Piercing gravitational radiation from domain walls

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The goal of the talk is to discuss physical effects in collisions of point particles with domain walls (DW)

Applications

- Cosmological DWs in the Early universe
- Randall Sundrum II braneworld scenario
- Primordial gravitational waves

Effects

- Perforation of DW by point particle
- Excitation of branons
- Gravitational Radiation

Perforation of domain wall by primordial black holes as destruction mechanism

In cosmology, (Vilenkin, Shellard) perforation of the domain walls by black holes as novel mechanism of domain walls destruction in the Early Universe (Chamblin, Eardley,S tojkovic, Freese, Starkman, Flachi,Tanaka)

If the theory admits both DWs and cosmic strings, the hole in DW will be surrounded by CSs, the subsequent evolution depends on competition between the tension of CS causing the holes to close and tension of DW causing expansion of the holes. The critical number of holes necessary to destroy DW is four (Eardley, Stojkovic)

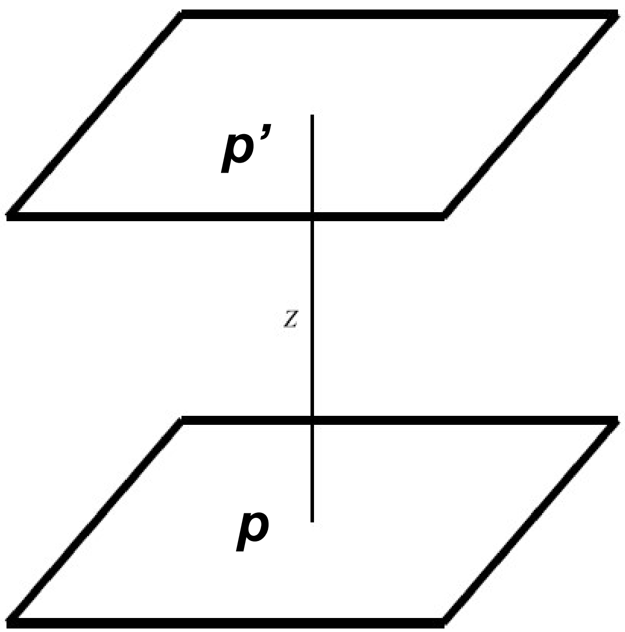
Gravitational waves

- The spectrum of gravitational waves, generated by the collapsing unstable DWs can be an important source of information about the early Universe (Gleiser, Roberts (1998)).
- In view of the forthcoming experimental studies of relict gravitational waves this subject attracted attention recently (T.Hiramatsu, M.Kawasaki, K.Saikawa(2010-2015)).
- The calculation, however, do not take into account gravitational interaction of DW with surrounding matter which may lead to additional generation of GW, may be subdominant, but with distinct spectral properties

Newtonian interaction between two parallel branes

Effective potential per unit volume of smaller brane

$p' \leqslant p$



$$U_{\text{eff}} = \frac{\varkappa_D^2 \mu \mu' (p'+1)(\tilde{d}-2)}{2(D-2)} \Phi_{\tilde{d}}(\mathbf{z})$$

$$\Phi_{\tilde{d}}(\mathbf{z}) = \begin{cases} |z|/2, & \tilde{d}=1 \\ (2\pi)^{-1} \ln |\mathbf{z}|, & \tilde{d}=2 \\ -(\tilde{d}-2)^{-1} \Omega_{\tilde{d}-1}^{-1} |\mathbf{z}|^{(2-\tilde{d})}, & \tilde{d} \geqslant 3 \end{cases}$$

$\tilde{d} \geqslant 3$-attraction
$\tilde{d} = 2$-no interaction
$\tilde{d} = 1$ -repulsion

