Geometry of String theory
And relative locality

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based on

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with F. Rudolph and D. Svoboda

Prelude

- I want to provide a quantum gravity perspective on String theory
- Both QG gravity and String theory have provided strong physical, mathematical and philosophical insights
- Both are suffering from profound shortcomings
- One can use the insight gain in one approach to upgrade the other

Physics and geometry

- Long and fruitful interplay between inventions of new geometrical structures and the discovery of new physical concepts: quantum mechanics is intimately tied up with symplectic geometry, general relativity with Riemannian geometry, and the gauge principle with the geometry of principal bundles.
- The deeper reason behind the connection between physics and geometry is that each time a new geometrical concept was realized in mathematics, a new expression of the relativity principle was at play in fundamental physics. Unification of electric and magnetic, unification of wave and particle or unification of space and time always comes with a relativization of what was before understood as an absolute concept.
- The mathematical expression of such relativization is geometrical by essence and always reveals a new mathematical structure as the central element of the underlying geometry:
- What's next? What geometry for QG? What relativity principle, What unification ? What new geometrical structure?

Road map for QG

Fundamental

- What is the fundamental new relativity principle ?
- What is the simplest implementation of that idea ?
- What geometry represents that idea ?
- Is there a model that can guide us through the maze of new concepts
- Are there any generic predictions ?

Technical

- Can we address several shortcomings of DFT as an effective string description: No unique connection,
  The fact that the section condition a is a kinematical choice
  Link DFT with generalised geometry,
  Can we include T-dual backgrounds?
- Can we curve T-duality?

What is Relative locality?

- Absolute locality is the hypothesis that the concept of spacetime is independent of the nature of probe used. That it is a universal notion. The x in $\Phi(x)$ doesn't depend on the field or the process
- Relative locality is on the contrary, exploring the idea that spacetime is a notion which depends on the quantum nature of probe used i-e energy and quantum numbers.

The usual spacetime notion is adapted to probes which are point-like and classical.

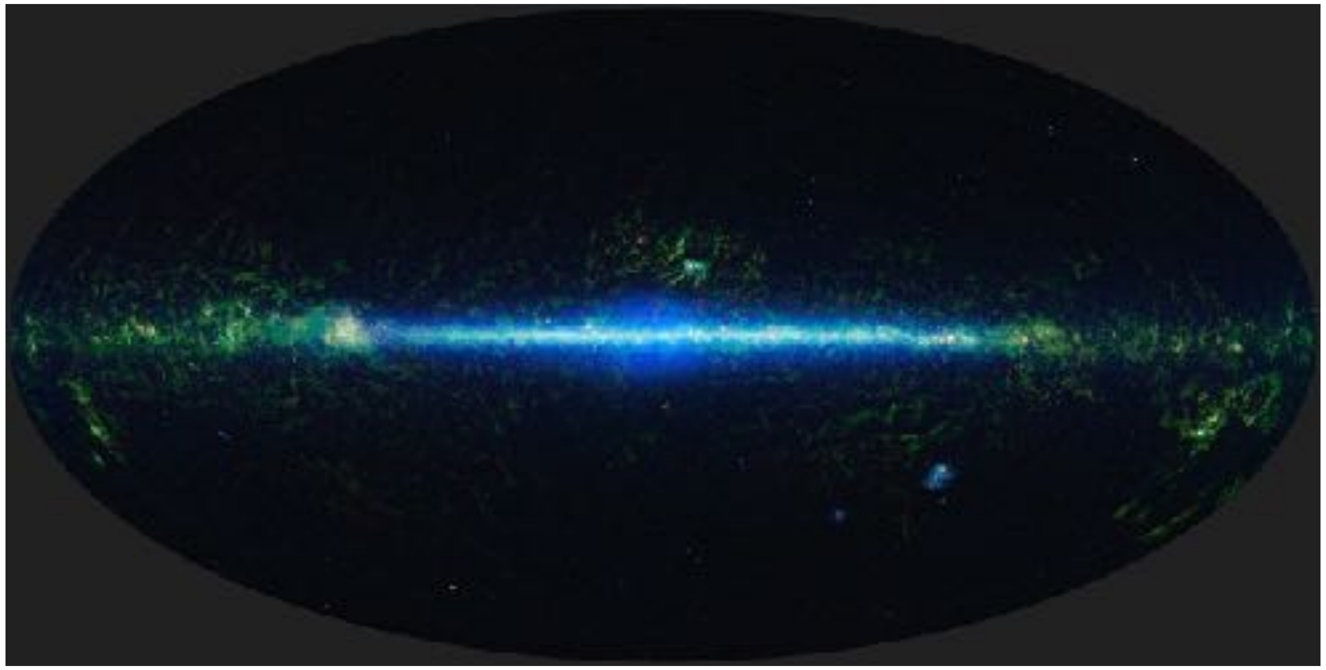
What is the proper notion of Home space, (the x in $\Phi(x)$) which is adapted to quantum and non-local probes ?

Why? Reconcile fundamental scale with the relativity principle
How to implement it? What are the elements?

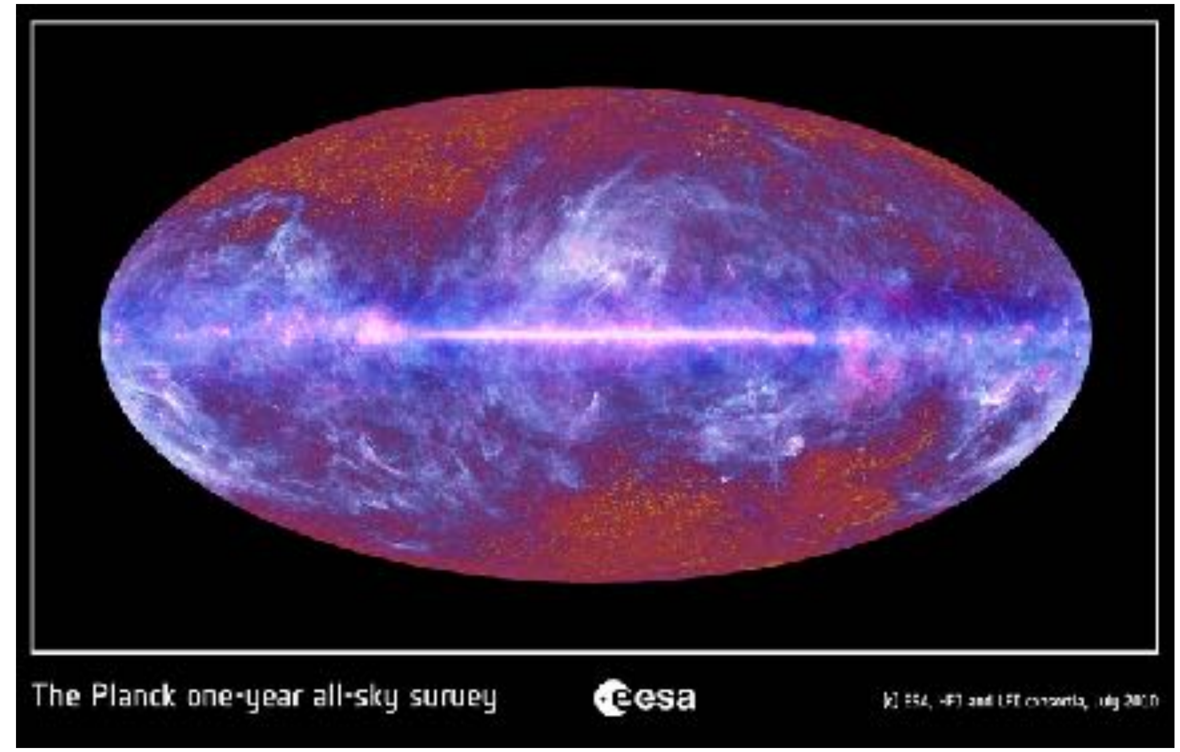
Lets start with an image then a model leading to an example.

