamma_5$$

the curvature two-form (field strength)

$$\boldsymbol{F}(\omega) = \frac{1}{2}F^{IJ}(\omega)\Gamma_{IJ} = \mathrm{d}\boldsymbol{\omega} - i\boldsymbol{\omega}\wedge\boldsymbol{\omega}$$

the lie algebra valued connection one-form

$$\boldsymbol{\omega} = \frac{1}{2}\omega^{IJ}\Gamma_{IJ}$$

$$\Gamma_{IJ} = \tfrac{i}{4}[\gamma_I,\gamma_J]$$

the way tetrads are associated to vector valued one-forms

$$\boldsymbol{e} = e^I\gamma_I$$

palatini-holst action in the commutative case

$$\boxed{S[e,\boldsymbol{\omega}] = -\frac{i}{32\pi G}\int \mathrm{Tr}\left[\boldsymbol{e}\wedge\boldsymbol{e}\wedge\left(*_{\mathrm{H}}\boldsymbol{F}+\frac{1}{\beta}\boldsymbol{F}\right)\right]}$$

invariant under gauge transformations Λ (lorentz transformations in the internal space)

$$\boldsymbol{e} \rightarrow \boldsymbol{\Lambda} \boldsymbol{e} \boldsymbol{\Lambda}^{-1}$$

$$\omega \rightarrow \boldsymbol{\Lambda}\omega\boldsymbol{\Lambda}^{-1} + i\boldsymbol{\Lambda}\mathrm{d}\boldsymbol{\Lambda}^{-1}$$

$$\boldsymbol{F} \rightarrow \boldsymbol{\Lambda}\boldsymbol{F}\boldsymbol{\Lambda}^{-1}$$

with the gauge transformation Λ given by:

$$\boldsymbol{\Lambda}(x) = \exp(-i\boldsymbol{\epsilon}(x)) \quad , \quad \text{with} \;\; \boldsymbol{\epsilon}(x) = \frac{1}{2}\epsilon^{IJ}(x)\Gamma_{IJ}$$

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a gauge connection is said to be self-dual (anti self-dual) if it is a solution to the eigenvalue equation

$$\boxed{*_{\mathrm{H}}\, \omega_{\pm} = \pm i\, \omega_{\pm}}$$

with $$*_{\mathrm{H}}\omega := -i\omega\gamma_5$$

analogous definition for the field strength

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decompose gauge connection and field strength into their self-dual and anti self-dual:

$$\omega = \omega_{+} + \omega_{-} \ , \quad \boldsymbol{F} = \boldsymbol{F}_{+} + \boldsymbol{F}_{-}$$

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decompose gauge connection and field strength into their self-dual and anti self-dual:

$$\omega = \omega_{+} + \omega_{-}\ , \quad \boldsymbol{F} = \boldsymbol{F}_{+} + \boldsymbol{F}_{-}$$

$$\Longrightarrow \quad \boldsymbol{F}_{\pm}(\omega) = \mathrm{d}\omega_{\pm} - i\omega_{\pm}\wedge\omega_{\pm} = \boldsymbol{F}_{\pm}(\omega_{\pm})$$

the self-dual (anti self-dual) part of the field strength is given by the field strength of the self-dual (anti self-dual) of the gauge connection for β = - i the anti self-dual component of the spin connection is projected out

the action reduces to a functional of the tetrad and the self-dual connection only:

$$\boxed{S[e,\omega_+] = -\frac{i}{32\pi G}\int \mathrm{Tr}\left[e\wedge e\wedge(*_{\mathrm{H}}F + iF)\right] = -\frac{i}{16\pi G}\int \mathrm{Tr}\Big[e\wedge e\wedge\big(*_{\mathrm{H}}\,F_+(\omega_+)\big)\Big]}$$

this is formally equal to twice the palatini action for a self-dual connection for β = - i the anti self-dual component of the spin connection is projected out

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the solution space of the theory is much enlarged;
equivalence with the standard palatini theory is obtained only after imposing suitable reality conditions

tetrad e^I be real

ω_+^{IJ} be the self-dual part of a real connection one-form $\omega_+^{IJ} = \frac{1}{2}\left(\omega^{IJ} - i *_{\mathrm{H}} \omega^{IJ}\right)$

