Quantum Spacetime and Particle Dynamics from a Relativity Perspective

— talk at SGQG (Jun 2018)

OTTO C. W. KONG
— Nat'l Central U, Taiwan

Prologue : Quantum Gravity

(Geometro-)

Dynamics of Quantum Spacetime

rather than

Quantum Dynamics of (Classical)

Spacetime

Configuration Vs Phase Space (? doubled geo.):

- ? quantum/noncommutative space behind Q.M.

  – solid basis for noncommutativity" $\Leftarrow$ Q.M.

- physical space : configuration space - free particle

- why phase space ?

- ? configuration space for quantum particle

- Hindsight: Phase Space is the Physical Space (model)

  : unifications of X and P (Born R.) and g & ω

? Physical Space for Quantum Mechanics :

- **quantum phase space is ∞-D symplectic manifold**

— **from unitary irrep. of $\tilde{G}(3)$ (or simply $H(3)$)**

- $|\phi\rangle = \sum (q_n + ip_n)|n\rangle$

— (q_n, p_n) **gives a set of 2∞ canonical coordinates**

- q_n **gives a set of ∞ coordinates for the physical space ?**
- ***BUT* irrep. – unlike classical case**

- **quantum/ noncommutative ?**
- **? classical approx. : coherent states $\longleftarrow$ coset**

Geometry — metric and curvature :

— story of projective Hilbert space (CP^∞)

- a **Kähler manifold** with hol. sect. curvature of $\frac{2}{\hbar}$
- symplectic Vs (FS) metric — $\omega = g(JX, Y)$; $J^2 = -1$
- $g \longrightarrow$ dispersion structure for uncertainty
- Kählerian function as observables

— Hamiltonian flows that preserve Kähler structure

- $cosD_{FS} = |\langle\psi|\phi\rangle| \longrightarrow$ coherent states : $ds^2 = dp^2 + dx^2$

- caution : $\mathcal{H} \longrightarrow$ sections of $U(1)$ bundle

Relativity Symmetry :

— particle dynamics (special Vs general)

- **central to fundamental physics – examples . . .**
- **symmetry of reference frame transformations**
- **symmetry of physical space(-time) model**
- ***symmetry of (free particle) configuration space***

— which is the physical space

- **symmetry of (free particle) phase space**

⋆ (center of mass for) any system behaves as a free particle

⋆ Newtonian model only as good as Newtonian mechanics

Coset Spaces as Homogeneous Spaces :

— **Lie group and Lie algebra**

- **coset space = group/subgroup as a representation**
- **Lie group $\Rightarrow$ homogeneous spaces, symplectic manifolds**

- **Einstein/Poincaré $\rightarrow$ Galilei/Newton ($c \rightarrow \infty$)**
- **Minkowski spacetime** $= ISO(1,3)/SO(1,3)$

  $\{L_{\mu,\nu}, P_{\mu}\}/\{L_{\mu,\nu}\}$, $\qquad [K_i, K_j] \sim \frac{1}{c^2} L_{ij}$
- **Newtonian space-time** $= G(0,3)/ISO(0,3)$, $\quad L_{0,i} \rightarrow K_i$
- **Newtonian phase space** $= G(0,3)/[SO(0,3) \times \{T\}]$

  $K_i = mX_i$

Fundamental (Special) Quantum Relativity:

$SO(2,4)$ (*cf.* deformed S.R.)

cf. Kowalski-Glikman & Smolin ; Chryssomalakos & Okon ; O.K.

- contains noncommuting X_μ and P_μ , I
- contains Lorentz symmetry $J_{\mu\nu}$
- stable symmetry, no deformation
- G, $\hbar$, c in structural constants
- (Lie algebra) contractions as approximations

$$
\begin{array}{ccccc}
SO(2,4) \longrightarrow H_R(1,3) & \longrightarrow & H_{GH}(3) \supset \tilde{G}(3) \supset H_R(3) & \\
\downarrow & (\frac{1}{c^2} \to 0) & \downarrow & (\hbar \downarrow 0) \\
ISO(1,3) \subset S(1,3) & \longrightarrow & S_G(3) \supset G(3) &
\end{array}
$$

