Open-string T-duality and applications to non-geometric backgrounds

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This talk is based on work together with F. Cordonier-Tello and D. Lüst ::

Open-string T-duality and applications to non-geometric backgrounds [arXiv:1806.01308]

Dualities are an integral part of string theory.

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S-duality

U-duality

T-duality

AdS/CFT correspondence

open-/closed-string duality

type IIA/M-theory duality

mirror symmetry

heterotic/type IIA duality

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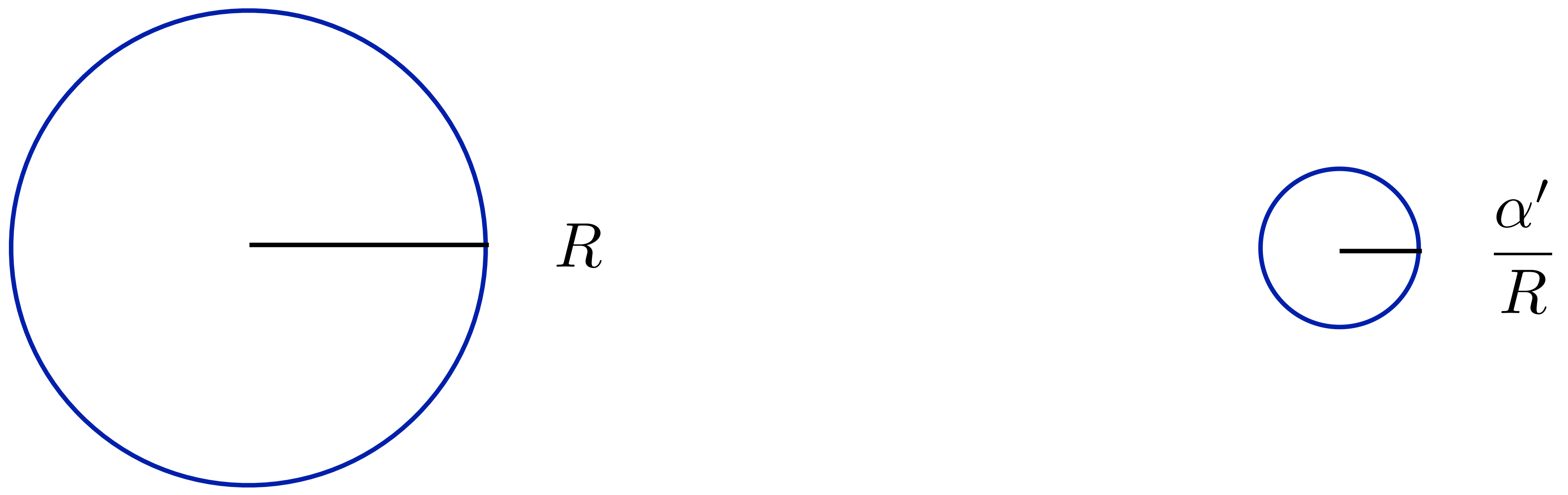
T-duality ::

- String-theory compactified on two T-dual circles cannot be distinguished.



- For flat backgrounds, CFT techniques give exact duality transformations.
- For curved backgrounds, one employs Buscher's procedure.

The T-dual background can be obtained by following Buscher's procedure ::

1) Identify a global symmetry (isometry) of the world-sheet action.
2) Gauge the global symmetry by introducing a gauge field.
3) Integrate-out the gauge field.

The resulting transformation rules

- agree with the CFT expressions for flat backgrounds,
- but apply also to curved backgrounds.

This talk ::

1) Discuss Buscher's procedure for open strings (including technical details).

Alvarez, Barbon, Borlaf - 1996
Dorn, Otto - 1996
Förste, Kehagias, Schwager - 1996
Albertsson, Lindström, Zabzine - 2004

2) Apply results to D-branes on non-geometric backgrounds.

outline

1. introduction
2. t-duality — closed string
3. t-duality — open string
4. d-branes & non-geometry
5. summary

outline

The world-sheet action for the closed string takes the form (Σ is a 2d manifold with $\partial\Sigma = \varnothing$)

$$\mathcal{S} = -\frac{1}{4\pi\alpha'}\int_\Sigma \Big[G_{ij}\, dX^i \wedge \star dX^j - i\, B_{ij}\, dX^i \wedge dX^j \Big] .$$

For Buscher's procedure, one assumes that $\mathcal{S}$ is invariant under a global transformation

$$\delta_\epsilon X^i = \epsilon\, k^i(X)\,, \qquad\qquad \epsilon = \text{const.} \ll 1\,.$$

The variation of the action vanishes provided that (for v a globally-defined one-form on Σ)

$$\mathcal{L}_k\, G = 0\,, \qquad\qquad \mathcal{L}_k\, B = dv\,.$$

The global symmetry can be gauged by introducing a gauge field A (and a Lagrange multiplier χ) as

$$\hat{\mathcal{S}} = -\frac{1}{2\pi\alpha'}\int_\Sigma \Big[\tfrac{1}{2}G_{ij}(dX^i + k^i A)\wedge\star(dX^j + k^j A) - \tfrac{i}{2}B_{ij}\,dX^i\wedge dX^j - i\,(v - \iota_k B + d\chi)\wedge A\Big]\,.$$

The local symmetry transformations take the form

$$\hat{\delta}_\epsilon X^i = \epsilon\, k^i\,, \qquad \hat{\delta}_\epsilon A = -d\epsilon\,, \qquad \hat{\delta}_\epsilon \chi = -\epsilon\,\iota_k v\,.$$

For multiple gauged (non-abelian) symmetries, additional restrictions apply.