$D - p - 1 = \tilde{d}$ *Codimension of "bigger" brane*

Singular for codimension three and bigger, non-singular for DW

The model **DW:** $x^M = X^M(\sigma^\mu), \quad M = 0,1,2,...,D-1 \quad \mu = 0,1,2, \; \dots \; , D-2$

particle: $x^M = z^M(\tau)$

The action:

$$S = -\frac{\mu}{2}\int \left[X^M_\mu X^N_\nu g_{MN}\gamma^{\mu\nu} - (D-3)\right]\sqrt{-\gamma}\, d^{D-1}\sigma - \frac{1}{2}\int\left(e\, g_{MN}\dot{z}^M\dot{z}^N + \frac{m^2}{e}\right)d\tau -$$
$$-\frac{1}{\varkappa^2}\int R_D\sqrt{-g}\; d^D x$$

Equations of motion

$$\left(X^M_\mu X^N_\nu - \frac{1}{2}\gamma_{\mu\nu}\gamma^{\lambda\tau}X^M_\lambda X^N_\tau\right)g_{MN} + \frac{D-3}{2}\gamma_{\mu\nu} = 0$$ **Nambu-Goto**

$$\frac{d}{d\tau}\left(e\dot{z}^N g_{MN}\right) = \frac{e}{2}\, g_{NP,M}\,\dot{z}^N\dot{z}^P$$ **geodesic**

Einstein equations:

$$T^{MN} = \mu\int X^M_\mu X^N_\nu \gamma^{\mu\nu}\,\frac{\delta^D\left(x - X(\sigma)\right)}{\sqrt{-g}}\,\sqrt{-\gamma}\; d^{D-1}\sigma$$

$$G^{MN} = \frac{1}{2}\varkappa^2\left[T^{MN} + \bar{T}^{MN}\right]$$

$$\bar{T}^{MN} = e\int \frac{\dot{z}^M\dot{z}^N\delta^D\left(x - z(\tau)\right)}{\sqrt{-g}}\, d\tau$$

Solved perturbatively, expandind X, z and the metric in terms of gravitational constant on Minkowski background: $g_{MN} = \eta_{MN} + \varkappa h_{MN}$

Iterative solution

Expand in terms of gravitational coupling $\Phi = {}^0\Phi + {}^1\Phi + {}^2\Phi + \ldots$

all the variables $z^M(\tau),\ e(\tau),\ X^\mu(\sigma)$ and $h_{MN}(x)$.

In the first order the superposition principle holds ${}^1h_{MN} = h_{MN} + \bar{h}_{MN}$

DW at rest: ${}^0X^M = \Sigma^M_\mu \sigma^\mu$ $\qquad \Sigma^M_\mu = \delta^M_\mu$ $\qquad \Xi_{MN} \equiv \Sigma^\mu_M \Sigma^\nu_N \eta_{\mu\nu}$

$$h_{MN} = \frac{\varkappa\mu}{2}\left(\Xi_{MN} - \frac{D-1}{D-2}\eta_{MN}\right)|z| = \frac{\varkappa\mu|z|}{2(D-2)}\,\mathrm{diag}\,(-1, 1, \ldots, 1, D-1)$$

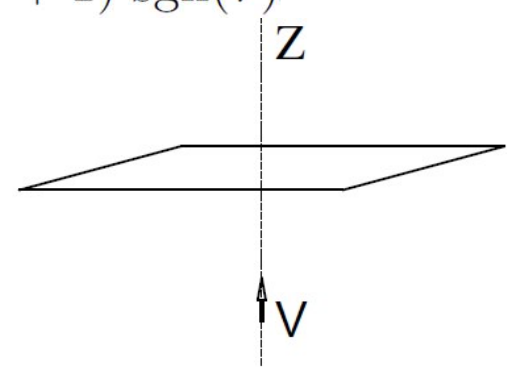
linearly growing potentials

particle:

$$\bar{h}_{MN}(x) = -\frac{\varkappa m \Gamma\left(\frac{D-3}{2}\right)}{4\pi^{\frac{D-1}{2}}}\left(u_M u_N - \frac{1}{D-2}\eta_{MN}\right)\frac{1}{[\gamma^2(z-vt)^2 + r^2]^{\frac{D-3}{2}}}$$

$${}^1\ddot{z}^{D-1} = k\,(D\gamma^2 v^2 + 1)\,\mathrm{sgn}(\tau)$$

Particle moves from negative to positive z with constant deceleration till perforation, Then acceleration changes sign. The velocity is continuous



