Towards Cosmology of Double Field Theory

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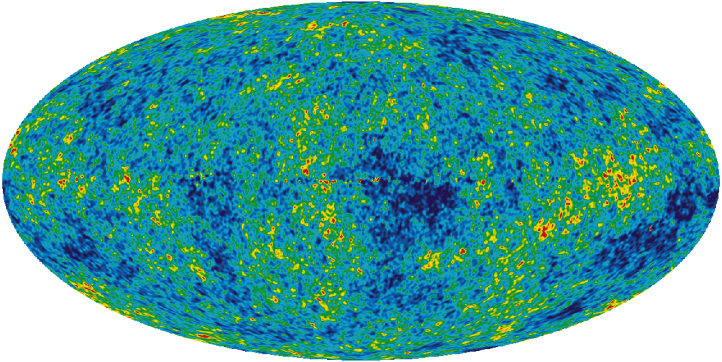
Outline

Plan

Inflation: the Standard Model of Early Universe Cosmology

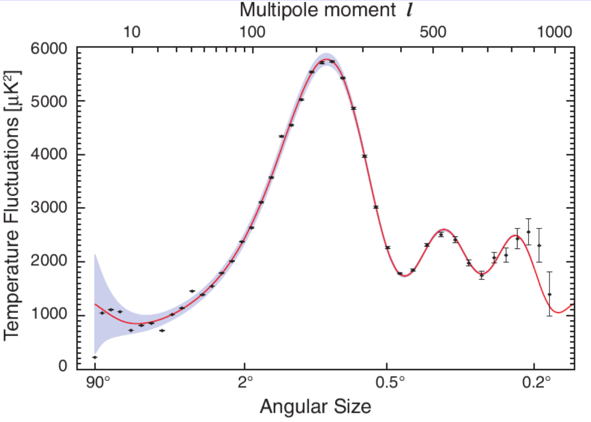
- Inflation is the standard paradigm of early universe cosmology.
- Inflation solves conceptual problems of Standard Big Bang Cosmology.
- Inflation predicts an almost scale-invariant spectrum of primordial cosmological perturbations with a small red tilt (Chibisov & Mukhanov, 1981).
- Fluctuations are nearly Gaussian and nearly adiabatic.

Map of the Cosmic Microwave Background (CMB)



Credit: NASA/WMAP Science Team

Angular Power Spectrum of CMB Anisotropies



Credit: NASA/WMAP Science Team

Motivation

- No convincing embedding of inflation in string theory exists.
- Alternatives to cosmological inflation for producing the structure we observe exist.
- Question: what early universe scenario emerges from string theory?
- Key tool: symmetries of string theory.

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Criteria

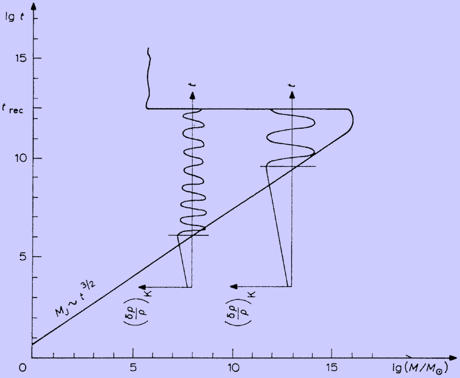

Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

Key Realization

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

- Given a scale-invariant power spectrum of adiabatic fluctuations on "super-horizon" scales before t_{eq}, i.e. standing waves.
- $\rightarrow$ "correct" power spectrum of galaxies.
- $\rightarrow$ acoustic oscillations in CMB angular power spectrum.

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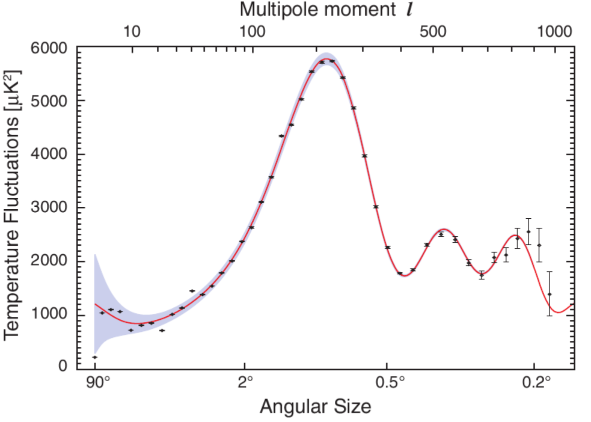