Relative Locality: Illustration

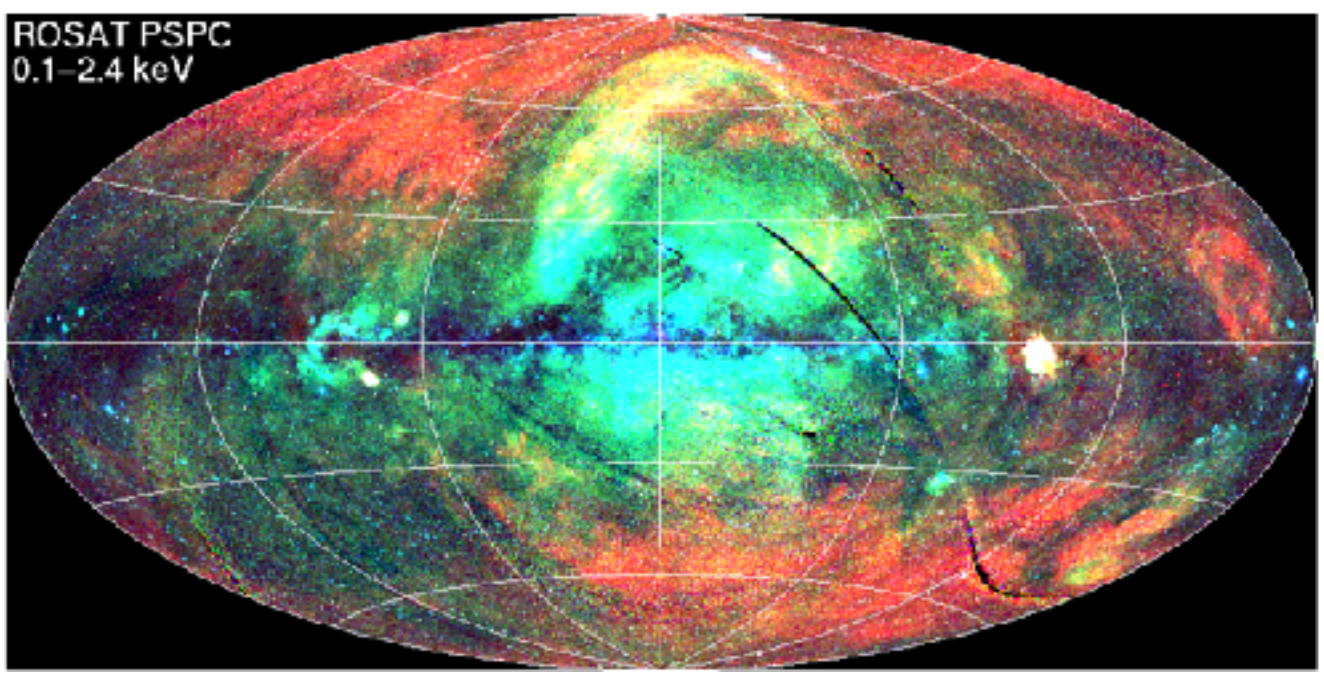
full sky survey:



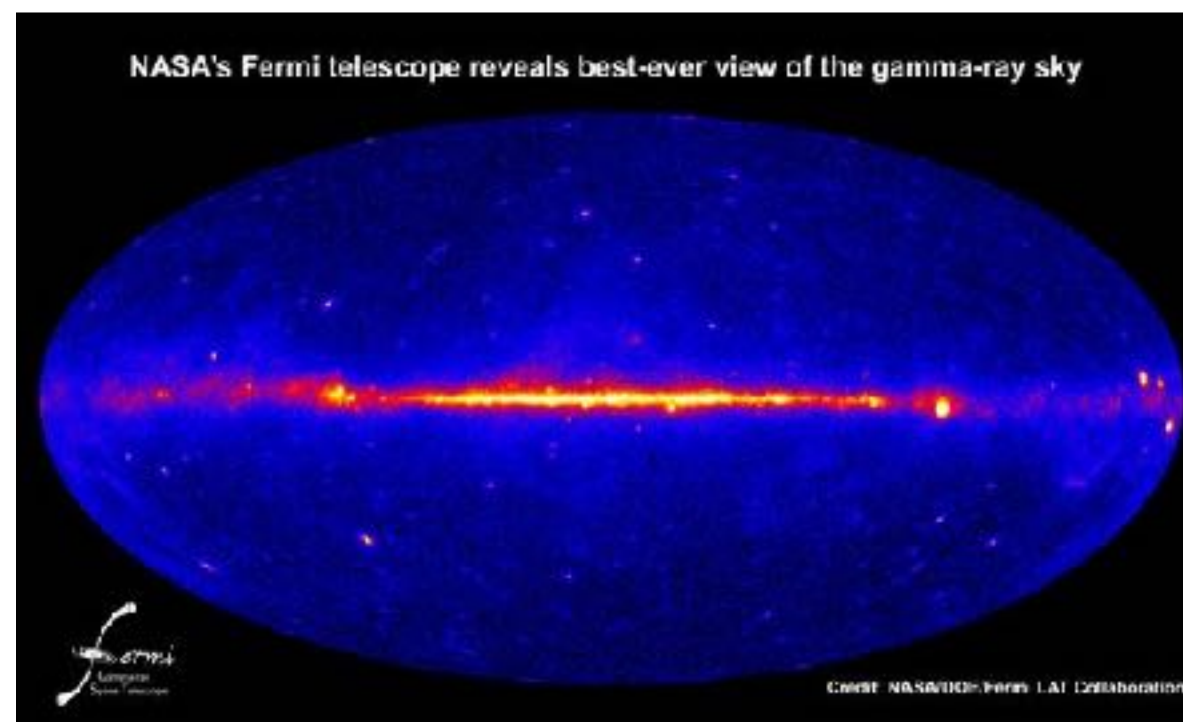
Wise infrared



Planck microwave



Rosat X-ray



Fermi Gamma ray

Geometry of String theory

Since Friedan we know that there exists a fundamental relation between 2d RG flow and spacetime geometry $\partial_t g_{ab} = R_{ab}$

The Zamolodchikov action governing 2d RG flow is then interpreted as the String Field action containing Einstein action.

$$S_0 + \sum_i \int g_i V_i \qquad\qquad \beta_i = \partial_\tau g_i = \partial_{g_i} Z$$

The problem is that this beautiful geometric picture usually breaks down, instabilities (tachyons), new marginal fields (dilaton, axions), modulis, extra fluxes, etc…

Recently this project has reopened we have more evidence that by generalizing the concept of geometry may we can simply write the CFT eq as a generalised Ricci Flow equation

Why would it even be possible?

Usual String Geometry

The usual picture is that if one has a CFT that can be written as a sigma model $X : \Sigma \to M$

Where M= the target is a non compact manifold then it is possible to effectively describe the same same system by a collection of fields $\Phi : M \to \mathbb{R}$ of diverse masses and spin

2 Organisations: 2d side= Conf invariance, EQFT = locality

And then we know that there is an infinite set of extra CFT that cannot be described like that and put into the limbo of Non-Geometrical Theories for which we have no handle:
Asymmetric orbifolds, Gepner points, R-flux compactification, Scherk-Schwarz twisted fluxes, self dual points, minimal models, etc…

Can we understand this untameable zoo in the language of quantum Gravity as quantum geometries ?

Metastring

Why would it even be possible?

Let's assume that one has a CFT that can be written as a sigma model $X : \Sigma \to M$ where target=M no longer a compact manifold

Then one wants to know whether it is is still possible to effectively describe the same same system by a collection of fields

$$\Phi : P \to \mathbb{R}$$

The label of the field, the $\mathbb{X}$ in $\Phi(\mathbb{X})$ belong to P= Home space

The basic hypothesis that we have to let go of is that in general Target space is not equal to Home space

$$P \neq M$$

If not then what is it ? A fundamentally QG question.