Deformation of the wall

Perturbing Nambu-Goto equations, one gets d'Alembert equation on the wall

$$\Pi_{MN}\,\Box_{D-1}\,\delta X^N = \Pi_{MN}\,J^N\,, \qquad \Pi^{MN} \equiv \eta^{MN} - \Sigma^M_\mu\,\Sigma^N_\nu\,\eta^{\mu\nu}$$

with projector onto one-dimensional subspace orthogonal to it. The source reads

$$J^N = \varkappa\,\Sigma^\mu_P\,\Sigma^\nu_Q\,\eta_{\mu\nu}\left(\frac{1}{2}\,\bar{h}^{PQ,N} - \bar{h}^{NP,Q}\right)_{z=0}$$

Define branon field $\quad \Phi(\sigma^\mu) \equiv \delta X^z \quad$, obeys $\quad \eta^{\mu\nu}\frac{\partial}{\partial\sigma^\mu}\frac{\partial}{\partial\sigma^\nu}\,\Phi(\sigma) = J(\sigma)$

$$J(\sigma) = -\varkappa\left[\frac{1}{2}\,\eta_{\mu\nu}\bar{h}^{\,\mu\nu,z} - \bar{h}^{\,z\,0,0}\right]_{z=0} = -\frac{\lambda v t}{\left[\gamma^2 v^2 t^2 + r^2\right]^{\frac{D-1}{2}}}$$

$$\lambda = \frac{\varkappa^2 m \gamma^2 \Gamma\left(\frac{D-1}{2}\right)}{4\pi^{\frac{D-1}{2}}}\left(\gamma^2 v^2 + \frac{1}{D-2}\right)$$

Retarded solution consist of time-asymmertic deformation (a) and free branon wave (b), starting at the moment of perforation and propagating with the velocity of light along the wall

$$\Phi(t,\mathbf{r}) = -\Lambda\,\mathrm{sgn}(t)\,I_{\mathrm{a}} + 2\,\Lambda\,\theta(t)\,I_{\mathrm{b}}$$

$$\Lambda \equiv \frac{\sqrt{\pi}\,\lambda}{2^{\frac{D-2}{2}}\gamma^3\Gamma\left(\frac{D-1}{2}\right)}$$

$$I_{\mathrm{a}}(t,r) = \frac{1}{r^{\frac{D-4}{2}}}\int_0^\infty dy\,J_{\frac{D-4}{2}}(yr)\,y^{\frac{D-6}{2}}\,\mathrm{e}^{-y\gamma v|t|}$$

$$I_{\mathrm{b}}(t,r) = \frac{1}{r^{\frac{D-4}{2}}}\int_0^\infty dy\,J_{\frac{D-4}{2}}(yr)\,y^{\frac{D-6}{2}}\,\cos yt$$