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similarly, for β = + i , a theory of an anti self-dual connection is obtained

let us concentrate on self-dual connections; similar results for anti self-dual case

noncommutative extension

noncommutative field theories are formulated in terms of fields which are elements of a noncommutative algebra over spacetime, with a noncommutative, associative, product

this is generally a deformation of a commutative algebra, which is recovered when the noncommutativity parameter is set to zero

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note: *it is a general feature of noncommutative gauge theories that the lie algebra of the structure group has to be extended in order to close*

aschieri, dimitrijevic, meyer, scgraml, wess (2006)

this is also the case for the lorentz algebra in the case of twist-deformed spacetimes

aschieri, castellani (2009)

noncommutative extension

deformed gauge symmetry and symmetry enlargement

ordinary gauge theories are modified by replacing the pointwise product of fields with a noncommutative product indicated with $\star$

noncommutative product

$$\boxed{f \star g = \mu \circ \mathcal{F}^{-1}(f \otimes g)}$$

abelian twist

deformation of ordinary pointwise multiplication

$$f \cdot g \equiv \mu \circ (f \otimes g)$$

$$f, g \in C^{\infty}(M)$$

spacetime manifold

bilinear operator μ denotes pointwise multiplication

$$\mu(f \otimes g) = f \cdot g$$

twist

$$\mathcal{F} = \mathrm{e}^{-\frac{i}{2}\theta^{\alpha\beta} X_\alpha \otimes X_\beta}$$

$\{X_\alpha\}$ *a set of mutually commuting vector fields*

constant antisymmetric matrix: $\theta^{\alpha\beta} = -\theta^{\beta\alpha}$

deformation parameter

noncommutative extension

deformed gauge symmetry and symmetry enlargement

the resulting field theories are invariant under the deformed gauge transformations

$$\phi(x) \longrightarrow g_\star(x) \triangleright_\star \phi(x) = \exp_\star\left(i\,\epsilon^i(x)T_i\right) \triangleright_\star \phi(x)$$

generic field in the theory

indicates the action of the group

lie algebra generators

gauge group elements defined as star exponentials:

$$g_\star(x) = \exp_\star\left(i\,\epsilon(x)^i T_i\right) = 1 + i\epsilon^i(x)T_i - \frac{1}{2}(\epsilon^i \star \epsilon^j)(x)T_iT_j + \dots$$

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at the infinitesimal level:

$$\phi(x) \longrightarrow \phi(x) + i(\epsilon \triangleright_\star \phi)(x)$$

with $(\epsilon \triangleright_\star \phi)(x) = \left(\epsilon^j \star (T_j \triangleright \phi)\right)(x)$

⇒ deformed lie bracket $[\epsilon_1, \epsilon_2]_\star(x) = (\epsilon_1 \star \epsilon_2)(x) - (\epsilon_2 \star \epsilon_1)(x)$

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consistency of the theory demands that the algebra of infinitesimal gauge transformations must close under this $\star$- commutator

noncommutative extension

deformed gauge symmetry and symmetry enlargement

however, in noncommutative field theory algebra closure is not guaranteed

since

$$(\epsilon_1 \star \epsilon_2)(x) - (\epsilon_2 \star \epsilon_1)(x) = \frac{1}{2}\left((\epsilon_1^i \star \epsilon_2^j)(x) + (\epsilon_2^j \star \epsilon_1^i)(x)\right)[T_i, T_j] + \frac{1}{2}\left((\epsilon_1^i \star \epsilon_2^j)(x) - (\epsilon_2^j \star \epsilon_1^i)(x)\right)\{T_i, T_j\}$$

which contains the anticommutator of the algebra generators

extend the universal covering of the lorentz group to include the anticommutators

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extend the universal covering of the lorentz group to include the anticommutators

enlarge the gauge algebra and consider more general infinitesimal gauge transformations of the form:

$$\epsilon(x) = \frac{1}{2}\epsilon^{IJ}(x)\Gamma_{IJ} + \epsilon(x)\mathbb{1} + \tilde{\epsilon}(x)\gamma_5$$