The original action is obtained using the Lagrange multiplier. First, perform a Hodge decomposition as

$$d\chi = d\chi_{(0)} + \chi_{(\mathsf{m})}\,\omega^{\mathsf{m}}$$

- with $\chi_{(0)}$ a globally-defined function on Σ ,
- $\chi_{(\mathsf{m})} \in \mathbb{R}$ with $\mathsf{m} = 1,\ldots,2g$,
- $\omega^{\mathsf{m}} \in H^1(\Sigma,\mathbb{R})$ a basis with $\int_\Sigma \omega^{\mathsf{m}} \wedge \omega^{\mathsf{n}} = J^{\mathsf{mn}} \in GL(2g,\mathbb{Z})$.

The equation of motion for $\chi_{(0)}$ leads to

$$\delta_{\chi_{(0)}}\hat{\mathcal{S}} = -\frac{i}{2\pi\alpha'}\int_\Sigma \delta\chi_{(0)}\, dA \overset{!}{=} 0 \qquad \longrightarrow \qquad F = dA = 0\,,$$

$$\longrightarrow \qquad A = da_{(0)} + a_{(\mathsf{m})}\,\omega^{\mathsf{m}}\,.$$

Using the gauge symmetry the exact part of the gauge field can be set to $a_{(0)} = 0$.

The equations of motion for $\chi_{(\mathsf{m})}$ are determined as

$$\delta_{\chi_{(\mathsf{m})}} \hat{\mathcal{S}} = \frac{i}{2\pi\alpha'} \delta\chi_{(\mathsf{m})} J^{\mathsf{mn}} a_{(\mathsf{n})} \stackrel{!}{=} 0 \qquad \longrightarrow \qquad a_{(\mathsf{m})} = 0\,.$$

Summary :: using χ, one can set $A = 0$ and the original action is recovered.

Integrating-out the gauge field (with $k^i = (1, 0, \ldots, 0)$ and $v = 0$) gives the dual action as

$$\check{\mathcal{S}} = -\frac{1}{2\pi\alpha'} \int_\Sigma \Bigg[\;\; \frac{1}{2}\left(G_{mn} - \frac{G_{m1}G_{n1} - B_{m1}B_{n1}}{G_{11}}\right) dX^m \wedge \star dX^n + \frac{1}{2}\frac{\alpha'^2}{G_{11}}\, d\tilde{X}^1 \wedge \star d\tilde{X}^1 \pm \alpha' \frac{B_{m1}}{G_{11}}\, d\tilde{X}^1 \wedge \star dX^m$$
$$- \frac{i}{2}\left(B_{mn} - \frac{B_{m1}G_{n1} - G_{m1}B_{n1}}{G_{11}}\right) dX^m \wedge dX^n \mp i\,\alpha' \frac{G_{m1}}{G_{11}}\, dX^m \wedge d\tilde{X}^1 \mp i\,\alpha'\, dX^1 \wedge d\tilde{X}^1 \Bigg] .$$

Interpreting $d\tilde{X}^1 = \pm\frac{1}{\alpha'}\, d\chi$ as the dual coordinate, the Buscher rules can be read-off

$$\check{G}_{11} = \frac{\alpha'^2}{G_{11}} ,$$

$$\check{G}_{m1} = \pm\alpha' \frac{B_{m1}}{G_{11}} , \qquad \check{B}_{m1} = \pm\alpha' \frac{G_{m1}}{G_{11}} ,$$

$$\check{G}_{mn} = G_{mn} - \frac{G_{m1}G_{n1} - B_{m1}B_{n1}}{G_{11}} , \qquad \check{B}_{mn} = B_{mn} - \frac{B_{m1}G_{n1} - G_{m1}B_{n1}}{G_{11}} .$$