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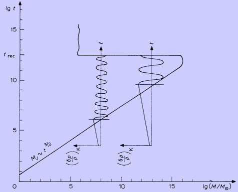
Early Work

1970 paper



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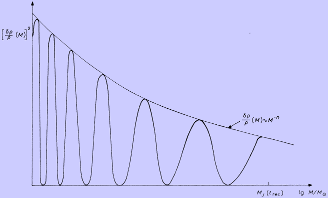

Fig. 1b. The dependence of the square of the amplitude of density perturbations of matter on scale. The fine line designates the usually assumed dependence $(\delta\varrho/\varrho)_M \sim M^{-n}$. It is apparent that fluctuations of relic radiation should depend on scale in a similar manner.

R. Sunyaev & Ya. Zeldovich, Astrophysics and Space Science 7 3-19 (1970)

Predictions from 1970

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

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- $\rightarrow$ "correct" power spectrum of galaxies.
- $\rightarrow$ acoustic oscillations in CMB angular power spectrum.
- $\rightarrow$ baryon acoustic oscillations in matter power spectrum.

Key Challenge

How does one obtain such a spectrum?

- Inflationary Cosmology is the first scenario based on causal physics which yields such a spectrum.
- But it is not the only one.

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Hubble Radius vs. Horizon

- Horizon: Forward light cone of a point on the initial Cauchy surface.
- Horizon: region of causal contact.
- Hubble radius: $l_H(t) = H^{-1}(t)$ inverse expansion rate.
- Hubble radius: local concept, relevant for dynamics of cosmological fluctuations.
- In Standard Big Bang Cosmology: Hubble radius = horizon.
- In any theory which can provide a mechanism for the origin of structure: Hubble radius $\neq$ horizon.

Criteria for a Successful Early Universe Scenario

- Horizon $\gg$ Hubble radius in order for the scenario to solve the "horizon problem" of Standard Big Bang Cosmology.
- Scales of cosmological interest today originate inside the Hubble radius at early times in order for a causal generation mechanism of fluctuations to be possible.
- Squeezing of fluctuations on super-Hubble scales in order to obtain the acoustic oscillations in the CMB angular power spectrum.
- Mechanism for producing a scale-invariant spectrum of curvature fluctuations on super-Hubble scales.

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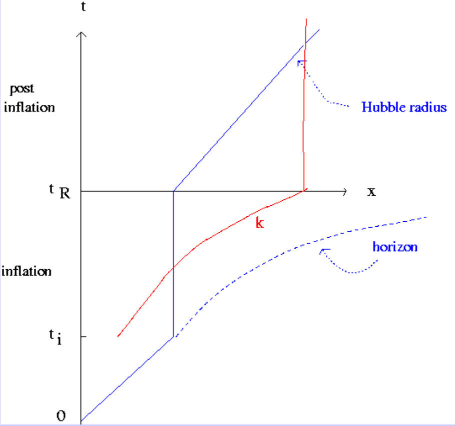
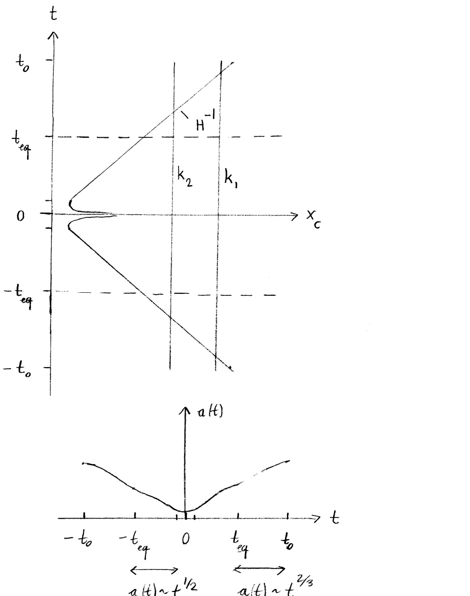
Inflation as a Solution