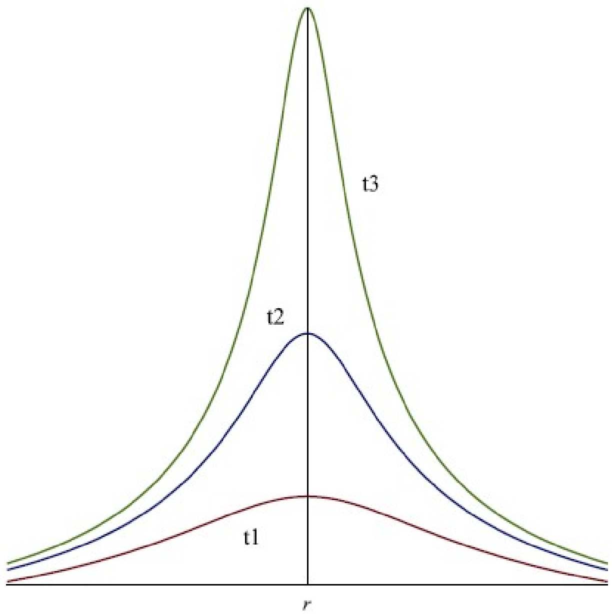
CASE D=5

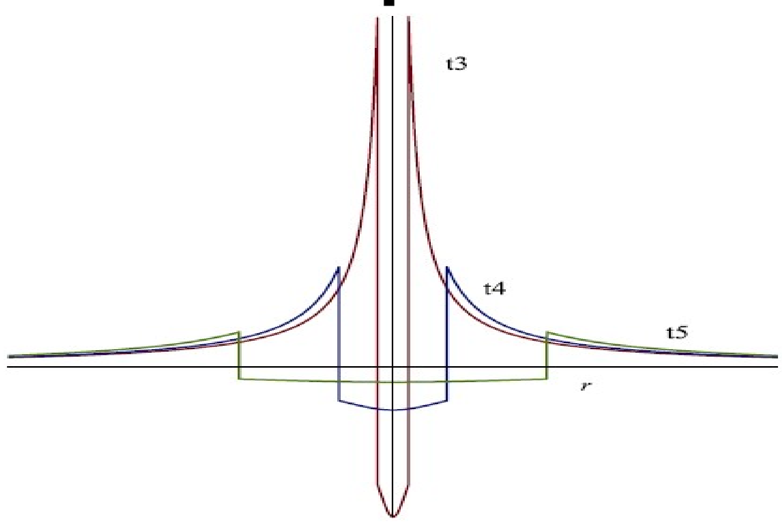
$$\Phi_a \sim -\mathrm{sgn}(t)\frac{1}{r}\arctan\left(\frac{r}{\gamma v|t|}\right)$$

Particle approaches the brane starting at large negative t , the spherical wave of small amplitude is created and moves forwards with an increasing amplitude

$$\Phi b \sim \frac{1}{r}\theta(t)\theta(r-t)$$

t>0 brane moves inversely while the particle moves away from the other side of the brane (Φa) at the moment t=0 (perforation) domain wall gets excited in the form of the spherical shock branon wave propagating along the brane with the speed of light. This wave is a backreaction of the wall to the change of the particle's acceleration by a finite amount at the moment of piercing (on the edge)





Second order

In the second order one obtains the leading contribution to gravitational radiation. The effective source of radiation consists of three ingredients. The first is due to particle which has constant acceleration before and after piercing. This has certain analogy with the Weinberg's computation of gravitational radiation from the system of particles colliding at a point: in that case one has the constant momenta before and after collision which instantaneously change on a finite amount. In our case it is the (proper) time derivatives of the momenta before and after collision which are constant and opposite, changing sign at the moment of perforation. The second contribution comes from the deformation of the brane world-volume caused by varying gravitational field of the moving particle. Finally, for consistency of calculations, the gravitational stresses have to be taken into account, these are described using Weinberg's expansion of the Einstein tensor up to the second order in the gravitational constant

$$G_{MN} = -\frac{\varkappa}{2}\Box\psi_{MN} - \frac{\varkappa^2}{2}S_{MN} + \sum_{n>2}\varkappa^n N_{MN}^{(n)}$$