Buscher - 1987, 1988

Integrating-out the gauge field (with $k^i = (1,0,\ldots,0)$ and $v=0$) gives the dual action as

$$\check{\mathcal{S}} = -\frac{1}{2\pi\alpha'}\int_\Sigma \Bigg[\;\frac{1}{2}\left(G_{mn} - \frac{G_{m1}G_{n1} - B_{m1}B_{n1}}{G_{11}}\right) dX^m \wedge \star dX^n + \frac{1}{2}\frac{\alpha'^2}{G_{11}}\, d\tilde{X}^1 \wedge \star d\tilde{X}^1 \pm \alpha' \frac{B_{m1}}{G_{11}}\, d\tilde{X}^1 \wedge \star dX^m$$
$$- \frac{i}{2}\left(B_{mn} - \frac{B_{m1}G_{n1} - G_{m1}B_{n1}}{G_{11}}\right) dX^m \wedge dX^n \mp i\,\alpha' \frac{G_{m1}}{G_{11}}\, dX^m \wedge d\tilde{X}^1 \boxed{\mp i\,\alpha'\, dX^1 \wedge d\tilde{X}^1} \;\Bigg].$$

Interpreting $d\tilde{X}^1 = \pm\frac{1}{\alpha'}\, d\chi$ as the dual coordinate, the Buscher rules can be read-off

$$\check{G}_{11} = \frac{\alpha'^2}{G_{11}}\,,$$

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$$\check{G}_{mn} = G_{mn} - \frac{G_{m1}G_{n1} - B_{m1}B_{n1}}{G_{11}}\,, \qquad \check{B}_{mn} = B_{mn} - \frac{B_{m1}G_{n1} - G_{m1}B_{n1}}{G_{11}}\,.$$

Buscher - 1987, 1988

For the original coordinate X^1 we perform the

- Hodge decomposition $dX^1 = dX^1_{(0)} + X^1_{(\mathsf{m})}\,\omega^{\mathsf{m}}$.
- If X^1 is compact and free, then $X^1_{(\mathsf{m})} \in 2\pi\mathbb{Z}$ contain the momentum/winding numbers.

The path integral over X^1 takes the form

$$
\begin{aligned}
&\int \frac{[\mathsf{D}X^1]}{\mathcal{V}_{\rm gauge}}\,\exp\left(\frac{i}{2\pi\alpha'}\int_\Sigma dX^1\wedge d\chi\right)\\
&= \int \frac{[\mathsf{D}X^1_{(0)}]}{\mathcal{V}_{\rm gauge}} \sum_{X^1_{(\mathsf{m})}\in 2\pi\mathbb{Z}} \exp\left(\frac{i}{2\pi\alpha'}\int_\Sigma X^1_{(\mathsf{m})}\,\omega^{\mathsf{m}}\wedge\chi_{(\mathsf{n})}\,\omega^{\mathsf{n}}\right)\\
&= \sum_{k^{(\mathsf{m})}\in\mathbb{Z}} \delta\left(\frac{1}{2\pi\alpha'}J^{\mathsf{mn}}\chi_{(\mathsf{n})} - k^{(\mathsf{m})}\right) \qquad \longrightarrow \qquad \chi_{(\mathsf{m})} \in 2\pi\alpha'\,\mathbb{Z}\,.
\end{aligned}
$$

Rocek, Verlinde - 1991

The dual coordinate $\tilde{X}^1 = \pm\frac{1}{\alpha'}\,\chi$ is therefore compact (and free).

Summary ::

- Buscher's approach to T-duality transformations has been reviewed.
- A Lagrange multiplier allows to recover the original model.
- The dual model is obtained via integrating-out the gauge field.

- Non-trivial world-sheet topologies have been taken into account.

outline

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2. t-duality — closed string
3. t-duality — open string
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3. **t-duality — open string**
    a) various actions
    b) neumann
    c) dirichlet
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The world-sheet action for the open string takes the form (Σ is a 2d manifold with $\partial\Sigma \neq \varnothing$)

$$\mathcal{S} = -\frac{1}{2\pi\alpha'}\int_{\Sigma}\left[\frac{1}{2}G_{ij}\,dX^i\wedge\star dX^j + \frac{i}{2}B_{ij}\,dX^i\wedge dX^j\right] -\frac{1}{2\pi\alpha'}\int_{\partial\Sigma}\left[2\pi i\,\alpha'\,a_i\,dX^i\right].$$

The possible boundary conditions for X^i are

Dirichlet $\quad 0 = \left(dX^{\hat{i}}\right)_{\mathsf{tan}}\,,$

Neumann $\quad 0 = G_{ai}\left(dX^i\right)_{\mathsf{norm}} + 2\pi\alpha' i\,\mathcal{F}_{ab}\left(dX^b\right)_{\mathsf{tan}}\,,$

$$\left(dX^i\right)_{\mathsf{tan}} \equiv t^{\mathsf{a}}\,\partial_{\mathsf{a}}X^i\,ds\big|_{\partial\Sigma}\,,$$

$$\left(dX^i\right)_{\mathsf{norm}} \equiv n^{\mathsf{a}}\,\partial_{\mathsf{a}}X^i\,ds\big|_{\partial\Sigma}\,,$$

$$2\pi\alpha'\mathcal{F} = 2\pi\alpha' F + B\,,$$

$$F = da\,.$$

The Hodge decomposition theorem for manifolds with boundaries can be expressed using

- closed forms $$C^p = \{\omega \in \Omega^p : d\omega = 0\}\,,$$
- exact forms $$E^p = \{\omega \in \Omega^p : \omega = d\eta,\, \eta \in \Omega^{p-1}\}\,,$$
- closed & co-closed, vanishing normal part $$CcC^p_N = \{\omega \in \Omega^p : d\omega = 0, d^\dagger\omega = 0, \omega_{\text{norm}} = 0\}\,.$$

For closed forms on then finds $C^p = E^p \oplus CcC^p_N$.

e.g. Capell, DeTurck, Gluck, Miller - 2005

This implies for Dirichlet directions $X^{\hat{i}}$ that $dX^{\hat{i}}$ is exact.