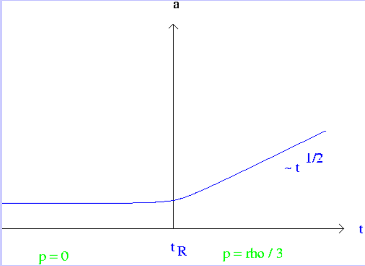
Matter Bounce as a Solution

F. Finelli and R.B., *Phys. Rev. D65, 103522 (2002)*, D. Wands, *Phys. Rev. D60 (1999)*



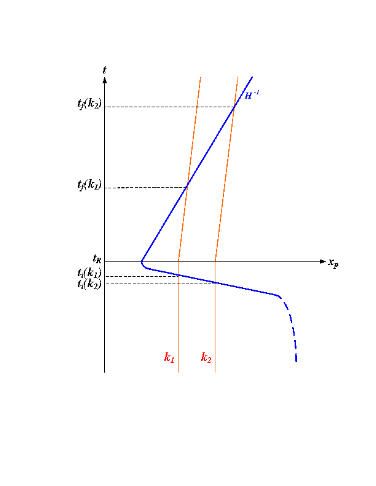
Emergent Universe

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*



Emergent Universe as a Solution

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



Which paradigm arises from string theory?

Plan

String States

Assumption: All spatial dimensions toroidal, radius R.

String states:

- momentum modes: $E_n = n/R$
- winding modes: $E_m = mR$
- oscillatory modes: E independent of R

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- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \to 1/R \quad (n, m) \to (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\to$ existence of D-branes

Position Operators

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Position operators (dual to momenta)

$$|x> = \sum_p \exp(ix \cdot p)|p>$$

Dual position operators (dual to windings)

$$|\tilde{x}> = \sum_w \exp(i\tilde{x} \cdot w)|w>$$

Note:

$$|x> = |x + 2\pi R>, \quad |\tilde{x}> = |\tilde{x} + 2\pi \frac{1}{R}>$$

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Heavy vs. Light Modes

- $R \gg 1$: momentum modes light.
- $R \ll 1$: winding modes light.
- $R \gg 1$: length measured in terms of $|x>$.
- $R \ll 1$: length measured in terms of $|\tilde{x}>$
- $R \sim 1$: both $|x>$ and $|\tilde{x}>$ important.

Conclusion: At string scale densities usual effective field theory (EFT) based on supergravity will break down.

Conclusion: If an effective field theory description is valid, it must be an EFT in 18 spatial dimensions.

Double Field Theory: Promising candidate for string cosmology.

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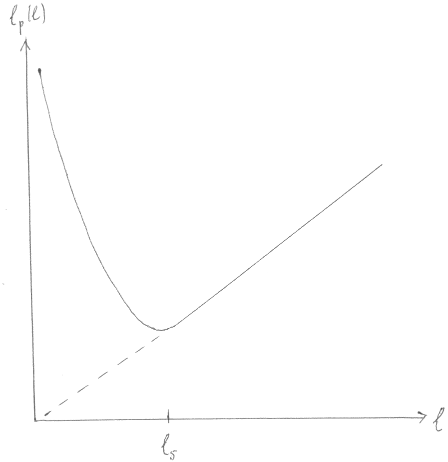
Physical length operator

$$
\begin{aligned}
l_p(R) &= R \quad R \gg 1 \\
l_p(R) &= \frac{1}{R} \quad R \ll 1
\end{aligned}
$$

Physical length



Plan

Geodesic Completeness

R.B., R. Costa, G. Franzmann and A. Weltman, arXiv:1710.02412 [hep-th]

Recall: For each dimension of the underlying topological space there are two position operators [R.B. and C. Vafa]:

- x: dual to the momentum modes
- $\tilde{x}$: dual to the winding modes

We measure **physical length** in terms of the **light** degrees of freedom.

$$
\begin{aligned}
l(R) &= R \quad \text{for} \quad R \gg 1\,, \\
l(R) &= \frac{1}{R} \quad \text{for} \quad R \ll 1\,.
\end{aligned}
$$

Doubled Space Approach

$$dS^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j - a^{-2}(t)\delta_{ij}d\tilde{x}^i d\tilde{x}^j$$

Point particle geodesic:

$$\frac{d}{dS}\left(\frac{dx^i}{dS}a^2\right) = 0$$
$$\frac{d}{dS}\left(\frac{d\tilde{x}^i}{dS}a^{-2}\right) = 0$$

Initial conditions: related by duality.

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Proper Time along Geodesic

Assume $a(t)$ as in Standard Big Bang Cosmology.

Proper distance into the future from some time t_0 to some time $t_2 \gg t_0$:

$$\Delta S = \int_{t_0}^{t_2} a(t)\gamma(t)^{-1} dt + T_2 \, ,$$

Proper distance into the past from some time t_0 to some time $t_1 \ll t_0$:

$$\Delta S = \int_{t_1}^{t_0} a(t)^{-1}\tilde{\gamma}^{-1}(t) dt + T_1 \, ,$$

Interpretation

- Expansion of the scale factor in the dual spatial directions as time decreases $\equiv$ expansion in the regular directions as time increases.
- Dynamics of the dual spatial dimensions as t decreases **is measured** as expansion when the dual time $t_d = \frac{1}{t}$ decreases.