- deformation is 'inverse' of contraction

Mathematical Scheme for Kinematics and Dynamics :-

relativity symmetry G	group C^*-algebra
unitary irrep. (on $\mathcal{H}$)	topo. cyclic irr. *-rep
– coherent states from G/H	– $\alpha(p,x)$ with $\star$-product
$\mathcal{P}(\mathcal{H})$ as space(time)	algebra of observables
Hamiltonian flows	'Heisenberg' flows
$\phi(p,x) \leftrightarrow$ (Wigner) $\rho(p,x) \Longleftarrow$	GNS construction
∞ Kähler z_n	NC $\hat{X} = x\star$, $\hat{P} = p\star$

G/H commutative : $\phi(p,x) \to \delta(p,x)$, $\hat{X} = x$, $\hat{P} = p$

'Heisenberg' $\to$ Poisson

- deformation of $G \Rightarrow$ deformation (quantization) $C^*(G)$

Observables, Dynamics, Phase Space, all from Symmetry :

$$i.e. \qquad H(3) \rtimes (SO(3) \times T) = \tilde{G}(3)$$

- **algebraic formulation from observables**
- **Connes' noncommutative geometry** $(\mathcal{A}, \mathcal{H}, \mathcal{D})$
- $C^*(H(3)) \longrightarrow \mathcal{A}$ **left regular rep.**

$$\alpha(p^i, x^i) \longrightarrow \alpha(p^i, x^i)\star = \alpha(p^i\star, x^i\star)$$

— **group** C^***-algebra** $C^*(H(3)) \qquad \Longleftarrow C^*(\tilde{G}(3))$

— **topological irreducible rep. from** $H(3)$ **on** $\mathcal{H}$

- **unitary flows** $\leftrightarrow$ **automorphisms** $U_\star \star \alpha \star \bar{U}_\star$

$$H(3) \rtimes (SO(3) \times T) = \tilde{G}(3) \quad \text{— spin 0 rep.}$$

- $\frac{d}{ds}\alpha = \frac{1}{i\hbar}\{\alpha, G_s\}_\star \quad \Longleftrightarrow \quad \frac{d}{ds}\alpha\star = [\alpha\star, G_s\star]$

- **Weyl-Wigner (WWGM) from coherent state basis**

— $C^*(H(3)) \longrightarrow \mathcal{H}$ **from** $\mathrm{Tr}[\rho_o \cdot]$

— $\rho \sim \phi \star \bar{\phi} \quad (\rho_o \sim \phi_o)$; $\mathrm{Tr} \longrightarrow \int d\mu$ **(← group metric)**

— $\alpha(p^i, x^i)\star$ **on** $\mathcal{H} = \{\alpha \star \phi_o(p^i, x^i); \alpha \in L^2(p^i, x^i)\}$

- $C(p^i\star, x^i\star)$ **: operators as nc coordinates (?)**
- **Geometry : (?) Dirac operator** $\mathcal{D} \sim (ds)^{-1}$ **on** $\mathcal{H}$

  typical example : sections of spin bundle on manifold

— $(p^i\star, x^i\star)$ **metric(?) Vs Kähler metric on "$\mathcal{H}$"** $(\subset \mathcal{A})$

- $\mathcal{A}$ **as** $C(X)$ — X **: unit ball in** $\mathcal{H}^*$**, compact Hausdorff**
- $\mathcal{A}$ **as** $C(H(3))^*$ — **space of irrep. , not Hausdorff**

Relativity Contraction as Dequantization :

$\longrightarrow$ **Newtonian limit (Poisson algebra; KvN Hilbert space)**

- $\mathcal{H} \to \mathcal{H}_{KvN} \sim (p^i, x^i)$ **reduces to 1-D reps.**
- $\alpha(p^i, x^i)\star \to \alpha(p^i, x^i)$; $\quad C^*(H(3)) \to C(p^i, x^i)$

- **Tomita rep.** $L^2(p^i, x^i)$ **of mixed states +**

— $\rho(p^i, x^i)$ **as wavefunctions (self-dual real cone)**

extra operators — $\tilde{G}_s = G_s\star - \star G_s$

$\longrightarrow$ **all** $G_s(p^i, x^i)$ **diagonal**

but $\tilde{G}_s\rho = \{\tilde{G}_s, \rho\}_\star \to$ **Hamiltonian vector field**