$(\Gamma_{IJ}, \mathbb{1}, \gamma_5)$ *is a set of generators of the reducible representation of* $\mathfrak{gl}(2,\mathbb{C})$ *on dirac spinors*

aschieri, castellani (2009)

noncommutative extension

new gravitational degrees of freedom

as a consequence of the enlargement of the gauge symmetry algebra, the field content of the theory is extended as:

polar component *axial component*

$$\boldsymbol{e} = e^I \gamma_I + \tilde{e}^I \gamma_I \gamma_5$$

$$\boldsymbol{\omega} = \frac{1}{2}\omega^{IJ}\Gamma_{IJ} + \omega \mathbb{I} + \tilde{\omega}\gamma_5$$

the generator γ_5 acts on e by exchanging its polar and axial components

field strength of the gauge connection: $$\boldsymbol{F} = \frac{1}{2}F^{IJ}\Gamma_{IJ} + r\mathbb{I} + \tilde{r}\gamma_5$$

field strength components

the lorentzian spin connection must be replaced by a $\mathfrak{gl}(2,\mathbb{C})$ gauge connection and similarly the tetrad must be replaced by a bitetrad

the action of internal symmetries on fields

under an infinitesimal $\star$-deformed gauge transformation, the fields representing the basic dynamical variables of the theory transform as

$$e \to e + i[e, \epsilon]_\star$$

$$\omega \to \omega - (\mathrm{d}\epsilon - i[\omega, \epsilon]_\star)$$

$$\boldsymbol{F} \to \boldsymbol{F} + i[\boldsymbol{F}, \epsilon]_\star$$

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$$F \to F + i[F, \epsilon]_\star$$

the physical meaning of the enlarged gauge symmetry becomes transparent in the commutative limit

under an infinitesimal gauge transformation, the bitetrad components transform as:

$$\boxed{\begin{aligned} \delta e^I &= \epsilon^I{}_J e^J + 2i\tilde{\epsilon}\,\tilde{e}^I \\ \delta \tilde{e}^I &= \epsilon^I{}_J \tilde{e}^J + 2i\tilde{\epsilon}\,e^I \end{aligned}}$$

where $\epsilon(x) = \frac{1}{2}\epsilon^{IJ}(x)\Gamma_{IJ} + \epsilon(x)\mathbb{1} + \tilde{\epsilon}(x)\gamma_5$

→ a generic gauge transformation acts on the bitetrad as the composition of a lorentz transformation and a transformation generated by γ_5

the action of internal symmetries on fields

the metric tensors can be defined using the two tetrad frames as:

$$\boxed{\begin{aligned} g_{ab} &= \eta_{IJ} e_a^I e_b^J \\ \tilde{g}_{ab} &= \eta_{IJ} \tilde{e}_a^I \tilde{e}_b^J \end{aligned}}$$

related through the no-shear condition:

$$\tilde{e}_a^I = \Omega \, \Lambda^I{}_J e_a^J$$

real function representing a weyl scaling

local lorentz transf

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under infinitesimal lorentz transformations the metric tensors transform as:

$$g_{ab} \to (\cosh\chi + \Omega \sinh\chi)^2\, g_{ab}$$

$$\tilde{g}_{ab} \to (\cosh\chi + \Omega^{-1} \sinh\chi)^2\, \tilde{g}_{ab}$$

where $\chi = 2i\tilde{\epsilon}$
and assuming it to be real

accordingly the relative scale of the two metrics transforms as:

$$\boxed{\Omega^2 \to \left(\frac{\Omega + \tanh\chi}{1 + \Omega\tanh\chi}\right)^2}$$

for $\chi \to \pm\infty :$ $\Omega \to 1$

noncommutative extension

action principle

palatini-holst action in the noncommutative case

$$\boxed{S[e,\omega] = -\frac{i}{32\pi G}\int \mathrm{Tr}\left[e \wedge_\star e \wedge_\star \left(*_{\mathrm{H}}\boldsymbol{F} + \frac{1}{\beta}\boldsymbol{F}\right)\right]}$$

dynamical variables

$$\boldsymbol{e} = e^I \gamma_I + \tilde{e}^I \gamma_I \gamma_5$$

bitetrad

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$\mathfrak{gl}(2,\mathbb{C})$ *gauge connection*