Proposal:

$$
\begin{aligned}
t_p(t) &= t \quad \text{for} \quad t \gg 1 \, , \\
t_p(t) &= \frac{1}{t} \quad \text{for} \quad t \ll 1 \, .
\end{aligned}
$$

Conclusion: Point particle geodesics can be extended in both time directions to infinite proper time.

Nonsingular String Cosmology

R.B., R. Costa, G. Franzmann and A. Weltman, arXiv:1805.06321 [hep-th]

Consider Dilaton gravity

$$
\begin{aligned}
\left(\dot{\phi} - dH\right)^2 - dH^2 &= e^{\phi}\rho \\
\dot{H} - H\left(\dot{\phi} - dH\right) &= \frac{1}{2}e^{\phi}p \\
2\left(\ddot{\phi} - d\dot{H}\right) - \left(\dot{\phi} - dH\right)^2 - dH^2 &= 0
\end{aligned}
$$

coupled to string gas matter.

$$
w(a) = \frac{2}{\pi d}\arctan\left(\beta \ln\left(\frac{a}{a_0}\right)\right),
$$

Limiting Solutions

Large radius limit:

$$\rho\,(a\text{ large}) \rightarrow \rho_0\,(a/a_0)^{-(d+1)}\,,$$

Small radius limit:

$$\rho\,(a\text{ small}) \rightarrow \rho_0\,(a/a_0)^{-d+1}$$

Ansatz:

$$\begin{aligned} a(t) &\sim (\frac{t}{t_0})^{\alpha} \\ \bar{\phi}(t) &\sim \beta\ln(t/t_0)\,, \end{aligned}$$

Where

$$\bar{\phi} \equiv \phi - d\ln(a)$$

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Limiting Solutions

Hagedorn phase, $w = 0$:

$$(\alpha, \beta) = (0, 2) .$$

Note: Static in string frame.

Large a phase, $w = 1/d$:

$$(\alpha, \beta) = (\frac{2}{D}, \frac{2}{D}(D-1)) .$$

Note: constant dilaton.

Small a phase, $w = -1/d$:

$$(\alpha, \beta) = (-\frac{2}{D}, \frac{2}{D}(D-1)) .$$

Interpretation

- Bouncing cosmology in the string frame $\rightarrow$ nonsingular.
- Contracting cosmology for $t \rightarrow 0$ in the Einstein frame.
- As $t \rightarrow 0$ the energy of the string gas drifts to winding modes.
- Physical space is measured in terms of winding modes.
- In terms of winding modes the contraction as $t \rightarrow 0$ corresponds to expansion.
- $t \rightarrow 0 \quad \equiv \quad t_d \rightarrow \infty$
- In terms of physical variables: bouncing cosmology.
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Next Step: Double Field Theory as a Background for String Gas Cosmology

Idea Describe the low-energy degrees of freedom with an action in doubled space in which the T-duality symmetry is manifest.

Candidate for dynamics in the Hagedorn phase: Double Field Theory [W. Siegel, 1993, C. Hull and B. Zwiebach, 2009, L. Freidel et al., 2017]

$$S = \int dx d\tilde{x} e^{-2d} \mathcal{R},$$

$$\begin{aligned}
\mathcal{R} = & \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL} \\
& + 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M d\partial_N d \\
& + 4\partial_M\mathcal{H}^{MN}\partial_N d + \frac{1}{2}\eta^{MN}\eta^{KL}\partial_M\mathcal{E}^A{}_K\partial_N\mathcal{E}^B{}_L\mathcal{H}_{AB}\,.
\end{aligned}$$

$$
\begin{aligned}
\mathcal{H}_{MN} &= \begin{bmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{bmatrix}. \\
X^M &= (\tilde{x}_i, x^i), \\
\eta^{MN} &= \begin{bmatrix} 0 & \delta_i{}^j \\ \delta^i{}_j & 0 \end{bmatrix}.
\end{aligned}
$$