The wave equation in the second order for the trace-reversed metric $\psi_{MN} \equiv h_{MN} - \frac{h}{2}\eta_{MN}$ read

$$\Box\,{}^2\psi_{MN} = -\varkappa\,\tau_{MN}$$

where

$$\tau_{MN} = {}^1\bar{T}_{MN} + {}^1T_{MN} + S_{MN}$$

the last term being the bilinear form of the first order metric deviations due to DW and the particle. Only the sum of three is conserved in the Minkowski sense

$$\partial_N\tau^{MN} = 0$$

Gravitational radiation formula revisited

Traditionally, both electromagnetic and gravitational radiation is computed in terms of fluxes of the field momentum in the wave zone, which is well-defined in asymptotically flat space-time. Our space-time is not asymptotically flat, so one has to revisit the derivation. In particular, the energy-momentum flux through the lateral surface of the world-tube turns out to be non-zero. The idea of new derivation is to start with radiation reaction work which is defined locally. To perform necessary transformation one has to consider expansions of Einstein equations up to the fourth order, however the leading radiation formula involves only the product of two second order terms

$$P_M = \frac{1}{2}\int {}^2h_{AB,M}\Box\, {}^2\psi^{AB} = -\frac{\varkappa}{2}\int {}^2h_{AB,M}\tau^{AB}\, d^D x$$

which finally transforms to the standard momentum space representation

$$E_{\rm rad} = \frac{\varkappa^2}{4(2\pi)^{D-1}}\sum_{\mathcal{P}}\int\limits_0^\infty \omega^{D-2} d\omega \int\limits_{S^{D-2}} d\Omega\, \left|\varepsilon^{SN}_{\mathcal{P}}\tau_{SN}(k)\right|^2$$

where the Fourier-transforms of the effective source are contracted with polarization tensors.

In any dimensions one can choose polarization tensors in such a way that only one on them (with two zz-legs) gives non-zero projections, so we actually deal with complex scalar amplitudes. One therefore has three complex amplitudes expressing contribution of the particle, the DW and gravitational stresses. This splitting is of course gauge dependent and refers to harmonic gauge together with certain additional specification of polarization states.

Introduce the unit space-like vector **n** on the unit sphere within the DW, and the angle ψ between **k** and the z-axis (the line of particle motion). The graviton wave-vector is then parametrized as **k** = ω (**n** sin ψ; cos ψ)

The wall amplitude:

$$T_z(k) = -\sqrt{\frac{D-3}{D-2}}\,\frac{\varkappa^2\mu m}{2\omega^2}\,\frac{\gamma v\sin^2\psi}{1+\gamma^2v^2\sin^2\psi} \times\left[\frac{\cos\psi}{v\left[\cos^2\psi+2i\epsilon k^0\right]}\left(\gamma^2v^2+\frac{1}{D-2}\right)+\gamma^2v^2-\frac{1}{D-2}\right]$$

Note the infrared divergence of this amplitude which is not surprising since our procedure did not take into account the finite depth of the piercing layer. Another interesting feature is that the amplitude remains non-zero in the limit v→0. This is related to branon shock wave which emerges independently of velocity. Also, the amplitude diverges at ψ=π/2, i.e. along the DW. This divergence is is due to the fact that the shock wave excitation which in out approach propagates to infinity without damping. Removing this contribution (this may correspond to Z2-symmetric braneworld models or to the case of two mirror particles impinging upon the wall), we obtain

$${}^{1}T_z(k)\Big|_{\Phi=0} = -\sqrt{\frac{D-3}{D-2}}\,\frac{\varkappa^2\mu\mathcal{E}}{2\omega^2}\,\frac{v\sin^2\psi}{1+\gamma^2v^2\sin^2\psi}\left[\gamma^2v^2-\frac{1}{D-2}\right]$$

In this case the amplitude does not blow up at ψ=π/2 and the angular distribution is finite

The particle amplitude

$$ {}^1\bar{T}_z(k) = -\left[\frac{D-3}{4(D-2)^3}\right]^{1/2} \frac{\varkappa^2 \mu m v}{\gamma\omega^2} \frac{\left[(D-2)\gamma^2 v^2 - 1\right] v\cos\psi + 2}{(1 - v\cos\psi)^3} \sin^2\psi $$