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$\mathfrak{gl}(2,\mathbb{C})$ *gauge connection*

this action is invariant under the infinitesimal $\star$ -deformed gauge transformations:

$$\boldsymbol{e}\to\boldsymbol{e}+i[\boldsymbol{e},\boldsymbol{\epsilon}]_\star$$

$$\boldsymbol{\omega}\to\boldsymbol{\omega}-(\mathrm{d}\boldsymbol{\epsilon}-i[\boldsymbol{\omega},\boldsymbol{\epsilon}]_\star)$$

$$\boldsymbol{F}\to\boldsymbol{F}+i[\boldsymbol{F},\boldsymbol{\epsilon}]_\star$$

this action is also invariant w.r.t. diffeomorphisms and $\star$ -diffeomorphisms

noncommutative extension

the enlargement of the internal gauge symmetry to $\mathfrak{gl}(2,\mathbb{C})$ is the minimal choice compatible with the twist

⇒ introduction of new gravitational degrees of freedom, represented by the extra components of **e** and **ω**

⇒ a bimetric theory of gravity (since the tetrad is replaced by a bitetrad)

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we shall **not** impose that the extra degrees of freedom vanish in the commutative limit, but we shall retain all extra fields as physical and provide an interpretation in the framework of this bimetric theory

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note: this theory will naturally feature higher-order derivatives as a consequence of the twist deformation

noncommutative gravity with a self-dual connection

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decompose connection $\boldsymbol{\omega}$ into its self-dual and anti self-dual parts: $\omega = \omega_+ + \omega_-$

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decompose connection ω into its self-dual and anti self-dual parts: $\omega = \omega_+ + \omega_-$

decompose the field strength as: $F(\omega) = \quad F_+(\omega_+) + F_-(\omega_-)$

where $$F_+(\omega_+) = \mathrm{d}\omega_+ - i\omega_+\wedge_\star\omega_+$$

$$F_-(\omega_-) = \mathrm{d}\omega_- - i\omega_-\wedge_\star\omega_-$$

noncommutative gravity with a self-dual connection

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decompose connection $\boldsymbol{\omega}$ into its self-dual and anti self-dual parts: $\boldsymbol{\omega} = \boldsymbol{\omega}_+ + \boldsymbol{\omega}_-$

decompose the field strength as: $\boldsymbol{F}(\boldsymbol{\omega}) = \boldsymbol{F}_+(\boldsymbol{\omega}_+) + \boldsymbol{F}_-(\boldsymbol{\omega}_-)$

where

$$\boldsymbol{F}_+(\boldsymbol{\omega}_+) = \mathrm{d}\boldsymbol{\omega}_+ - i\boldsymbol{\omega}_+ \wedge_\star \boldsymbol{\omega}_+$$

$$\boldsymbol{F}_-(\boldsymbol{\omega}_-) = \mathrm{d}\boldsymbol{\omega}_- - i\boldsymbol{\omega}_- \wedge_\star \boldsymbol{\omega}_-$$

decompose bitetrad **e** as: $e = P_+\, \boldsymbol{u}\, P_- + P_-\, \boldsymbol{v}\, P_+$

projectors

where $\boldsymbol{u} = u^I \gamma_I \ , \quad u^I = e^I + \tilde{e}^I$

$\boldsymbol{v} = v^I \gamma_I \ , \quad v^I = e^I - \tilde{e}^I$

noncommutative gravity with a self-dual connection

$$S[\boldsymbol{e},\boldsymbol{\omega}] = -\frac{i}{32\pi G}\int \mathrm{Tr}\left[\boldsymbol{e}\wedge_\star \boldsymbol{e}\wedge_\star \left(*_{\mathrm{H}}\boldsymbol{F} + \frac{1}{\beta}\boldsymbol{F}\right)\right]$$

as for the commutative palatini-holst theory, also in the noncommutative case, the choice β = - i leads to a theory of a self-dual connection

for β = - i :

$$S[e,\boldsymbol{\omega}_+] = -\frac{i}{8\pi G}\int u^I\wedge_\star v^J\wedge_\star\left[(F_+)_{IJ} + 2i\eta_{IJ}r_+\right]$$