Flat metastring

Lets study the simplest string with target a one dimensional circle S_R and ask what is Home space in that case. Is it S_R?

$$X(\tau,\sigma) = X_R(\tau+\sigma) + X_L(\tau-\sigma),$$

Is the coordinates field, while the dual coordinate field is

$$\tilde{X}(\tau,\sigma) = X_R(\tau+\sigma) - X_L(\tau-\sigma),$$

The string equation of motion can simply be written as a self duality condition $$dX = *d\tilde{X},$$

The zero modes are

$$\boxed{\begin{array}{rcl} X(\sigma,\tau) & = & x + 2\lambda^2 k\tau + 2\lambda^2\tilde{k}\sigma + \cdots \\ \tilde{X}(\sigma,\tau) & = & \tilde{x} + 2\lambda^2\tilde{k}\tau + 2\lambda^2 k\sigma + \cdots \end{array}}$$

Wave vector k and $\tilde{k}$ dual wave vector

$$\lambda \equiv \sqrt{\frac{\hbar\alpha'}{2}},$$

T-duality

If the compact target is one-dimensional then

$$\boxed{k = \frac{n}{R}, \quad \tilde{k} = \frac{w}{\tilde{R}}, \qquad R\tilde{R} = 2\lambda^2.}$$

Invariance under $\quad (n, R) \leftrightarrow (w, \tilde{R})$

What space does the string moves in? It depends !

$R \to \infty \quad \tilde{x}$ decouples EFT in $\phi(x)$

$R \to 0 \quad x$ decouples EFT in $\phi(\tilde{x})$

What happens in between? How do we go from space to dual space?

We see from this two extreme examples that the string moves in a subspace which is half the dimension of a `doubled space'. And that which half depends on the nature of the probe.

Commutators

Starting from

$$S = \frac{1}{4\pi\alpha'}\int d^2\sigma \left[(\partial_\tau X)^2 - (\partial_\sigma X)^2\right].$$

What are the commutators for the zero modes?

It easy to establish that momenta commute $[k, \tilde{k}] = 0$

$$[\tilde{k}, \tilde{x}] = i \qquad [k, x] = i$$

And that they are canonical generator of translations

It is usually assumed that the positions commute

$$[x, \tilde{x}] = 0.$$

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That is not correct!

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And that they are canonical generator of translations

Instead we have

$$[x, \tilde{x}] = 2i\pi\lambda^2$$

The doubled space of compact string is non-commutative.

Even in the absence of fluxes!

Proof I

First proof one starts from

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \left[(\partial_\tau X)^2 - (\partial_\sigma X)^2 \right].$$

And compute the symplectic structure $\delta L \hat{=} \mathrm{d}\theta$ On-shell

And the associated commutator via the covariant phase space method $\Omega = \delta\theta$

Important subtlety: $\mathrm{d}X(\sigma)$ Periodic

But $X(\sigma)$ is only quasi-periodic. This quasi-periodicity reveals the presence of edge modes: New non-local degrees of freedom that appears at the edge of the domain . A phenomenon generic to massless theories and gauge theories (QED, QCD, Gravity, massless scalar,…) which is related to the new understanding of IR physics.

Strominger, Balachandran, LF Donnelly, LF Pranzetti…

Proof I

First proof one starts from

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \left[(\partial_\tau X)^2 - (\partial_\sigma X)^2\right].$$

And compute the symplectic structure $\delta L \hat{=} \mathrm{d}\theta$ On-shell

And the associated commutator via the covariant phase space method $\Omega = \delta\theta$

$$\Omega = \delta k \wedge \delta x + \delta \tilde{k} \wedge (\delta \tilde{x} - \underbrace{2\pi\lambda^2 \delta k})$$

Extra flux due to the edge modes

After inversion responsible for $[x, \tilde{x}] = 2i\pi\lambda^2$

Proof II

First proof one starts the construction of vertex operators

A generic closed string vertex operator is the product of 3 entities: a zero mode an holomorphic left mover and an anti-holomorphic right mover: $z = e^{\tau + i\sigma}$

$$\boxed{W_{k,\tilde{k}} = e^{ik\hat{X}} e^{i\tilde{k}\hat{\tilde{X}}} = U_0 V_L(z) V_R(\bar{z})}$$

The OPE determine the algebra of the left chiral algebra and the right chiral algebras, Both are non-commutative VOA

The consistency of string theory (mod invariance, duality symmetry, consistent coupling higher genus, etc…) demands that vertex operators have to be Mutually local.

$$W_{(k_1,\tilde{k}_1)}(1) W_{(k_2,\tilde{k}_2)}(2) = W_{(k_2,\tilde{k}_2)}(2) W_{(k_1,\tilde{k}_1)}(1)$$

Mutual locality

$$W_{\mathbb{K}} = e^{ik\hat{X}} e^{i\tilde{k}\hat{\tilde{X}}} = U_{\mathbb{K}} V_{k_L}(z) V_{k_R}(\bar{z})$$

V_L and V_R form non-commutative VOA

Lattice of momenta $\mathbb{K} = (k, \tilde{k})$

The W's must form a commutative algebra by mutual locality.
How is that possible?

Restriction of the spectra $\eta(\lambda\mathbb{K}, \lambda\mathbb{K}) = 2\lambda^2(k \cdot \tilde{k}) \in \mathbb{Z}.$

$\hat{U}_{\mathbb{K}} = e^{ik\cdot\hat{x}} e^{i\tilde{k}\cdot\hat{\tilde{x}}},$ Satisfies a Heisenberg algebra

$$\boxed{U_{\mathbb{K}} U_{\mathbb{K}'} = e^{2i\pi\omega(\mathbb{K},\mathbb{K}')} U_{\mathbb{K}'} U_{\mathbb{K}}}$$

Symplectic form

Mutual locality

$$W_{\mathbb{K}} = e^{ik\hat{X}} e^{i\tilde{k}\hat{\tilde{X}}} = U_{\mathbb{K}}\, V_{k_L}(z) V_{k_R}(\bar{z})$$

Lattice of momenta $\mathbb{K} = (k, \tilde{k})$ $\hat{U}_{\mathbb{K}} = e^{ik\cdot\hat{x}} e^{i\tilde{k}\cdot\hat{\tilde{x}}}$,

Satisfies a Heisenberg algebra

$$\boxed{U_{\mathbb{K}} U_{\mathbb{K}'} = e^{2i\pi\omega(\mathbb{K},\mathbb{K}')} U_{\mathbb{K}'} U_{\mathbb{K}}}$$

Compatibility of the spectra and commutation

$$\begin{aligned} \eta(\mathbb{K}, \mathbb{K}') &= \lambda^2 (k \cdot \tilde{k}' + \tilde{k} \cdot k') \\ \omega(\mathbb{K}, \mathbb{K}') &= \lambda^2 (k \cdot \tilde{k}' - \tilde{k} \cdot k') \end{aligned}$$

O(d,d) metric

Symplectic

Compatibility $(\eta \pm \omega)(\mathbb{K}, \mathbb{K}') \in \mathbb{Z}$ Para-hermitian lattice

What is Home space?

$$[x,\tilde{x}] = 2i\pi\lambda^2$$

The zero mode of the string is a non commutative space P but that's not yet the Home space of the string

Mutual locality implies that the string has to project itself onto a commutative sub-lagrangian manifold inside the non-commutative phase space P.
The 2-form decides what Lagrangian the string projects itself onto.

Compatibility $(\eta \pm \omega)(\mathbb{K},\mathbb{K}') \in \mathbb{Z}$ Para-hermitian lattice

What is space from quantum?

In QM Euclidean space appears simply as a choice of polarization: That is in the argument of the wave function. This is the quantum analog of a choice of Lagrangian

$$\Psi(x) \rightarrow \Phi(x)$$

Similarly Lorentzian space appears simply as a field label.

Space appears in the statement of micro-causality as the locus at which field which are space like separated commute.

Can we define a notion of quantum space? quantum space-time?

Quantum spaces are?

We simply reverse the logic and define space as the maximal set of commutative operations allowed in our algebra. Taking the Heisenberg algebra as an example

$$[x^a, \tilde{x}_b] = 2i\pi\lambda^2\delta^a_b$$

By definition a quantum space is the spectra of a maximally commutative *-subalgebra of the Heisenberg algebra

In general a Schrodinger like polarization is associated with classical Lagrangian sub-manifold of phase space. $F(\hat{x})$

Is there more than Lagrangian and Schrodinger space ?