This amplitude, apart from the infrared, has also the angular divergence at $\psi = 0$ in the case of the massless particle $v = 1$. This is the well-known collinear divergence encountered in quantum perturbation theory for interacting massless particles. In classical theory this is the line divergence of the retarded potentials

The stress contribution

$$ S_z(k) = \sqrt{\frac{D-3}{D-2}} \frac{\varkappa^2 \mu m\, v}{\gamma\,\omega^2} \frac{\sin^2\psi}{(1 - v\cos\psi)^3 (1 + v\cos\psi)} \left[\gamma^2 v^4 \cos^2\psi - \frac{(1 - v\cos\psi)^2}{D-2}\right] $$

Here one also observes both the infrared and the angular divergences.

The destructive interference in the ultrarelativistic limit

In the ultrarelativistic limit both the particle and the stress amplitudes have similar behavior near the forward direction which could give the leading contribution to radiation. However, keeping the common singular factors and expanding the rest as

$$\sin\psi \approx \psi\,, \qquad 1 - v\cos\psi \approx \frac{\psi^2 + \gamma^{-2}}{2}$$

one finds for large γ

$${}^1\bar{T}_z(k) = -\frac{1}{2}\sqrt{\frac{D-3}{D-2}}\,\frac{\varkappa^2 \mu \mathcal{E}\,\sin^2\psi}{\omega^2\,(1 - v\cos\psi)^3}\;\left(1 + \mathcal{O}(\gamma^{-2})\right)$$

$$S_z(k) = \frac{1}{2}\sqrt{\frac{D-3}{D-2}}\,\frac{\varkappa^2 \mu \mathcal{E}\,\sin^2\psi}{\omega^2(1 - v\cos\psi)^3}\;\left(1 + \mathcal{O}(\gamma^{-2})\right)$$

where E = m γ is the particle energy. So in the leading order in γ, these two amplitudes exactly cancel. This is manifestation of the destructive interference which reflects the equivalence principle in the language of flat space, which was encountered earlier in the bremsstrahlung problem for point particles. (Khriplovich(1973), Galtsov, Spirin,Tomaras (2010))
After cancelation of the leading terms, the sum of two amplitudes has two orders less in γ .

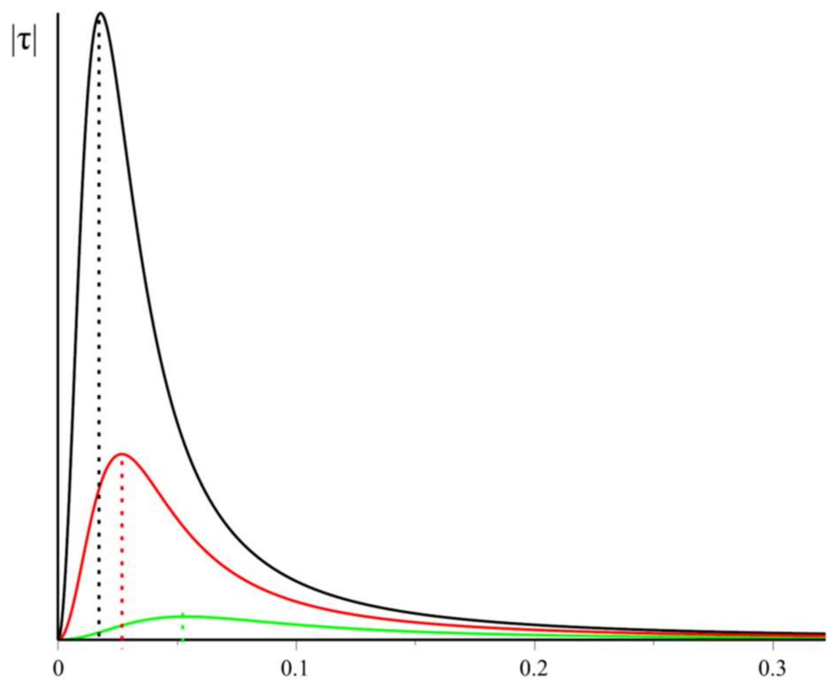
The subleading brane amplitude in the ultrarelativistic limit in the forward direction:

$${}^1T_z(k)\Big|_{\psi\ll 1} \approx -\sqrt{\frac{D-3}{D-2}}\,\frac{\varkappa^2 \mu \mathcal{E}}{\omega^2}\,\frac{\gamma^2 \sin^2\psi}{1 + \gamma^2 \sin^2\psi}$$