$\longrightarrow$ **correct limit under Heisenberg picture dynamics**

Relativity for Simple Quantum Mechanics :

- need $[X_i, P_j] = i(\hbar)\,\delta_{ij} I$

— I as central charge, commutes with all

- Heisenberg-Weyl $\subset$ extended Galileo

- (configuration) space coset : $\widetilde{G}(3)/[ISO(3) \times \{T\}]$

$$\begin{pmatrix} dx^i \\ d\theta \\ 0 \end{pmatrix} = \begin{pmatrix} \omega^i_j & 0 & \bar{x}^i \\ \bar{p}_j & 0 & \bar{\theta} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x^j \\ \theta \\ 1 \end{pmatrix} = \begin{pmatrix} \omega^i_j x^j + \bar{x}^i \\ \bar{p}_j x^j + \bar{\theta} \\ 0 \end{pmatrix}$$

★ θ as coordinates ?!

Quantum Phase Space from Coherent States :

$$\left|p^i, x^i\right\rangle \equiv e^{-i\theta} U(p^i, x^i, \theta)|0,0\rangle$$

— $U(p^i, x^i, \theta) \equiv e^{i(p^i \hat{X}_i - x^i \hat{P}_i + \theta \hat{I})}$ as unitary rep. of HW group

- $\hat{X}_i$ translates p^i and $\hat{P}_i$ translates x^i
- overcomplete basis $\longrightarrow$ (abstract) Hilbert space $\mathcal{H}$
- wavefunctions $\phi(p^i, x^i)$: — not delta function

— Gaussian centered on (p^i, x^i), minimal uncertainty

— x^i and p^i are *only* expectation values

— $|0,0\rangle$ explicit — ground state of SHO

— the 'classical states' described in QM

Coset to Coherent States :

- point on coset space $(p^i, x^i, \theta) \leftrightarrow e^{i\theta}|p^i, x^i\rangle$

— transformations of coset $\longrightarrow$ unitary rep. on $\mathcal{H}$

- works also for (configuration) space

$$|x^i\rangle \equiv e^{-i\theta} U'(x^i, \theta)|0\rangle$$

— $U'(x^i;\theta) \equiv e^{i(-x^i\hat{P}_i + \theta\hat{I})}$ from subgroup generated by $\{P_i, I\}$

— results on coset $\Rightarrow$ $|x^i\rangle$ as eigenstate of $\hat{X}_i$

— the Hilbert space as physical space for QM

Classical Limit — $\hbar \to 0$ approximation :

- **classical symmetry as approximation**

— **symmetry (algebra) contraction, rep. contracts**

- **on cosets I decouples : $[X_i, P_j] \to 0$**

— $d\theta = \bar{\theta}$, $\quad$ dx^i **and** dp^i θ**-independent**

- **Hilbert space $\to$ sum of 1-D subspaces**

— ***i.e.*** **only coherent states, no superpositions**

— **but set of coherent states $\leftrightarrow$ coset space**

$$X_i^c = \frac{1}{k} X_i \text{ and } P_i^c = \frac{1}{k} P_i \text{ with } k \to \infty \; (\frac{1}{k^2} \sim \hbar)$$

$$[X_i^c, P_j^c] = \frac{i}{k^2} \delta_{ij} I \to 0$$

— group parameters (also cosets) : $p_c^i = kp^i$ and $x_c^i = kx^i$

$$\begin{pmatrix} dp_c^i \\ dx_c^i \\ d\theta \\ 0 \end{pmatrix} = \begin{pmatrix} \omega_j^i & 0 & 0 & \bar{p}_c^i \\ 0 & \omega_j^i & 0 & \bar{x}_c^i \\ -\frac{1}{2k^2}\bar{x}_{cj} & \frac{1}{2k^2}\bar{p}_{cj} & 0 & \bar{\theta} \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p_c^j \\ x_c^j \\ \theta \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} dx_c^i \\ d\theta \\ 0 \end{pmatrix} = \begin{pmatrix} \omega_j^i & 0 & \bar{x}_c^i \\ \frac{1}{k^2}\bar{p}_{cj} & 0 & \bar{\theta} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_c^j \\ \theta \\ 1 \end{pmatrix}$$