Yes there are

Flat Modular space

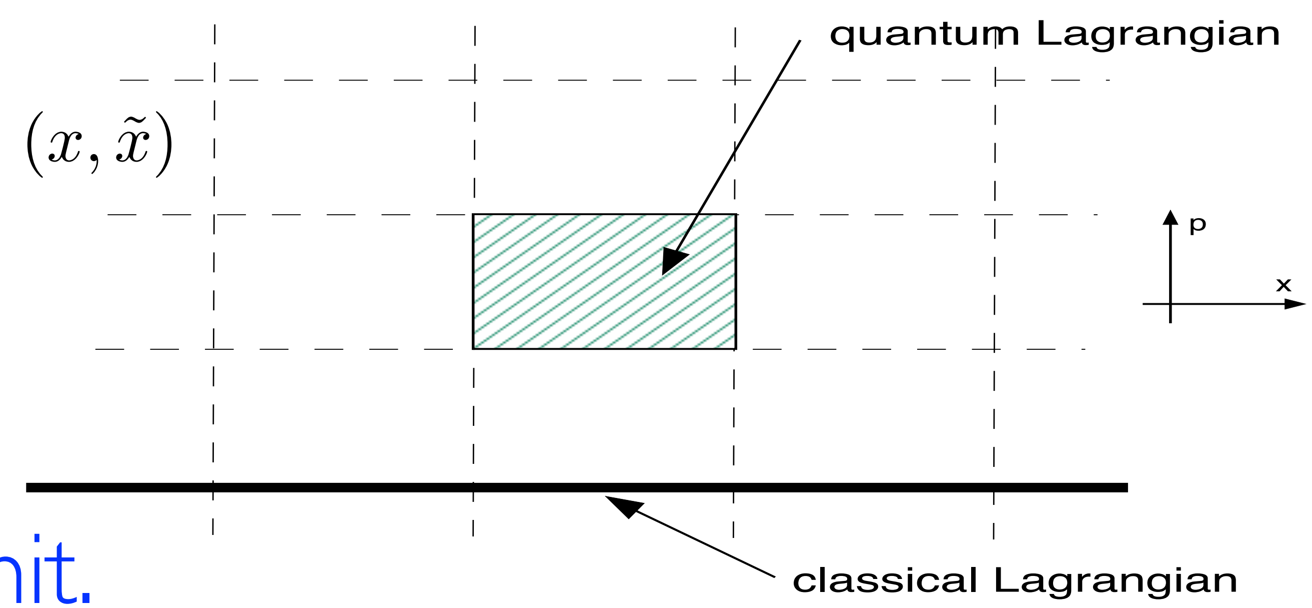
Flat Modular space are quantum space associated with abelian subgroup of the Heisenberg group. $[x^a, \tilde{x}_b] = 2i\pi\lambda^2\delta^a_b$

Such groups are generated by modular observables Aharanov

$$\boxed{\left[e^{i\frac{x}{R}}, e^{i\frac{\tilde{x}}{\tilde{R}}}\right] = 0} \qquad R\tilde{R} = \lambda^2$$

A quantum algebra possesses more commutative directions than a classical Poisson algebra $\{e^{i\frac{x}{R}}, e^{i\frac{\tilde{x}}{\tilde{R}}}\} \neq 0$

Modular uncertainty: can specify $(x, \tilde{x})$ within a cell of fixed area but no knowledge of which cell



Schroedinger rep is a singular limit.

1d modular line is a 2d torus compact and not simply-connected

Home space for compact string is a modular space

Geometry of String theory

(H, η, ω) Born geometry

$$\begin{aligned} H(\mathbb{K}, \mathbb{K}') &= \lambda^2(k \cdot k' + \tilde{k} \cdot \tilde{k}'), \\ \eta(\mathbb{K}, \mathbb{K}') &= \lambda^2(k \cdot \tilde{k}' + \tilde{k} \cdot k'). \\ \omega(\mathbb{K}, \mathbb{K}') &= \lambda^2(k \cdot \tilde{k}' - \tilde{k} \cdot k'). \end{aligned}$$

O(2,2d-2) metric

O(d,d) metric

Sp(2d) Symplectic

H defines the spectra, η level matching, (η, ω) mutual locality

$$\boxed{0 = H(\mathbb{K}, \mathbb{K}) + N_R + N_L - 2,}$$

$$\boxed{0 = \eta(\mathbb{K}, \mathbb{K}) + N_R - N_L.}$$

$$(\eta \pm \omega)(\mathbb{K}, \mathbb{K}') \in \mathbb{Z}$$

Para-hermitian lattice

= modes on Modular space

The new player compare to DFT is ω doubled space is a symplectic manifold

Born Geometry I

The structure group of Born Geometry:

Satisfy compatibility conditions:

$K = \eta^{-1}\omega$ Bilagrangian structure $K^2 = 1$

$J = \eta^{-1}H$ Chiral structure $J^2 = 1$

(η, ω) Defines the field commutator $\theta'(\sigma) = 2\pi\delta(\sigma)$, $\theta(\sigma) \in \pi\mathbb{Z}$

$$\boxed{[\mathbb{X}^A(\sigma_1), \mathbb{X}^B(\sigma_2)] = 2i[\pi\omega^{AB} - \eta^{AB}\theta(\sigma_{12})]}$$

J Defines the string equations of motion $\mathbb{X} = (x^a, \tilde{x}_a)$

$$\boxed{\partial_\tau \mathbb{X} = J\partial_\sigma \mathbb{X}.}$$

Curved Born Geometry

The structure group of Born Geometry is Lorentz

$$\boxed{\mathrm{Sp}(2d) \cap \mathrm{O}(d,d) \cap \mathrm{O}(2, 2(d-1)) = O(1,(d-1))}$$ Lorentz

Curving Born geometry amounts to 2 different operations:
It turns on non trivial fluxes: It promote the bilagrangian (η, K) structure into a para-Hermitian structure $\eta K = \omega$

$$\mathrm{d}\omega \neq 0$$

Fluxes

$$\mathrm{d}\omega = H_{abc}\mathrm{d}x^a \wedge \mathrm{d}x^b \wedge \mathrm{d}x^c + \cdots$$

The doubled space still admit a decomposition in its Lagrangian

$$TP = L \oplus \tilde{L}$$

L $\tilde{L}$ Are Lagrangian eigenbundles of K $\quad \eta|_L = \omega|_L = 0$

They generalises notion of spacetime and momentum space

This provides a gravitization of the quantum LF, Leigh, Minic

Curved Born Geometry

The structure group of Born Geometry is Lorentz

$$\mathrm{Sp}(2d) \cap \mathrm{O}(d,d) \cap \mathrm{O}(2, 2(d-1)) = O(1,(d-1))$$

Lorentz

Curving Born geometry amounts to 2 different operations:
It turns on non trivial fluxes: $\mathrm{d}\omega \neq 0$

It also means that we have a metric on each Lagrangian

$$TP = L \oplus \tilde{L}$$

$$H = \begin{pmatrix} g & 0 \\ 0 & g^{-1} \end{pmatrix}$$

Completely different philosophy from DFT we assign the B-fields and flux to ω

Explains why Lorentz, the structure group of g on L

Fluxes and non commutativity

B-transform

$$\begin{aligned} x^a &\to x^a \\ \tilde{x}_a &\to \tilde{x}_a + B_{ab}x^b \end{aligned}$$

Preserves space

Commutators

$$[x^a, x^b] = 0, \qquad [x^a, \tilde{x}_b] = 2\pi i \lambda^2 \delta^a_b, \qquad [\tilde{x}_a, \tilde{x}_b] = 0$$

Fluxes and non commutativity

B-transform
$$\begin{aligned} x^a &\rightarrow x^a \\ \tilde{x}_a &\rightarrow \tilde{x}_a + B_{ab}x^b \end{aligned}$$
Preserves space

Commutators becomes

$$[x^a, x^b] = 0, \qquad [x^a, \tilde{x}_b] = 2\pi i \lambda^2 \delta^a_b, \qquad [\tilde{x}_a, \tilde{x}_b] = B_{ab}$$

Dual non-commutativity

Satisfies Jacobi if dB=0.