Thus in the small-angle region the main contribution still comes from the sum of ${}^{1}\bar{T}(k^M)$ and $S(k^M)$. Expanding these with more accuracy and keeping the subleading terms, one finds to the main order:

$$\tau_z(k)\Big|_{\psi\ll 1} \approx -\frac{1}{8}\sqrt{\frac{D-3}{D-2}}\,\frac{\varkappa^2\mu\mathcal{E}\,\sin^2\psi}{\gamma^2\,\omega^2(1-v\cos\psi)^3}\,\left[\frac{D+2}{D-2}+\gamma^2\sin^2\psi\right]$$

The total amplitude is peaked at $\psi\sim 1/\gamma$ in any dimensions



The angular dependence of the radiation amplitude in D=4 for γ =15 in four (black) γ=30 (red) and γ=45 (green)

Spectral and angular distribution of PGR

$$E_{\rm rad} = \frac{\varkappa^2}{(4\pi)^{D/2}\Gamma\left(\frac{D-2}{2}\right)} \int\limits_{\omega_{\rm min}}^{\omega_{\rm max}} d\omega\, \omega^{D-2} \int\limits_{\psi_{\rm min}}^{\pi} d\psi\, \sin^{D-3}\psi\, |\tau_z(\omega,\psi)|^2$$

- Forward peaking is D-dependent, and may be divergent (e.g. for photons in D=4)
- IR divergence for all D<6
- UV divergent for D>4

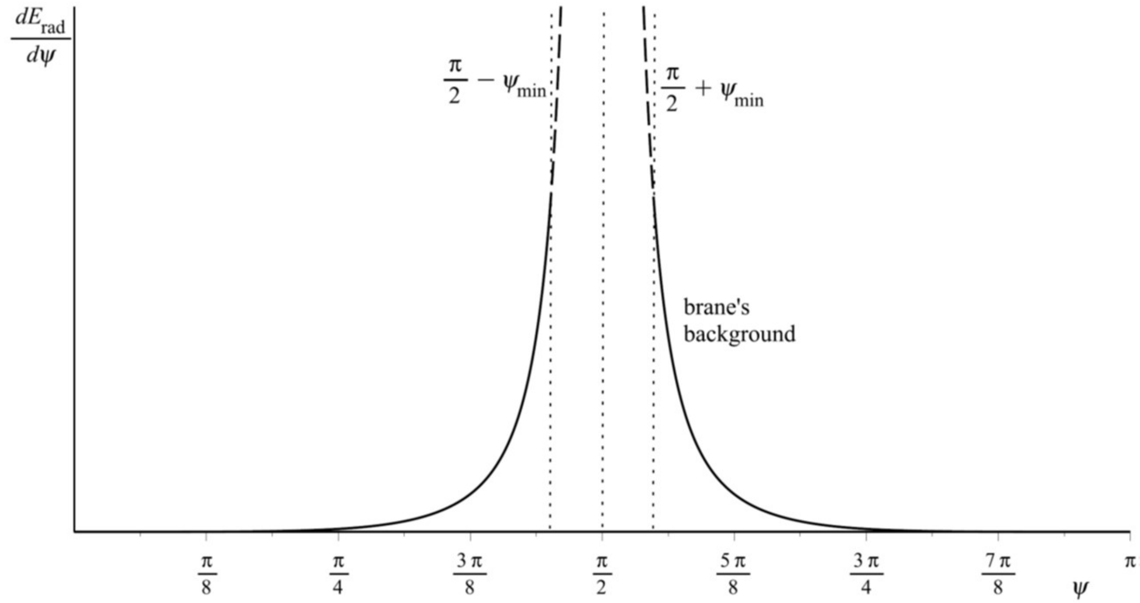
D=4 with IR cutoff

The frequency cut-offs are obtained by the inverse coordinate ones.