For Buscher's procedure, one assumes that the action is invariant under a global transformation

$$\delta_\epsilon X^i = \epsilon\, k^i(X)\,, \qquad\qquad \epsilon = \text{const.} \ll 1\,.$$

The variation of the action vanishes provided that

$$\mathcal{L}_k G = 0\,,$$

$$\mathcal{L}_k B = dv\,,$$

$$2\pi\alpha' \mathcal{L}_k a\big|_{\partial\Sigma} = (-v + d\omega)\big|_{\partial\Sigma}\,.$$

v globally-defined one-form on Σ,

ω globally-defined function on $\partial\Sigma$,

The global transformation is not compatible with Dirichlet boundary condition $\delta X^{\hat{i}}\big|_{\partial\Sigma} = 0$.

The global symmetry can be gauged by introducing a gauge field A (and a Lagrange multiplier χ)

$$\hat{\mathcal{S}} = -\frac{1}{2\pi\alpha'}\int_\Sigma \Big[\tfrac{1}{2}G_{ij}(dX^i + k^i A)\wedge\star(dX^j + k^j A) \\ - \tfrac{i}{2}B_{ij}\,dX^i\wedge dX^j - i\,(v - \iota_k B + d\chi)\wedge A\Big] \\ -\frac{1}{2\pi\alpha'}\int_{\partial\Sigma}\Big[2\pi i\,\alpha'\,a_a\,dX^a - i\,\Omega_{\partial\Sigma}\Big]\,.$$

The local symmetry transformations take the form

$$\hat{\delta}_\epsilon X^i = \epsilon\, k^i\,, \qquad \hat{\delta}_\epsilon A = -d\epsilon\,, \qquad \hat{\delta}_\epsilon \chi = -\epsilon\,\iota_k v\,.$$

The possible boundary conditions for the gauge field are

Dirichlet $\quad 0 = A_{\mathsf{tan}}\big|_{\partial\Sigma}\,,$

Neumann $\quad 0 = G_{ai}\,k^i A_{\mathsf{norm}} + 2\pi\alpha' i\,\mathcal{F}_{ab}\,k^b A_{\mathsf{tan}}\big|_{\partial\Sigma}\,.$

Albertsson, Lindström, Zabzine - 2004

For Dirichlet boundary conditions the variation parameter satisfies $\epsilon|_{\partial\Sigma} = 0$ and one finds $\Omega_{\partial\Sigma} = 0$.

For Neumann boundary conditions a second Lagrange multiplier is needed and

$$\Omega_{\partial\Sigma} = \big(\chi + \phi + \omega - 2\pi\,\alpha'\iota_k a\big)\,A\,,$$

χ globally-defined function on $\partial\Sigma$,

ϕ constant function on $\partial\Sigma$.

For Dirichlet boundary conditions

- equation of motion for χ $\qquad F = dA = 0\,,$
- boundary condition $\qquad 0 = A_{\mathsf{tan}}\big|_{\partial\Sigma}\,.$

For Neumann boundary conditions

- equation of motion for χ $\qquad F = dA = 0\,,$
- equation of motion for ϕ $\qquad 0 = A_{\mathsf{tan}}\big|_{\partial\Sigma}\,.$

Using Hodge decomposition for manifolds with boundary the original action is recovered via ($\omega^{\mathsf{m}} \in CcC^1_N$)

$$\xrightarrow{dA=0} \quad A = da_{(0)} + a_{(\mathsf{m})}\,\omega^{\mathsf{m}} \quad \xrightarrow{A_{\mathsf{tan}}=0} \quad a_{(\mathsf{m})} = 0$$

$$\xrightarrow{\hat{\delta}_\epsilon A} \quad a_{(0)} = 0$$

Integrating-out the gauge field (with $k^i = (1, 0, \ldots, 0)$) gives the action

$$\check{\mathcal{S}} = -\frac{1}{2\pi\alpha'}\int_\Sigma \Bigg[\;\; \frac{1}{2}\left(G_{mn} - \frac{G_{m1}G_{n1} - \tilde{B}_{m1}\tilde{B}_{n1}}{G_{11}}\right) dX^m \wedge \star dX^n + \frac{1}{2}\frac{1}{G_{11}}\,d\chi \wedge \star d\chi + \frac{\tilde{B}_{m1}}{G_{11}}\,d\chi \wedge \star dX^m$$

$$- \frac{i}{2}\left(B_{mn} - \frac{\tilde{B}_{m1}G_{n1} - G_{m1}\tilde{B}_{n1}}{G_{11}}\right) dX^m \wedge dX^n - i\,\frac{G_{m1}}{G_{11}}\,dX^m \wedge d\chi + i\,dX^1 \wedge (d\chi + v) \;\; \Bigg]$$

$$- \frac{1}{2\pi\alpha'}\int_{\partial\Sigma}\Big[2\pi i\,\alpha'\,a_a\,dX^a\Big]\,,$$

with components $\tilde{B}_{m1} = B_{m1} - v_m$.