Fluxes and non commutativity

beta-transform
$$\begin{aligned} x^a &\rightarrow x^a + \beta^{ab}\tilde{x}_b \\ \tilde{x}_a &\rightarrow \tilde{x}_a \end{aligned}$$
Preserves dual space

Commutators becomes

$$[x^a, x^b] = 2i\pi\lambda^2\beta^{ab}, \qquad [x^a, \tilde{x}_b] = 2\pi i\lambda^2\delta^a_b, \qquad [\tilde{x}_a, \tilde{x}_b] = 0$$

Space is non-commutative even in the limit $R \rightarrow \infty$

Explains clearly in what way this is a non-geometrical background: It is a non commutative one.

$$\mathrm{d}\omega \neq 0$$

Does it mean non associativity ?

D- bracket

Associated with a non necessarily integrable or symplectic para-hermitian structure. It is possible to construct the notion of a D-bracket on TP that generalises the Lie bracket. Moreover even if the Lie bracket $[\,,]$ is not associative the projected D-bracket $[\![\,,]\!]$ can be associative when projected onto its Lagrangian even when $\mathrm{d}\omega \neq 0$

D-bracket provides a dynamical generalisation of the notion of Courant bracket used in generalized geometry.

LF, Rudolph F., Svoboda, D.

Curved Born Geometry

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$$\boxed{\mathrm{Sp}(2d) \cap \mathrm{O}(d,d) \cap \mathrm{O}(2,2(d-1)) = O(1,(d-1))}$$ Lorentz

This suggest the following theorem proven recently:
There exists a unique connection-The Born connection-which preserves (H, η, ω) and is Torsionless in a generalised sense. This connection reduces to the Levi-Civita connection when projected onto its Lagrangian.

This resolves an old puzzle of DFT

This gives an interesting and new perspective on Levi-Civita which appears simply as the projection of the D-bracket

$$\nabla_x y = 4P[\![x_+, y_-]\!]_- \qquad x_\pm = x \pm g(x)$$

LF, Rudolph F., Svoboda, D.

Summary

- We have reviewed very briefly the idea of relative locality and showed how this concept give a powerful new perspective on the geometry of string theory.
- We have seen that the zero mode positions of the compact string are non-commutative. While the Home space of the compact string is a modular space: The string folds itself onto one of its quantum Lagrangian.
- The consistency of this structure leads at the classical level to the construction of a Born geometry that includes a para-hermitian structure encoded the mutual locality of vertex operators plus a generalized metric encoding the spectra of this operators.

- The new ingredient ω missed by DFT promotes double space to a phase space and allows the dynamical selection of the section condition and the construction of a unique connection

Road map for QG/ST Geometry

- What is the fundamental new relativity principle ?
  Relative locality
- What is the simplest implementation of that idea ?
  Modular space
- What geometry represents that idea ?
  Born geometry
- Is there a model that can guide us through the maze of new concepts ? Metastring theory
- There is in addition to the pairing and metric an additional structure a 2 form ω that geometrically control the deformation of the differential structure and the choice of section condition.

Conclusion

We have seen that the dynamicalisation of the section condition naturally involves a new notion of space that incorporate

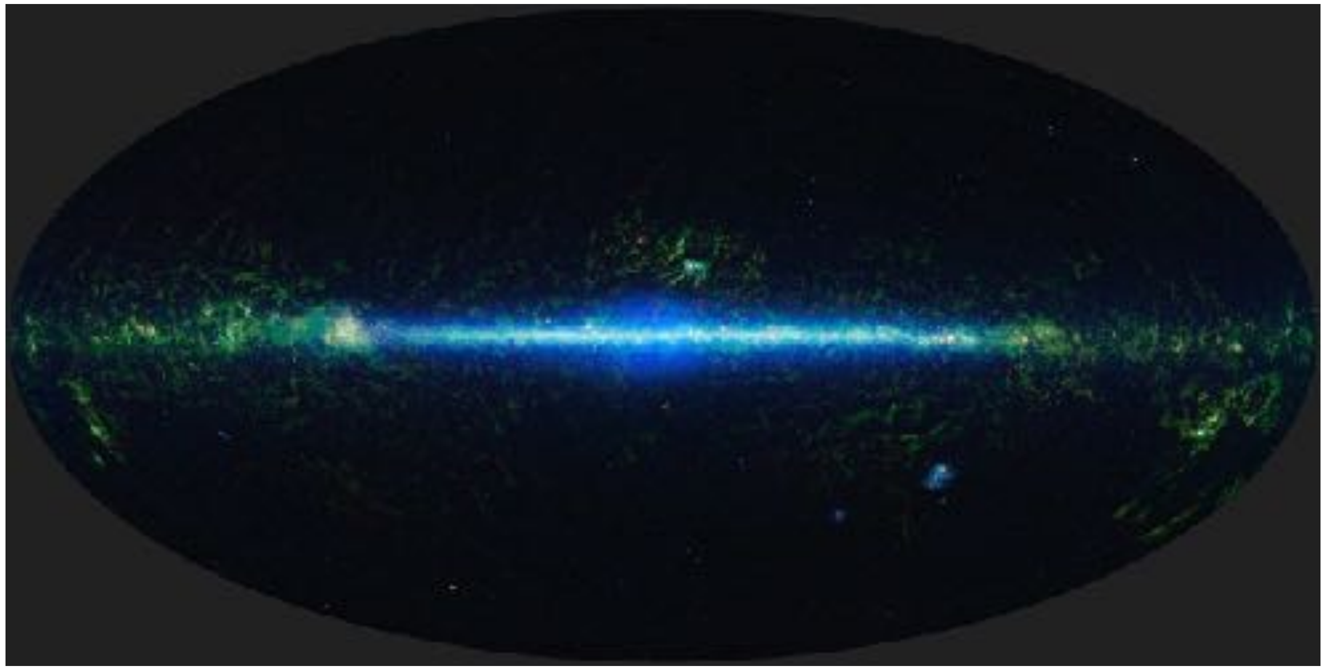
- fundamentally quantum
- fundamentally non-local
- respecting the principle of relative locality
- reconciling discreetness with relativity

It is tempting to think that it implies generalization of the concept of fields that goes beyond DFT

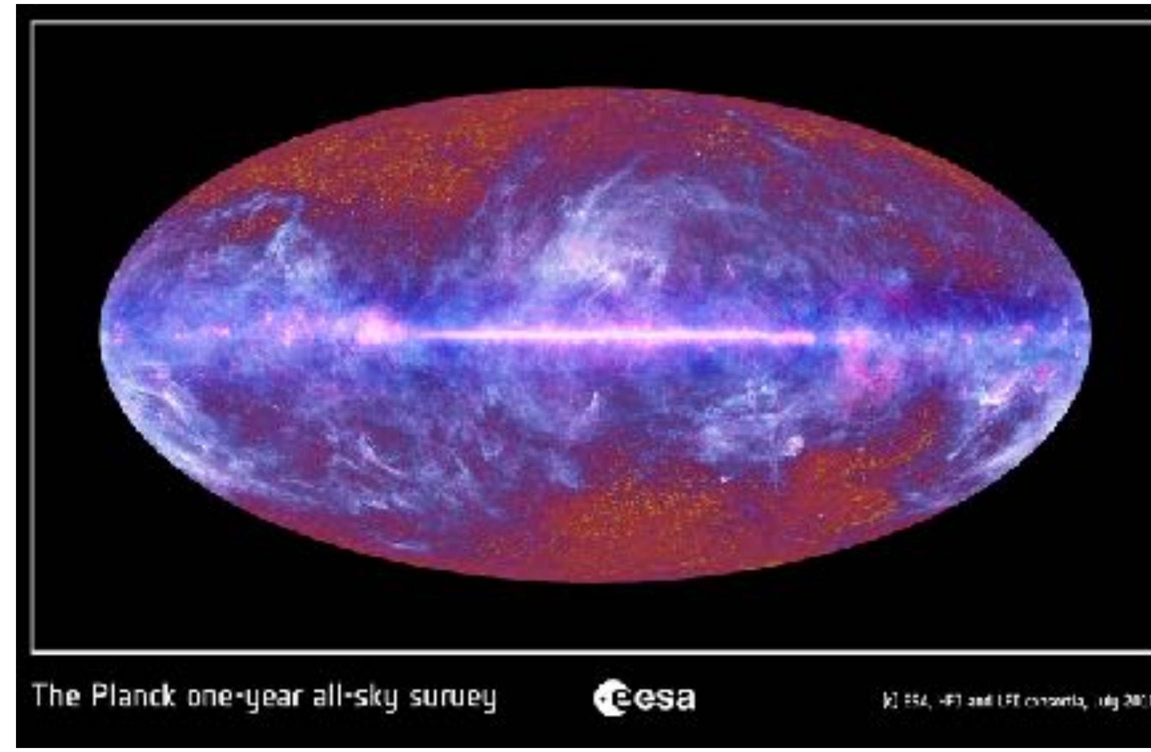
Generic prediction: Fundamental UV-IR mixing
To be tested in Quantum cosmology: First opportunity to finally adress the fundamental problem of quantum cosmology