— only $d\theta = \bar{\theta}$, $dx_c^i = \omega_j^i x_c^j + \bar{x}_c^i$, $dp_c^i = \omega_j^i p_c^j + \bar{p}_c^i$ (Newtonian) for the coherent states : $\tilde{p}_i^c = \sqrt{\hbar} p_i$ and $\tilde{x}_i^c = \sqrt{\hbar} x_i$

$$\left\langle \tilde{p}_i'^c, \tilde{x}_i'^c \right| \hat{X}_i^c \left| \tilde{p}_i^c, \tilde{x}_i^c \right\rangle = \frac{(\tilde{x}_i'^c + \tilde{x}_i^c) - i(\tilde{p}_i'^c - \tilde{p}_i^c)}{2} \left\langle \tilde{p}_i'^c, \tilde{x}_i'^c | \tilde{p}_i^c, \tilde{x}_i^c \right\rangle$$

$$\left\langle \tilde{p}_i'^c, \tilde{x}_i'^c \right| \hat{P}_i^c \left| \tilde{p}_i^c, \tilde{x}_i^c \right\rangle = \frac{(\tilde{p}_i'^c + \tilde{p}_i^c) + i(\tilde{x}_i'^c - \tilde{x}_i^c)}{2} \left\langle \tilde{p}_i'^c, \tilde{x}_i'^c | \tilde{p}_i^c, \tilde{x}_i^c \right\rangle$$

$$\left\langle \tilde{p}_i'^c, \tilde{x}_i'^c | \tilde{p}_i^c, \tilde{x}_i^c \right\rangle = \exp\left[i \frac{\tilde{x}_i'^c \tilde{p}_i^c - \tilde{p}_i'^c \tilde{x}_i^c}{2\hbar} \right] \exp\left[- \frac{(\tilde{x}'^c - \tilde{x}^c)^2 + (\tilde{p}'^c - \tilde{p}^c)^2}{4\hbar} \right]$$

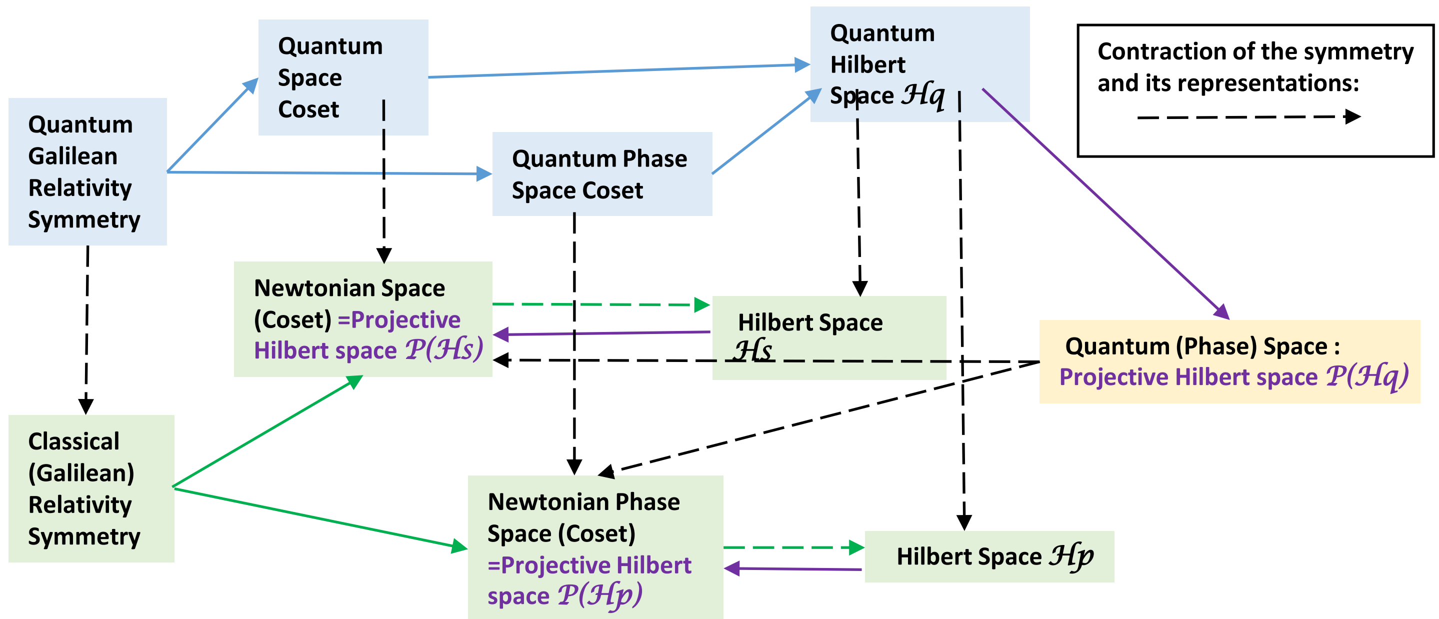
$$\left\langle \tilde{p}_i^c, \tilde{x}_i^c | \tilde{p}_i^c, \tilde{x}_i^c \right\rangle = 1$$