The variation on the boundary introduces a constraint

Dirichlet	$\varnothing\,,$
Neumann	$0 = 2\pi\alpha'\,\iota_k a - (\chi + \phi + \omega)\Big\rvert_{\partial\Sigma}\,.$

outline

1) Integrating-out the gauge field ::

- The boundary constraint can be implemented in the path integral through

$$\delta(\phi - \tilde{\chi})_{\partial\Sigma}\,, \qquad\qquad \tilde{\chi} = \chi + \omega - 2\pi\alpha'\iota_k a\,.$$

- The Neumann boundary condition for A becomes $0 = d\tilde{\chi}\big|_{\partial\Sigma}$.

2) Integrating-out the Lagrange multiplier ϕ ::

- The path-integral takes the form

$$\mathcal{Z} = \int \frac{[\mathcal{D}X^i]\,[\mathcal{D}\chi]}{\mathcal{V}_{\text{gauge}}} \int [\mathcal{D}\phi]\,\delta(\phi - \tilde{\chi})_{\partial\Sigma}\,\exp\Big(\check{\mathcal{S}}[X^i,\chi]\Big)\,,$$

- and integration over ϕ is trivially performed.

3) Integrating-out the original coordinate ::

- The relevant terms in the action read (with $k^i = (1,0,\ldots,0)$)

$$+\frac{i}{2\pi\alpha'}\int_\Sigma (d\chi + v)\wedge dX^1 - \frac{i}{2\pi\alpha'}\int_{\partial\Sigma} 2\pi\alpha' a_1 dX^1 = +\frac{i}{2\pi\alpha'}\int_{\partial\Sigma} \tilde\chi\, dX^1 \,.$$

- Expand $dX^1 = dX^1_{(0)} + X^1_{(\mathsf{m})}\,\omega^{\mathsf{m}}$. For X^1 compact and free $X^1_{(\mathsf{m})} \in 2\pi\mathbb{Z}$, and

$$\int \frac{[\mathcal{D}X^1]}{\mathcal{V}_{\text{gauge}}} \exp\left[\frac{i}{2\pi\alpha'}\int_{\partial\Sigma}\tilde\chi\, dX^1\right]$$

$$= \int \frac{[\mathcal{D}X^1_{(0)}]}{\mathcal{V}_{\text{gauge}}} \sum_{X^1_{(\mathsf{m})}\in 2\pi\mathbb{Z}} \exp\left[\frac{i}{2\pi\alpha'}\int_{\partial\Sigma}\tilde\chi\, X^1_{(\mathsf{m})}\,\omega^{(\mathsf{m})}\right]$$

$$= \sum_{m_{(\mathsf{m})}\in\mathbb{Z}} \delta\left[\frac{1}{2\pi\alpha'}\tilde\chi - m_{(\mathsf{m})}\right]_{\partial\Sigma} \longrightarrow \qquad \tilde\chi\big|_{\partial\Sigma} \in 2\pi\alpha'\,\mathbb{Z}\,.$$

→ The dual coordinate $\tilde X^1 = \pm\frac{1}{\alpha'}\tilde\chi$ is quantized on the boundary and thus compact.

Summary ::

- T-duality along a Neumann direction results in a T-dual Dirichlet direction.
- A Wilson loop along X^1 shifts the dual coordinate as $\tilde{X}^1 = \pm\frac{1}{\alpha'}(\chi + \omega - 2\pi\alpha' a_1)$.
- Momentum modes of X^1 determine winding modes via $\tilde{X}^1|_{\partial\Sigma} \in 2\pi\mathbb{Z}$.

- The dual metric and *B*-field can be identified as (contain open-string gauge flux)

$$
\begin{aligned}
\check{G}_{11} &= \frac{\alpha'^2}{G_{11}}\,, \\
\check{G}_{m1} &= \pm\alpha'\,\frac{\tilde{B}_{m1}}{G_{11}}\,, & \check{B}_{m1} &= \pm\alpha'\,\frac{G_{m1}}{G_{11}}\,, \\
\check{G}_{mn} &= G_{mn} - \frac{G_{m1}G_{n1} - \tilde{B}_{m1}\tilde{B}_{n1}}{G_{11}}\,, & \check{B}_{mn} &= B_{mn} - \frac{\tilde{B}_{m1}G_{n1} - G_{m1}\tilde{B}_{n1}}{G_{11}}\,.
\end{aligned}
$$

- The dual gauge field reads $\check{a} = a_m dX^m$.