See Brandenberger talk.

Quantum Gravity in the sky??



Wise infrared



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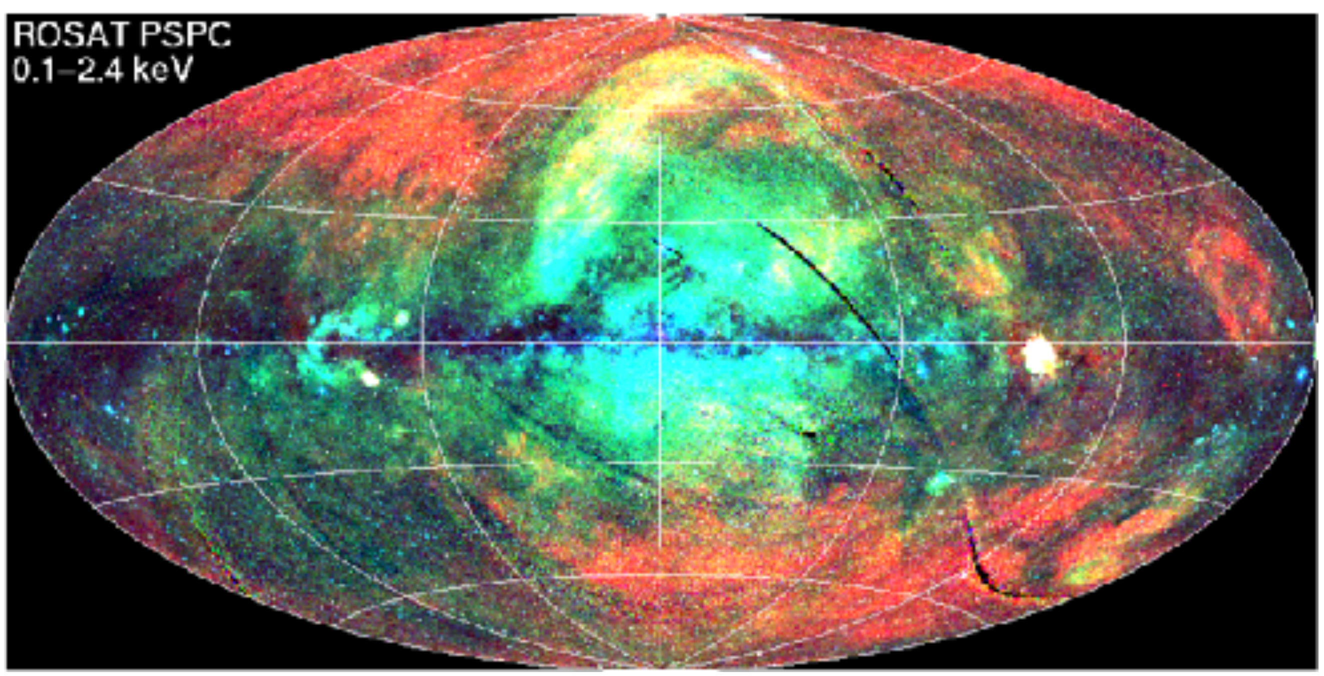

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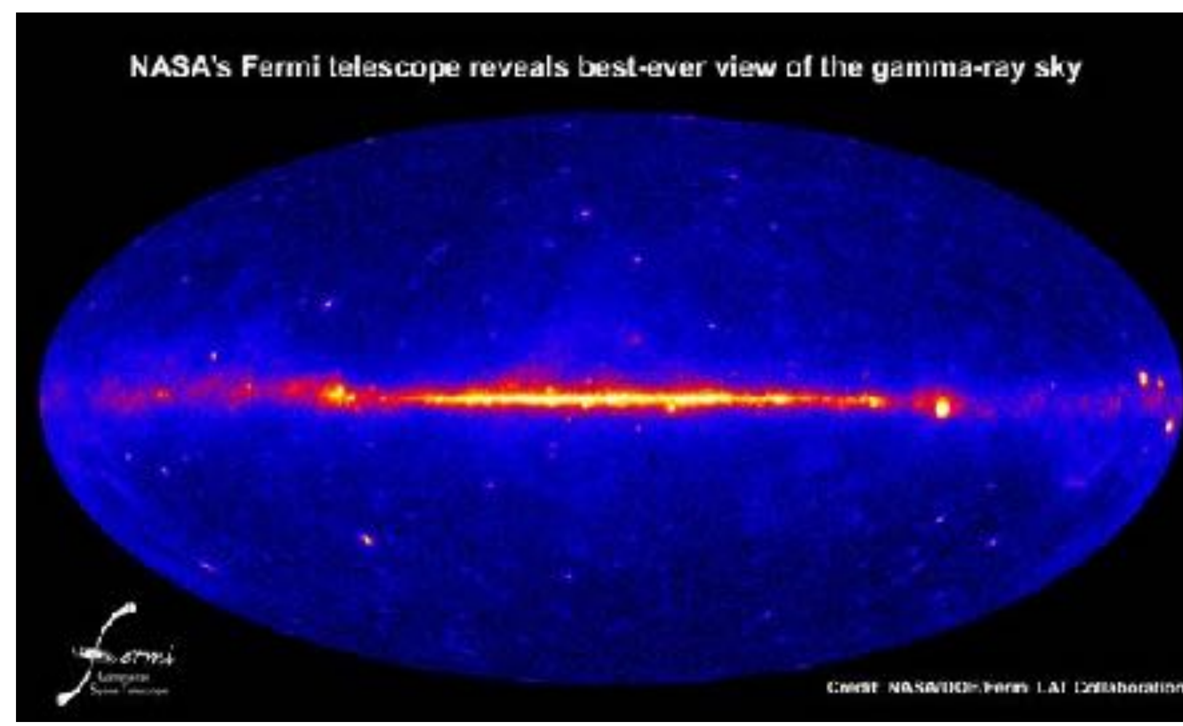


Fermi Gamma ray

Lessons from Quantum-Gravity

- We have many different approaches to the problem.
- Strings, Loops, Emergent gravity, AdS/CFT, Non Gaussian fixed point, CDT etc…
- What have we learned so far? What do they all have in common?
  - The only common theme between all of them is non-locality
  - non-local observables in background independent approach
  - non-local probes
  - non-locality of holography
  - discreteness
  - non-local fixed point, etc…

The Challenges of non-Locality

- We expect that any theory of quantum gravity will involve some non-locality. How do we deal with non-locality without opening Pandora's Box?

- Locality is built in Field Theory and General Relativity:

locality of asymptotic states,
locality of interactions,
locality of RG =separation of scales.

These are the foundations of modern physics.

We need to specify what type of non-locality is viable, we need a new principle to tame non-locality.

What kind of non locality?

A new take on quantum gravity: It should emerge from a theory which is quantum has a fundamental delocalisation scale and satisfies the relativity principle. Non-locality cannot be arbitrary.