Cosmology of DFT

R.B., R. Costa, G. Franzmann and A. Weltman, in preparation

Add matter action S_m to the background action of SGC:

$$S = \int dx d\tilde{x} e^{-2d} \mathcal{R} + S_m$$

Consider generalized Friedmann metric:

$$ds^2 = dt^2 + d\tilde{t}^2 - a(t)^2 dx^2 - \frac{1}{a^2(t)} d\tilde{x}^2$$

Physical time constraint:

$$|\tilde{t}| = \frac{1}{|t|}$$

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Equations of Motion

$$\begin{aligned} 2\bar{\phi}^{`'} - (\bar{\phi}')^2 - (D-1)\tilde{H}^2 + 2\ddot{\bar{\phi}} - & \\ (\dot{\bar{\phi}})^2 - (D-1)H^2 &= 0 \\ (D-1)\tilde{H}^2 - \bar{\phi}^{`'} - (D-1)H^2 + \ddot{\bar{\phi}} &= \frac{1}{2}e^{\bar{\phi}}\bar{\rho} \\ \tilde{H}^{`} - \tilde{H}\bar{\phi}' + \dot{H} - H\dot{\bar{\phi}} &= \frac{1}{2}e^{\bar{\phi}}\bar{p} \end{aligned}$$

where

$$\begin{aligned} \bar{\phi} &= \phi - (D-1)\ln a \\ ` &= \frac{\partial}{\partial \tilde{t}} \\ \tilde{H} &= \frac{a'}{a} \end{aligned}$$

Plan

String Gas Cosmology

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings.

Assumption: $g_s \ll 1$.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

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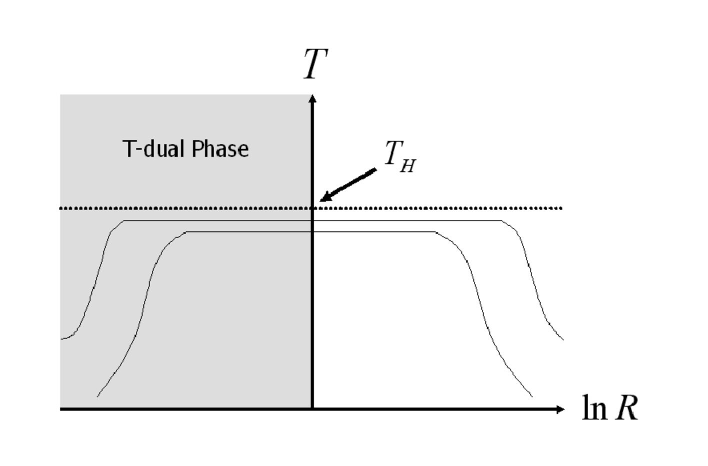
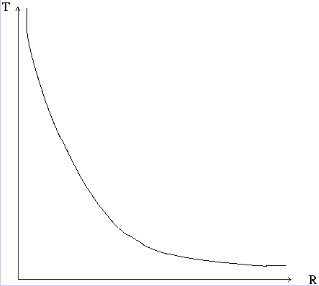
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

Temperature-size relation in string gas cosmology



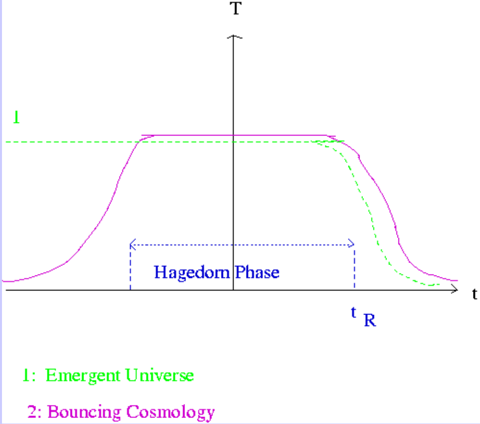
Singularity Problem in Standard and Inflationary Cosmology

Temperature-size relation in standard cosmology



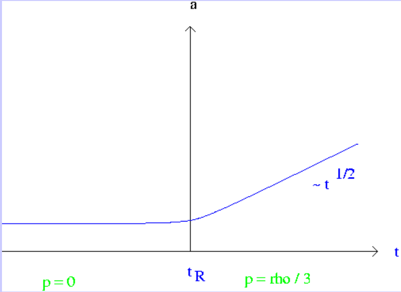
Dynamics

Assume some action gives us $R(t)$



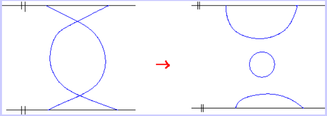
Dynamics

We will thus consider the following background dynamics for the scale factor $a(t)$:



Dynamical Decompactification

- Begin with all 9 spatial dimensions small, initial temperature close to T_H $\rightarrow$ winding modes about all spatial sections are excited.
- Expansion of any one spatial dimension requires the annihilation of the winding modes in that dimension.