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similarly, for β = + i the action depends only on the anti self-dual part of the gauge connection

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let us focus on the self-dual case and obtain the equations of motion

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symmetries: deformed gauge invariance and duality

palatini-holst action in the noncommutative case

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noncommutative generalisations of the symmetries of the standard palatini theory

invariant under spacetime symmetries (diffeomorphisms, $\star$- diffeomorphisms), and deformed $\mathrm{GL}(2,\mathbb{C})$ *gauge transformations*

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  - transformation of the couplings of the model: $G \to \beta G$ $\qquad \beta \to -\dfrac{1}{\beta}$

    (effect of exchanging palatini and holst terms in the action)

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  - transformation of the couplings of the model: $G \to \beta G \qquad \beta \to -\frac{1}{\beta}$

- exchange of polar and axial components of the bitetrad: $e \to e\gamma_5$

  (action invariant up to a sign and dynamics of pure gravity theory is invariant)

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  - exchange of polar and axial components of the bitetrad: $e \to e\gamma_5$
  - **new duality symmetry that leaves the e.o.m. invariant while flipping the sign of the barbero-immirzi parameter**: $\beta \to -\beta$

self-dual theory in the commutative limit

formally obtained by letting the deformation parameter tend to zero: $\theta^{\alpha\beta} \to 0$

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- the gravitational field equation has the same form as the field equation in vacuum einstein-cartan's theory

$$G_{ab} = 0 \qquad G_{ab} := R_{ab} - \frac{1}{2} g_{ab} R$$

einstein tensor includes torsion contributions

$$R_{ab} := R_{cab}{}^{c} = \overline{R}_{ab} + \overline{\nabla}_c K^c{}_{ab} + \overline{\nabla}_a T_b - T_c K^c{}_{ab} - K^c{}_{ae} K^e{}_{cb}$$

$$R := g^{ab} R_{ab} = \overline{R} + 2\overline{\nabla}_a T^a - T^a T_a - K_{abc} K^{cab} \; .$$

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- the gravitational field equation has the same form as the field equation in vacuum einstein-cartan's theory

the extra component in the $\mathfrak{gl}(2,\mathbb{C})$ gauge connection is identified with the weyl one-form and acts as a source for torsion

$$\nabla_{[a}T^{d}{}_{bc]} - T^{e}{}_{[ab}T^{d}{}_{c]e} = (\mathrm{d}w)_{[ab}\tilde{e}^{I}_{c]}e^{d}_{I}$$

weyl vector $w = -4i\,\omega_{+}$

torsion is sourced by non-metricity, thus even in vacuo, torsion would be dynamical in general
an important departure from einstein-cartan's theory

perturbative expansion of the twist operator in the deformation parameter θ

$$\tilde{e}^I = \tilde{e}^I_{(0)} + \tilde{e}^I_{(1)} + \dots \ ,$$

$$T^I = T^I_{(0)} + T^I_{(1)} + \dots \ .$$

valid approximation at scales much larger than the noncommutativity scale

obtain correction terms to the e.o.m. of the commutative theory perturbative expansion of the twist operator in the deformation parameter θ

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$$\tilde{e}^I_{(0)} = \Omega \, e^I$$ with Ω constant and $\neq 1$

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$$T^I_{(0)} = 0$$

first order perturbative corrections to the torsion from: $e^{[I} \wedge T^{J]}_{(1)} = -\frac{1}{2}\theta^{\alpha\beta}\mathcal{L}_{X_\alpha}\omega_+ \wedge \mathcal{L}_{X_\beta}(e^I \wedge e^J)$

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<u>simple example</u>: conformally related unperturbed tetrads

$$\tilde{e}^I_{(0)} = \Omega\, e^I$$ with Ω constant and $\neq 1$

$$\Rightarrow \begin{cases} T^I_{(0)} = 0 \\ \text{first order perturbative corrections to the torsion from: } e^{[I} \wedge T^{J]}_{(1)} = -\frac{1}{2}\theta^{\alpha\beta}\mathcal{L}_{X_\alpha}\omega_+ \wedge \mathcal{L}_{X_\beta}(e^I \wedge e^J) \end{cases}$$