One of the fundamental challenges is to reconcile having a fundamental scale with Lorentz invariance

Instead of assuming the existence of an absolute space time one define it through localization/interaction of fundamental probes. Each probes might define its own notion of space-time: Locality is relative

- Relative locality

Relative Locality is taken as the organizational feature allowing us to tame non locality.

- Born Duality

ability to change polarization

Modular space

A generic polarization is in fact a modular space. Modular space have a built-in length and energy scales, they are fundamentally quantum

Is there a physical system where modular spacetime is realized ?
Yes there is: In string theory
We can show that the Home space of closed string is a modular space. It is not doubled. It is not compactified.
Modularity is the target space realization of T-duality.

Modular space

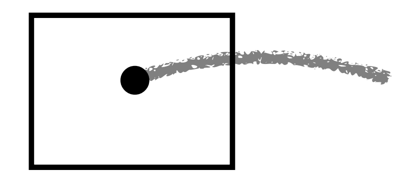
$$[e^{2i\pi x}, e^{2i\pi \tilde{x}}] = 0$$

A generic commutative subalgebra is associated with a lattice in phase space $\Lambda \in \mathbf{P}$

A modular wave function is quasi-periodic

$$\Psi(x + a, \tilde{x}) = e^{2i\pi a\tilde{x}}\Psi(x, \tilde{x}) \qquad \Psi(x, \tilde{x} + \tilde{a}) = \Psi(x, \tilde{x})$$

The quasi-periods correspond to the tails of an Aharonov-Bohm potential attached to a unit flux



The Hilbert space corresponds to sections of a U(1) bundle

$$\mathcal{H}_\Lambda = \Gamma(L_\Lambda) \qquad L_\Lambda \to T_\Lambda \quad = \mathbf{P}/\Lambda$$

1d Euclidean space is non compact, simply connected
1d modular space is 2d, compact and not simply-connected

It carries flux and the doubling is a choice of polarisation.

Lorentz covariance of modular space

In order to construct $\mathcal{H}_\Lambda = \Gamma(L_\Lambda)$ we need to choose a lift

$$T_\Lambda \to L_\Lambda$$

A lift determines a polarisation metric η
A vacuum determines a quantum metric H

$$(\boldsymbol{P}, \omega, H, \eta)$$

There are no translational invariant vacua, since translations do not commute, space corresponds to a broken phase of the Heisenberg group viewed as a translational group.

The unbroken symmetry group is the lattice translation.
Space corresponds to a Cartan subgroup. G_Λ

This is why we can reconcile for the first time fundamental discreetness and translational and Lorentz symmetries.

Lorentz covariance of modular space

Besides the translations the Heisenberg group is invariant under

$$\mathrm{Sp}(2d)$$

The choice of modular polarisation η break it down to

$$\mathrm{Sp}(2d) \cap \mathrm{O}(d,d) = GL(d)$$

a frame

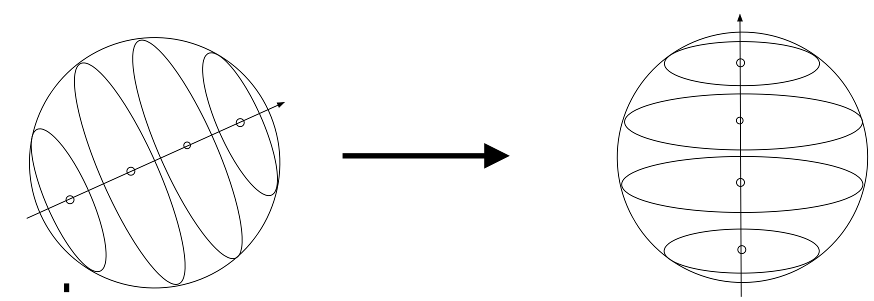
The choice of vacua H break it further to

$$\boxed{\mathrm{Sp}(2d) \cap \mathrm{O}(d,d) \cap \mathrm{O}(2,2(d-1)) = O(1,(d-1))}$$

Lorentz

Under a boost $O\Lambda \neq \Lambda$ but $[G_\Lambda, G_{O\Lambda}] \neq 0$

A boost is a change of polarization



Analog to rotation of the frame of a spin $\sigma_x \to \sigma_z$

A boosted state is a superposition of unboosted ones $\Psi_\Lambda \to \Psi_{O\Lambda} = U_{O\Lambda}\Psi_\Lambda$

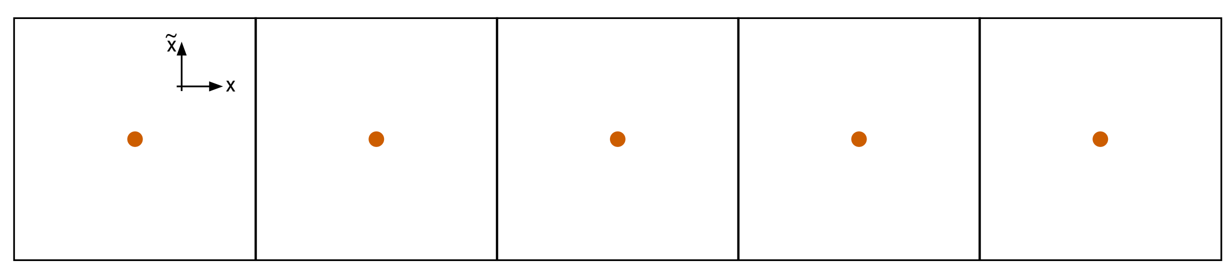
Many is Large

Under a boost $\Psi_\Lambda \to \Psi_{O\Lambda} = U_{O\Lambda}\Psi_\Lambda$

This is the essence of Relative locality: Different boosted observers experience different space-times

Space is definitely not compactified: it is modular

We get from quantum to classical through a many body limit of extensification



$\left(\mathcal{H}_\Lambda^{(1)}\right)^{\otimes N}$

It leads to a form of unification between matter-like dof and pure geometry dof : flux unit $\mathcal{H}_{N\Lambda} \simeq \mathcal{H}^{\otimes \Lambda}$

Singular limits

In any breakthrough, invisible phenomena become visible:
The lower order description is a singular limit of the higher one (M-Berry). That is, a mathematically consistent description which cannot reveal certain observables.

- Eulerian fluid is a singular limit of the viscous fluid: planes can't fly
- geometrical optic-wave optics: No central bright spot
- Classical-Quantum: Aharonov-Bohm phases are invisible
- Non relativivistic-relativistic Quantum Field Theory: No anti-particle
- Newton-GR: No gravity waves

Duality and Unification

In any breakthrough, invisible phenomena become visible, but also a fundamental form of unification takes place. Seemingly opposite concepts in the original picture are unified in the more advanced one. Each time a fundamental constant is understood as a conversion factor.

- h: Unification of wave and particle
- c: Unification of space and time
- G: Unification of Inertial and gravitational mass
- k: Unification of Energy and information
- h,c: Unification of quanta and fields
- G,c: Unification of matter and geometry

$$G_{ab} = 8\pi G T_{ab}$$

What's next?

What is Home space ?

So far we have presented the classical side of relative locality. At the quantum level phase space is promoted to a non-commutative Heisenberg algebra.

In QM Euclidean space appears simply as a choice of polarization: That is in the argument of the wave function. This is the quantum analog of a choice of Lagrangian

$$\Psi(x) \rightarrow \Phi(x)$$

Similarly Lorentzian space appears simply as a field label. Classical locality is built in the field definition.

$$(i\partial_t - H)\Psi = 0 \rightarrow \Box_g \Phi = 0$$

Can we define a notion of quantum space? quantum space-time?

Quantum + Gravity

- Should we care about putting together gravity and the quantum?