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1) Integrating-out the gauge field ::

- The Dirichlet condition for A becomes a Neumann condition for $\tilde{X}^1 = \pm\frac{1}{\alpha'}\chi$

$$0 = \check{G}_{1i}\big(d\tilde{X}^i\big)_{\text{norm}} + i\,\check{B}_{1i}\big(d\tilde{X}^i\big)_{\text{tan}}\,.$$

- No boundary constraint to be imposed.

2) Integrating-out the Lagrange multiplier ϕ :: not present.

3) Integrating-out the original coordinate ::

- The relevant term in the action reads (with $k^i = (1,0,\ldots,0)$ and $v=0$)

$$-\frac{i}{2\pi\alpha'}\int_\Sigma dX^1\wedge d\chi = -\frac{i}{2\pi\alpha'}\int_{\partial\Sigma} 2\pi\alpha'\left[\frac{X^1|_{\partial\Sigma}}{2\pi\alpha'}\,d\chi\right].$$

- Expand $d\chi = d\chi_{(0)} + \chi_{(\mathsf{m})}\,\omega^{\mathsf{m}}$, and for X^1 compact perform the path-integral

$$\int\frac{[\mathcal{D}X^1]}{\mathcal{V}_{\text{gauge}}}\exp\left[-\frac{i}{2\pi\alpha'}\int_{\partial\Sigma}X^1|_{\partial\Sigma}\,d\chi\right]$$

$$=\int\frac{[\mathcal{D}X^1_0]}{\mathcal{V}_{\text{gauge}}}\sum_{n_{\partial\Sigma}\in\mathbb{Z}}\exp\left[-\frac{i}{2\pi\alpha'}\int_{\partial\Sigma}\left(X^1_0|_{\partial\Sigma}+2\pi\,n_{\partial\Sigma}\right)d\chi\right]$$

$$=\sum_{m_{(\mathsf{m})}\in\mathbb{Z}}\delta\left[\frac{1}{2\pi\alpha'}\,\chi_{(\mathsf{m})}-m_{(\mathsf{m})}\right]\exp\left[-\frac{i}{2\pi\alpha'}\int_{\partial\Sigma}2\pi\alpha'\,\frac{X^1_0|_{\partial\Sigma}}{2\pi}\,\frac{d\chi}{\alpha'}\right] \longrightarrow \chi_{(\mathsf{m})}\in 2\pi\alpha'\mathbb{Z}\,.$$

→ Wilson loop and quantized momenta for the dual coordinate $\tilde{X}^1 = \pm\frac{1}{\alpha'}\chi$.

Summary ::

- T-duality along a Dirichlet direction results in a T-dual Neumann direction.
- The position of $X^1|_{\partial\Sigma}$ determines a Wilson loop for $\tilde{X}^1$.
- Winding modes of X^1 determine momentum modes of $\tilde{X}^1$.
- The dual metric and *B*-field can be identified as

$$
\begin{aligned}
\check{G}_{11} &= \frac{\alpha'^2}{G_{11}}\,, \\
\check{G}_{m1} &= \pm\alpha'\,\frac{B_{m1}}{G_{11}}\,, & \check{B}_{m1} &= \pm\alpha'\,\frac{G_{m1}}{G_{11}}\,, \\
\check{G}_{mn} &= G_{mn} - \frac{G_{m1}G_{n1} - B_{m1}B_{n1}}{G_{11}}\,, & \check{B}_{mn} &= B_{mn} - \frac{B_{m1}G_{n1} - G_{m1}B_{n1}}{G_{11}}\,.
\end{aligned}
$$

- The dual gauge field reads $\check{a} = \frac{X^1_0|_{\partial\Sigma}}{2\pi}\,d\tilde{X}^1 + a_m dX^m$.

Summary ::

Neumann boundary conditions	T-duality $\longleftrightarrow$	Dirichlet boundary conditions
▪ momentum modes		▪ winding modes
▪ Wilson loop		▪ D-brane position

Here ::

- CFT results are reproduced for curved backgrounds.
- T-duality along Dirichlet directions.
- Inclusion of non-trivial world-sheet topologies.

outline

The duality group for toroidal compactifications is $O(D,D;\mathbb{Z})$.