$\boxed{\text{torsion is sourced by spacetime noncommutativity, provided that } \omega_+ \text{ is not a constant}}$

<u>further extensions of the model</u>: bitetrad interactions

the simplest (polynomial) interaction terms in 4-dim, compatible with the symmetries of the model, and do not give rise to higher-order derivatives in the commutative limit

$$S_{\rm int}[e^I, \tilde{e}^I] = \int \left\{ \underbrace{i\, c_1 {\rm Tr}\, [e \wedge_\star e \wedge_\star e \wedge_\star e\, \gamma_5]}_{} + \underbrace{c_2 {\rm Tr}\, [e \wedge_\star e \wedge_\star e \wedge_\star e]}_{} + \underbrace{c_3 \left({\rm Tr}\, [e \wedge_\star e]\right)^2}_{} + \underbrace{c_4 \left({\rm Tr}\, [e \wedge_\star e\, \gamma_5]\right)^2}_{} \right\}$$

there are only four possible such terms that are polynomial and compatible with the gauge symmetries of the model

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there are only four possible such terms that are polynomial and compatible with the gauge symmetries of the model

commutative limit: a one-parameter bigravity model with interaction terms given by:

$$S_{\rm int}^{\theta=0} = 4\, c_1 \int \epsilon_{IJKL} \left(e^I \wedge e^J \wedge e^K \wedge e^L + \tilde{e}^I \wedge \tilde{e}^J \wedge \tilde{e}^K \wedge \tilde{e}^L - 2\, e^I \wedge e^J \wedge \tilde{e}^K \wedge \tilde{e}^L\right)$$

interaction terms typical of ghost-free bimetric theories of gravity

further extensions of the model: higher-order curvature invariants

there are only two possible monomial invariants that can be built using only the field strength and its dual

the corresponding actions are quadratic in the curvature

pontryagin action

$$S_{\mathrm{P}} = \int \mathrm{Tr}\,[\boldsymbol{F} \wedge_\star \boldsymbol{F}] = -\frac{1}{2}\int F^{IJ} \wedge_\star F_{IJ} + 4\int (r \wedge_\star r + \tilde{r} \wedge_\star \tilde{r})$$

macdowell-mansouri action

$$S_{\mathrm{MM}} = \int \mathrm{Tr}\,[\boldsymbol{F} \wedge_\star *_{\mathrm{H}} \boldsymbol{F}] = \frac{1}{2}\int F^{IJ} \wedge_\star *_{\mathrm{H}} F_{IJ} - 4\,i\int (r \wedge_\star \tilde{r} + r \wedge_\star \tilde{r})$$

conclusions

we generalized the model of *aschieri and castellani (2009*) and build a noncommutative extension of tetrad palatini-holst gravity, based on an abelian twist

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⟹ the theory is inherently bimetric

conclusions /discussion

our universe consists of various particles with different spins and masses

SM of particle physics: massive and massless fields with spin 0, ½ and 1

gravitational interactions are attributed to a spin-2 field which, within standard GR, is massless and possesses nonlinear self-interactions

SM and GR are experimentally and observationally well-tested, but unsolved problems do remain: unknown nature of dark matter and dark energy

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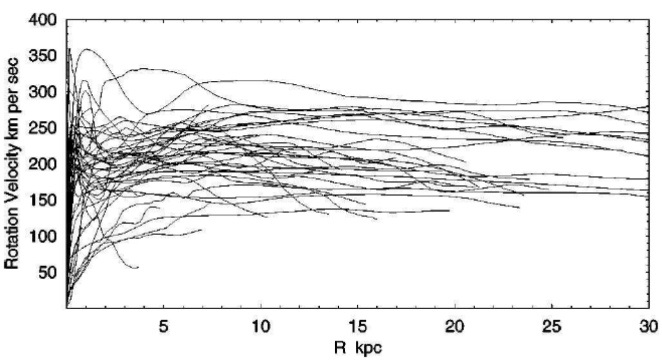