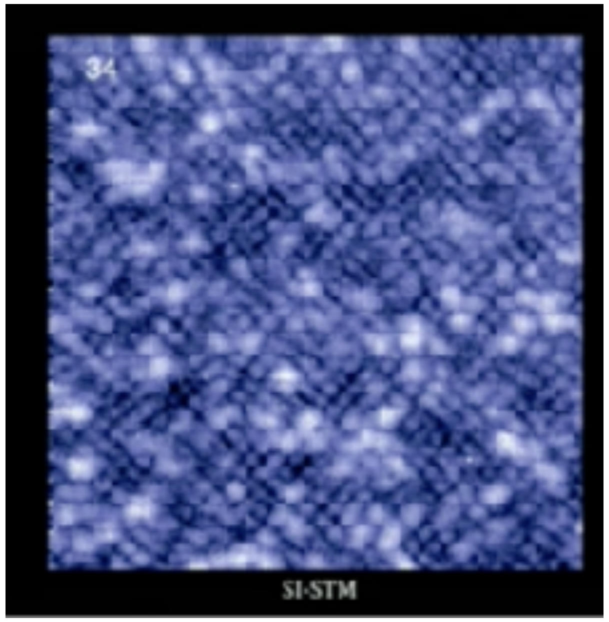
  Yes: We expect radically new phenomena to become visible, not just small corrections to known phenomena, more than EFT.

- Quantising gravity? : Doesn't work — non-renormalisable, Asymptotic Safety
- Quantising geometry ? : Background independence and non local observables, space is fundamentally discrete, built-in Hilbert space bases. But the challenge is reconciliation with the General relativity principle outside the classical limit.
- String Theory ?: The probe is fundamental, delocalising the probe, consistent with relativity, but it hasn't changed yet our understanding of space and time at the fundamental level.
- Emergent models ?: CDT, Causal sets, Horava Gravity, CMT inspired or Holography AdS/CFT

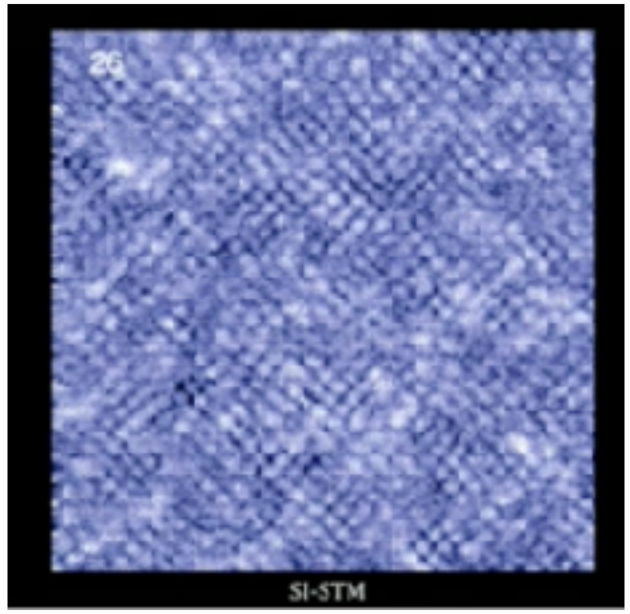
- What have we learned? what are we missing? what haven't we tried?

Visualization of wave function

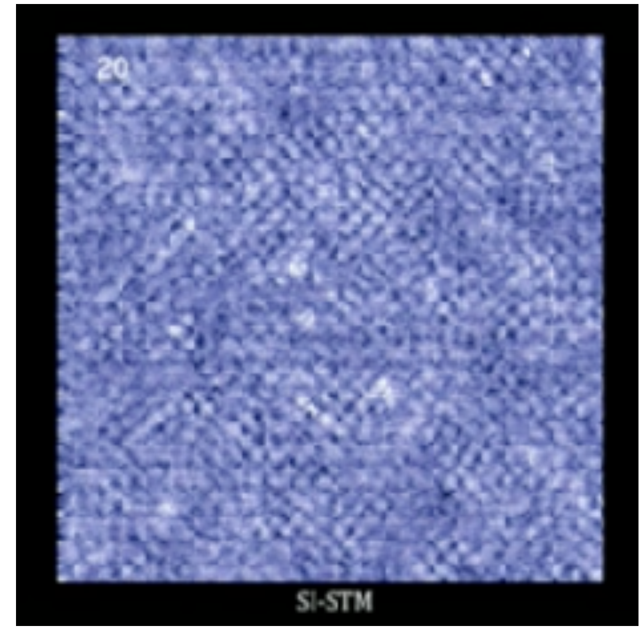
S. Davis



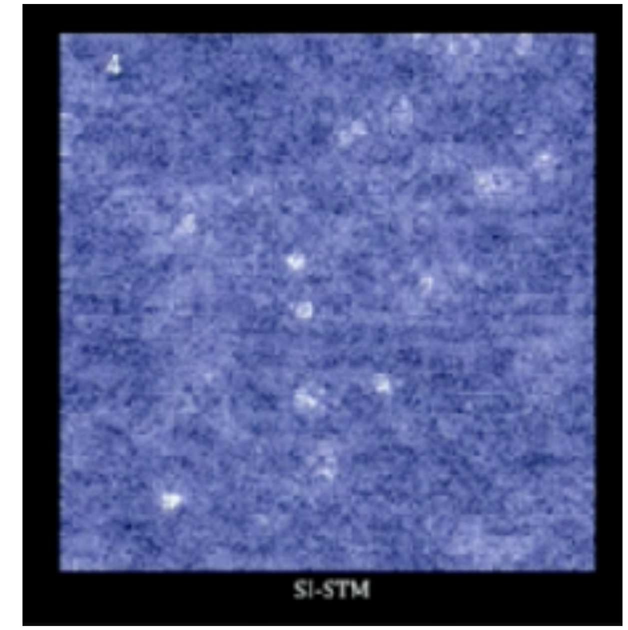
High E
disordered



Quasi-Particle interferences:
Friedel oscillations



ordered



Low E
translation invariant

The classical question: is it ordered or disordered? is ill-defined in QM. It depends on the observation not just the system!

In the same region of space we can have different eigenstates of different energy. It is disordered at a given energy and ordered at another.

localization is in beholder's eye

key element: lattice scale

Analogy: Quantum crystal = spacetime
electrons= probes.

Locality is relative but special relativity is missing

Notion of spaces

Our concepts of space and time have radically evolved over history

- Euclid: Notion of absolute space
- Galileo: Relativity of observers in space : velocity is relative
- Newton: Motion generated by forces : action at a distance
- Einstein: Time is relative
- Einstein: Relativity of general observers

Quantum mechanic hasn’t affected our fundamental picture of space and time yet

Notion of Matter

The history of the concept of matter is more intertwined, it is a constant dialogue between the idea of individual objects versus continuum fields

- Democritus: Atomicity
- Aristotle: Continuum hypothesis : `There is no void' Descartes
- Newton: there are individual macroscopical objects acting on each other, due to their charges, mass.
- Faraday : Fields are real
- Maxwell: Fields in space are dynamical
- Heisenberg-Born-Jordan: Discovery of Quantum mechanics: Atoms are stable after all, fundamental discreteness .
- Dirac: Quantum Fields are relativistic: Anti-particles
- Kramers-Heisenberg-Mandelstam-Chew: S-matrix: scattering of asymptotic states are the only observables (fields are not real)
- Veneziano, Nambu,…: String theory, probes are non local

Classical space still appears as wave function, fields or string labels. $\Phi(x)$
Quantum matter on classical space-time

What is Relative locality?

Simply using Heisenberg uncertainty relation but also demanding that the equivalence principle holds in quantum mechanics means that Home space is at least relative to energy-momentum in phase space.

The geometry of relative locality allows to unify mathematically the two notions of proximity: Close in space or close in state.

Why? How to implement it? What are the elements?

What is the geometry of relative locality

Road map for QG

- What is the fundamental new relativity principle ?

Road map for QG/ST geometry

- What is the fundamental new relativity principle ?
  Relative locality
- What is the simplest implementation of that idea ?
  Modular space
- What geometry represents that simple idea ?
  Born geometry
- Is there a model that can guide us through the maze of new concepts ?
  Metastring theory (dual symmetric string) and string geometry
- Is there any generic predictions ?

Road map for QG

- What is the fundamental new relativity principle ?
  Relative locality

Road map for QG

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