$$E_{\rm rad} = \frac{1}{80\pi^2} \frac{(\varkappa_4^3 \mu \mathcal{E})^2 \gamma^2}{\omega_{\rm min}}$$

Non-relativistic velocities

Angular cutoff required



$$\tau_z(k)\Big|_{\psi\approx\pi/2} = -\sqrt{\frac{D-3}{D-2}}\,\frac{\varkappa^2 \mu m}{2\omega^2\cos\psi}\,\frac{\gamma}{1+\gamma^2 v^2}\left(\gamma^2 v^2 + \frac{1}{D-2}\right)$$

$$E^{\mathrm{perf}} \simeq (\varkappa_D^3 \mu m)^2 \frac{\omega_{\mathrm{max}}^{D-4}}{k^z_{\mathrm{min}}}$$

Cut-off $$r_{\max} = (\varkappa_D^2 \mu)^{-1} \qquad r_{\min}^{D-3} = \varkappa_D^2 \mathcal{E}$$

The ultra-relativistic case
(beaming in the bulk direction):

$$E_{\text{rad}} \simeq (\varkappa_D^3 \mu \mathcal{E})^2 \gamma^{6-D} \cdot \begin{cases} 1/\omega_{\min}, & D = 4; \\ \ln \dfrac{\omega_{\max}}{\omega_{\min}}, & D = 5; \\ \omega_{\max}^{D-5}/(D-5), & D > 5. \end{cases}$$

The frequencv cut-offs are obtained bv the inverse coordinate ones.

$$D > 5 \qquad E^{\text{rad}} \simeq (\varkappa_D^3 \mu \mathcal{E})^2 \gamma^{6-D} \omega_{\max}^{D-5} \sim \mathcal{E} \frac{r_{min}^2}{r_{max}^2} \gamma^{6-D}$$

$$D = 5 \qquad E^{\text{rad}} \simeq (\varkappa_D^3 \mu \mathcal{E})^2 \gamma \ln \frac{\omega_{\max}}{\omega_{\min}}$$

for the massless limit taking into account the existence of r_{min} preventing the angle ψ from approaching zero $\psi_{min} \sim \frac{r_{\min}}{r_{max}}$ the next restriction is obtained

$$E^{\text{rad}} \simeq (\varkappa_D^3 \mu \mathcal{E})^2 \gamma \ln \frac{r_{\max}}{r_{\min}} \simeq \mathcal{E} \frac{r_{\min}}{r_{\max}} \ln \frac{r_{\max}}{r_{\min}}$$

$$\boldsymbol{D = 4 \qquad E^{\text{rad}} \simeq (\varkappa_D^3 \mu \mathcal{E})^2 \gamma^2 \frac{1}{\omega_{\min}} \simeq \mathcal{E} \frac{r_{\max}}{r_{\min}}.}$$

Branon wave radiation: $$E^{\text{perf}} \simeq \mathcal{E} \frac{r_{\min}}{r_{\max}}$$

Conclusions

Piercing gravitational radiation (PGR) is novel and universal mechanism of GW emission by DWs independent on their particular nature, in particular, on the issue of their stability

The effect is especially efficient in the massless limit (photons piercing DWs). In this case no velocity (quadrupole) factor in the gravitational radiation power

PGR predicts an excess of relict gravitons in the low frequency region

Present model is too simplified to make definitive quantitative predictions, but on general grounds the expected effect is not small compared to GW generation by collapsing walls

PGR may serve a novel mechanism for DW destruction. For more definitive predictons further calculations within the cosmological setting are necessary.

Thank you for attention!