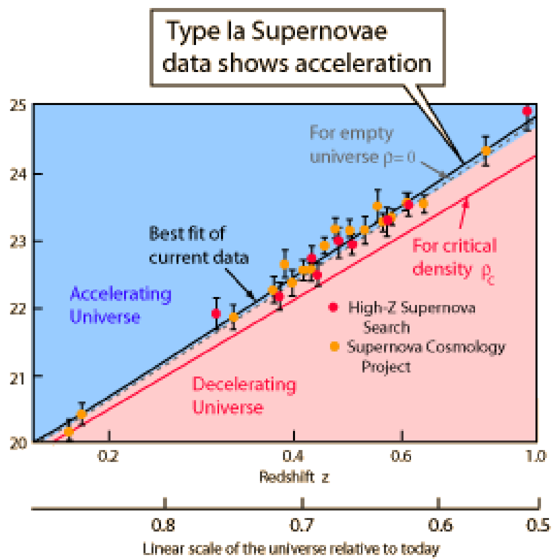

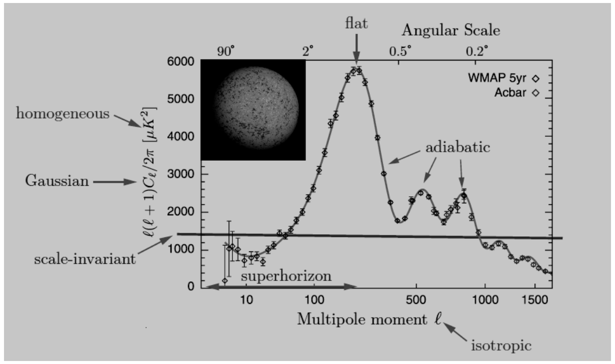

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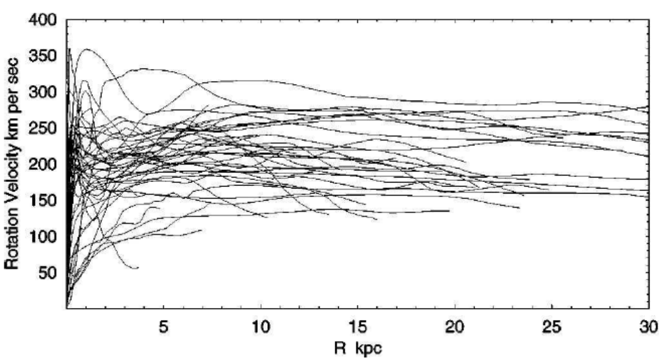

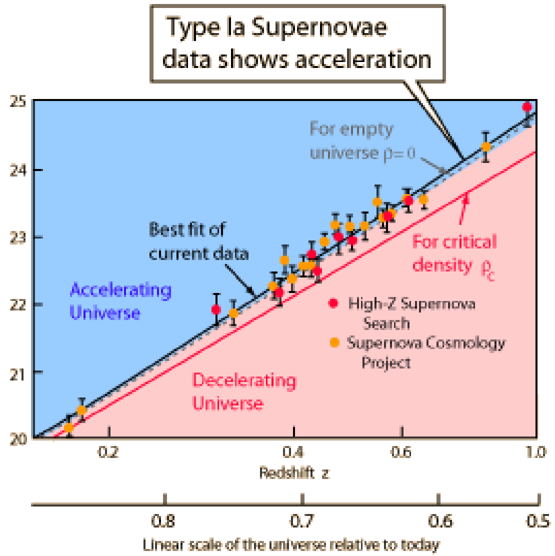

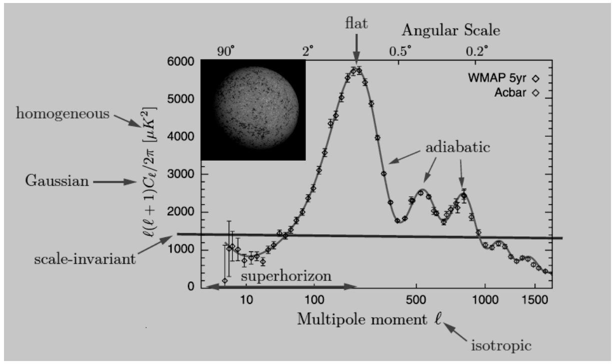

ΛCDM model: phenomenological model

conclusions /discussion

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⇒ *new physics is required*

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proposal: add a massive spin-2 field whose presence is expected to mostly affect the gravitational sector