$\exp[\cdot] \rightarrow e^{-\infty} = 0 \quad \Longrightarrow \quad \hat{X}_i^c$ and $\hat{P}_i^c$ diagonal on $\left| \tilde{p}_i^c, \tilde{x}_i^c \right\rangle$

— eigenstates of all observables (rep. reducible)

$|x_i\rangle \longrightarrow \left| \tilde{x}_i^c \right\rangle$ also eigenstates of all observables

Quantum Model of the Physical Space



Quantum Mechanics is Particle Dynamics on the Quantum Space

Newtonian Space is only a model of our physical space, the model behind Newtonian mechanics. Our quantum relativity approach gives a quantum model of the physical space as behind quantum mechanics which allows as intuitive a description for quantum theory as the classical theory.

A Quantum Space behind Simple Quantum Mechanics
C.S. Chew, O.C.W. Kong*, J. Payne.
Adv. High Energy Phys. 2017 (2017) 4395918

Quantum Mechanics

can and should be seen as

Particle Dynamics on the Quantum Space

rather than

Quantized Dynamics on the Newtonian space

Phase Space as Physical Spacetime :-

- **Quantum : NC coordinates *or* ∞-real numbers**

- **metric : $dx^2 + dp^2$ — proper units (Planck ?)**
  familiar — $x \gg 1$ and $p \ll 1$

'spacetime' & energy-momentum
$\longrightarrow$ SPACETIME
(Einstein : space & time $\rightarrow$ 'spacetime')

The Physical World is *Quantum* !

we (still) describe Quantum Mechanics

with *Classical Concepts*

Intuitive Concepts *are not* Classical !

– Quantum Concepts *no less Intuitive*

– the Classical ones *only more familiar*

the main culprit : 'Quine's *convenient fiction*' – real numbers

NEW PICTURE OF QUANTUM PHYSICS :

- Newtonian space (3D Euclidean space) model is the classical approximation of the quantum space model, as the contraction limit of the quantum relativity symmetry
- the quantum physical space can be described as ∞-D Kähler manifold or a 6 'dimensional' noncommutative geometry
- a quantum particle has a location given by a point in the quantum physical space, coordinates of which can be determined (in principle) with arbitrary precision

- **fixing a state fixed values of all observables without uncertainty**
- **value of an observable should be seen as an infinite set of real numbers, or a noncommutative number, a piece of quantum information about the system**
- **uncertainties as in the Heisenberg uncertainty principle *apply only* to the best single real number value description of an observable**
- **Born probability *is only* a statement about results of von Neumann (eigenvalue-answered) measurement, a consequence of decoherence induced by the measuring process**

More Concluding Remarks :

- **understanding the spacetime — key issue in physics**
- **Lie group for relativity symmetry amazingly powerful**

- **quantum geometry = noncommutative geometry**

  starting from the simplest and most solid — Q.M.

- **? quantum field theory says about spacetime**

— one system, quantum fields as degrees of freedom

- **? quantum gravity**

★ quantum measurement deals with quantum information
— a change of perspective similiar to Copernicus' ?

THANK YOU !