The duality group for toroidal compactifications is $O(D,D;\mathbb{Z})$ — which is generated by ::

- A-transformations ($\mathsf{A} \in GL(D,\mathbb{Z})$)

$$\mathcal{O}_{\mathsf{A}} = \begin{pmatrix} \mathsf{A}^{-1} & 0 \\ 0 & \mathsf{A}^T \end{pmatrix} \longrightarrow \text{diffeomorphisms}$$

- B-transformations (B_{ij} an anti-symmetric matrix)

$$\mathcal{O}_{\mathsf{B}} = \begin{pmatrix} \mathbb{1} & 0 \\ \mathsf{B} & \mathbb{1} \end{pmatrix} \longrightarrow \text{gauge transformations } b \to b + \alpha' \mathsf{B}$$

- β-transformations (β^{ij} an anti-symmetric matrix)

$$\mathcal{O}_{\beta} = \begin{pmatrix} \mathbb{1} & \beta \\ 0 & \mathbb{1} \end{pmatrix}$$

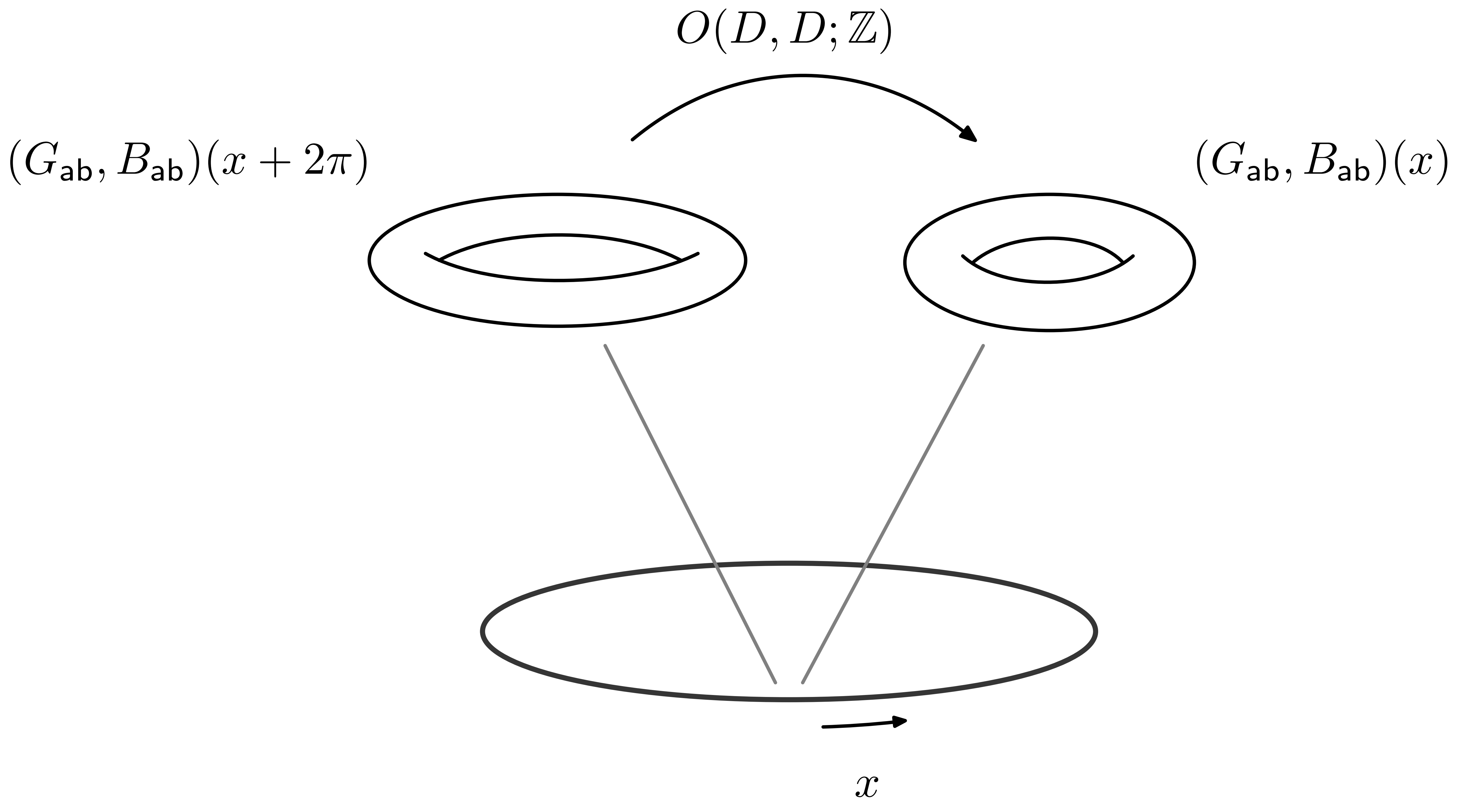
- factorized duality (E_{i} with only non-zero $E_{\mathsf{ii}} = 1$)

$$\mathcal{O}_{\pm \mathsf{i}} = \begin{pmatrix} \mathbb{1} - E_{\mathsf{i}} & \pm E_{\mathsf{i}} \\ \pm E_{\mathsf{i}} & \mathbb{1} - E_{\mathsf{i}} \end{pmatrix} \longrightarrow \text{T-duality transformations } g_{\mathsf{ii}} \to \frac{\alpha'^2}{g_{\mathsf{ii}}}$$

(Non-geometric) torus fibrations can be specified by

$$G_{ij} = \begin{pmatrix} G_{\mathsf{ab}}(x) & 0 \\ 0 & R_3^2 \end{pmatrix}$$

$$B_{ij} = \begin{pmatrix} B_{\mathsf{ab}}(x) & 0 \\ 0 & 0 \end{pmatrix}$$



The standard example for a non-geometric background is a $\mathbb{T}^2$-fibration ::