assuming lorentz invariance, weinberg's theorem in 1964 and its extensions exclude more than one interacting massless graviton in 4dim minkowski spacetime

those theorems do not exclude massive graviton(s) interacting with a massless graviton

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modifying gravity is motivated by the fact that the SM of particle physics is based on the very solid framework of renormalisable QFT
however, a theory of QG does not yet exist (hence GR is yet not complete)

moreover, **both DE and DM problems are intimately related to gravity but cannot be solved in the context of GR without raising additional questions**

conclusions /discussion

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a bimetric theory of gravity is a physical setup involving two dynamical metrics interacting with each other

in such a setup, after diagonalising the mass matrix for metric perturbations around minkowski background, one finds a massive graviton and a massless graviton

conclusions /discussion

bimetric theory of massive gravity:

$g_{\mu\nu}$: physical metric (the metric based on which all usual distances are defined)

$f_{\mu\nu}$: a dynamical rank-two tensor; this metric coupled to the physical one, gives mass to gravitons

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$$\boxed{\begin{aligned} S = & -\frac{M_g^2}{2}\int d^4x\sqrt{-\det g}R(g) - \frac{M_f^2}{2}\int d^4x\sqrt{-\det f}R(f) \\ & + m^2 M_g^2 \int d^4x\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right) + \int d^4x\sqrt{-\det g}\mathcal{L}_m\left(g,\Phi\right) \end{aligned}}$$

the graviton mass m and the five quantities β's are free parameters to be determined observationally

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the graviton mass m and the five quantities β's are free parameters to be determined observationally

$\Downarrow$

derive equations of motion to determine the time-evolution of the universe (if properties of matter and energy are known)

+ constraints from Bianchi identities + constraint from conservation of $T_{\mu\nu}$

conclusions /discussion

generic theories of massive gravity and bimetric gravity contain *a boulware-deser ghost mode* which renders these theories unstable

→ two constraints are needed to eliminate the two phase-space degrees of freedom of the *boulware-deser ghost*

in non-linear massive gravity and bimetric gravity theories, there exist a hamiltonian constraint and a secondary constraint rendering the boulware-deser ghost absent

hassan, rosen (2011)

↓

a ghost-free theory for nonlinear interactions between massive and massless spin-2 fields has been found

conclusions /discussion

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hassan, rosen (2011)

a ghost-free theory for nonlinear interactions between massive and massless spin-2 fields has been found

bimetric theory is a candidate to explain the late accelerated expansion of the universe

bimetric gravity has been shown to possess FLRW solutions which can match observations of the cosmic expansion history, even in the absence of vacuum energy

conclusions

we generalized the model of *aschieri and castellani (2009*) and build a noncommutative extension of tetrad palatini-holst gravity, based on an abelian twist

the noncommutative deformation leads to the enlargement of the internal gauge symmetry of the model, which is extended from the lorentz group to $\mathrm{GL}(2, \mathbb{C})$

consistency requirements demand that the metric degrees of freedom of the theory must be also augmented, thus replacing the tetrad by a bitetrad

$\Rightarrow$ the theory is inherently "bimetric"

remark*:*

in a bimetric theory, the two metrics are dynamical and come with their own equations of motion

in the model we have been discussing, there is however only ***one spin connection*** *and* ***invariance is only under one copy of the gauge group****; the degrees of freedom of the second metric are not dynamic*

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$\Rightarrow$ the theory is inherently "bimetric"

our standpoint: *the extra d.o.f. required by the noncommutative extension are physical*

$\Rightarrow$ a modified theory of gravity entailing both higher-order derivatives and new gravitational degrees of freedom is obtained, which naturally encodes modifications of spacetime structure at the planck scale


hvala