$\mathbb{T}^3$ with H-flux

- patch with B-transform
- non-triviality via H-flux

$\xleftrightarrow{\mathcal{O}_E}$

twisted $\mathbb{T}^3$

- patch with A-transform
- non-triviality via F-flux

$\xleftrightarrow{\mathcal{O}_E}$

T-fold

- patch with β-transform
- non-triviality via Q-flux

$\mathcal{O}_{\mathsf{B}}$ $\qquad$ $\mathcal{O}_{\mathsf{A}}$ $\qquad$ $\mathcal{O}_{\beta}$

The open-string boundary conditions can be expressed using (restriction to $\partial\Sigma$ is understood)

$$\begin{pmatrix} \mathsf{D} \\ \mathsf{N} \end{pmatrix} = \begin{pmatrix} \alpha' & 0 \\ 2\pi\alpha'\mathcal{F} & G \end{pmatrix} \begin{pmatrix} i\,(dX)_{\mathsf{tan}} \\ (dX)_{\mathsf{norm}} \end{pmatrix} .$$

A particular type of D-brane is selected using a projection operator

$$\Pi = \begin{pmatrix} \Delta & 0 \\ 0 & \mathbb{1} - \Delta \end{pmatrix} , \qquad\qquad \Delta^2 = \Delta \, .$$

Question :: are D-branes globally well-defined on non-geometric backgrounds?

The coordinate differentials behave under transformations $\mathcal{O} \in O(D,D;\mathbb{Z})$ as

$$\begin{pmatrix} i\,(dX)_{\mathsf{tan}} \\ (dX)_{\mathsf{norm}} \end{pmatrix} \xrightarrow{\quad\mathcal{O}\quad} \begin{pmatrix} i\,(d\tilde{X})_{\mathsf{tan}} \\ (d\tilde{X})_{\mathsf{norm}} \end{pmatrix} = \Omega \begin{pmatrix} i\,(dX)_{\mathsf{tan}} \\ (dX)_{\mathsf{norm}} \end{pmatrix},$$

where

$\mathbb{T}^3$ with H-flux
$$\Omega_{\mathsf{B}} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix},$$

twisted $\mathbb{T}^3$
$$\Omega_{\mathsf{A}} = \begin{pmatrix} \mathsf{A}^{-1} & 0 \\ 0 & \mathsf{A}^{-1} \end{pmatrix},$$

T-fold
$$\Omega_{\beta} = \begin{pmatrix} \mathbb{1} + 2\pi\,\beta\,\mathcal{F} & \frac{1}{\alpha'}\,\beta\,G \\ \frac{1}{\alpha'}\,\beta\,G & \mathbb{1} + 2\pi\,\beta\,\mathcal{F} \end{pmatrix}.$$

Remark :: for the T-fold, tangential and normal part are mixed.

Boundary conditions for previous examples are well-defined using $O(D,D;\mathbb{Z})$ transformations

$$\begin{aligned}
\begin{pmatrix} \mathsf{D} \\ \mathsf{N} \end{pmatrix}_{x+2\pi} &= \begin{pmatrix} \alpha' & 0 \\ 2\pi\alpha'\mathcal{F} & G \end{pmatrix}_{x+2\pi} \begin{pmatrix} i\,(d\tilde{X})_{\mathsf{tan}} \\ (d\tilde{X})_{\mathsf{norm}} \end{pmatrix} \\
&= \mathcal{O}_\star \begin{pmatrix} \alpha' & 0 \\ 2\pi\alpha'\mathcal{F} & G \end{pmatrix}_{x} \Omega_\star^{-1} \begin{pmatrix} i\,(d\tilde{X})_{\mathsf{tan}} \\ (d\tilde{X})_{\mathsf{norm}} \end{pmatrix} \\
&= \mathcal{O}_\star \begin{pmatrix} \mathsf{D} \\ \mathsf{N} \end{pmatrix}_{x} , \qquad\qquad \star = (\mathsf{B}, \mathsf{A}, \beta)\,.
\end{aligned}$$

The projection onto a particular D-brane has to be performed after the transformation

$$\Pi\left[\begin{pmatrix} \mathsf{D} \\ \mathsf{N} \end{pmatrix}_{x+2\pi}\right] = \Pi\left[\mathcal{O}_\star \begin{pmatrix} \mathsf{D} \\ \mathsf{N} \end{pmatrix}_{x}\right].$$

1. introduction
2. t-duality — closed string
3. t-duality — open string
4. d-branes & non-geometry
5. summary

Summary ::

- Open-string T-duality via Buscher's procedure has been discussed,
- taking into account non-trivial world-sheet topologies.

- Using T-duality, D-branes on various flux-backgrounds are obtained.
- Boundary conditions are well-defined using $O(D,D;\mathbb{Z})$ transformations.