- Decay only possible in three large spatial dimensions (see also M. Sakellariadou).
- $\rightarrow$ dynamical explanation of why there are exactly three large spatial dimensions.

Note: For $R \rightarrow 0$ there is an analogous decompactification mechanism which only allows three dual dimensions to be large.

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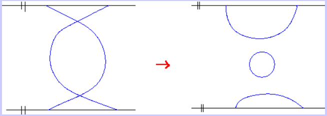

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Moduli Stabilization in SGC

Size Moduli [S. Watson, 2004; S. Patil and R.B., 2004, 2005]

- winding modes prevent expansion
- momentum modes prevent contraction
- $\rightarrow \ V_{eff}(R)$ has a minimum at a finite value of $R, \ \rightarrow \ R_{min}$
- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at R_{min}
- $\rightarrow V_{eff}(R_{min}) = 0$
- $\rightarrow$ size moduli stabilized in Einstein gravity background

Shape Moduli [E. Cheung, S. Watson and R.B., 2005]

- enhanced symmetry states
- $\rightarrow$ harmonic oscillator potential for θ
- $\rightarrow$ shape moduli stabilized

Dilaton stabilization in SGC

- The only remaining modulus is the dilaton.
- Make use of gaugino condensation to give the dilaton a potential with a unique minimum.
- $\rightarrow$ diltaton is stabilized.
- Dilaton stabilization is consistent with size stabilization [R. Danos, A. Frey and R.B., 2008].
- Gaugino condensation induces (high scale) supersymmetry breaking [S. Mishra, W. Xue, R.B. and U. Yajnik, 2012].

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Plan

Theory of Cosmological Perturbations: Basics

Cosmological fluctuations connect early universe theories with observations

- Fluctuations of matter $\rightarrow$ large-scale structure
- Fluctuations of metric $\rightarrow$ CMB anisotropies
- N.B.: Matter and metric fluctuations are coupled

Key facts:

- 1. Fluctuations are small today on large scales
- $\rightarrow$ fluctuations were very small in the early universe
- $\rightarrow$ can use linear perturbation theory
- 2. Sub-Hubble scales: matter fluctuations dominate
- Super-Hubble scales: metric fluctuations dominate

Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep. 215:203 (1992)*

Step 1: Metric including fluctuations

$$ds^2 = a^2[(1 + 2\Phi)d\eta^2 - (1 - 2\Phi)d\mathbf{x}^2]$$

$$\varphi = \varphi_0 + \delta\varphi$$

Note: Φ and $\delta\varphi$ related by Einstein constraint equations

Step 2: Expand the action for matter and gravity to second order about the cosmological background:

$$S^{(2)} = \frac{1}{2}\int d^4x\left((v')^2 - v_{,i}v^{,i} + \frac{z''}{z}v^2\right)$$

$$v = a\left(\delta\varphi + \frac{z}{a}\Phi\right)$$

$$z = a\frac{\varphi_0'}{\mathcal{H}}$$

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Step 3: Resulting equation of motion (Fourier space)

$$v_k'' + (k^2 - \frac{z''}{z})v_k = 0$$

Features:

- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales $v_k \sim z$

Quantum vacuum initial conditions:

$$v_k(\eta_i) = (\sqrt{2k})^{-1}$$

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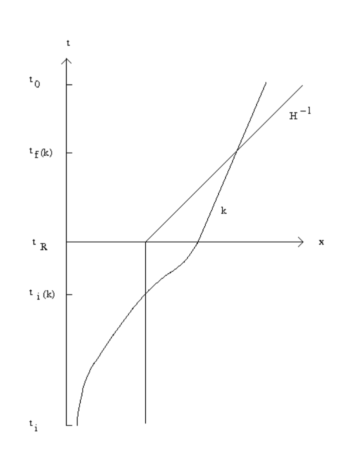
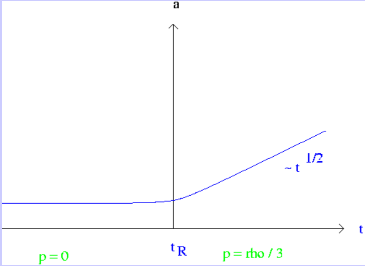
Structure formation in inflationary cosmology



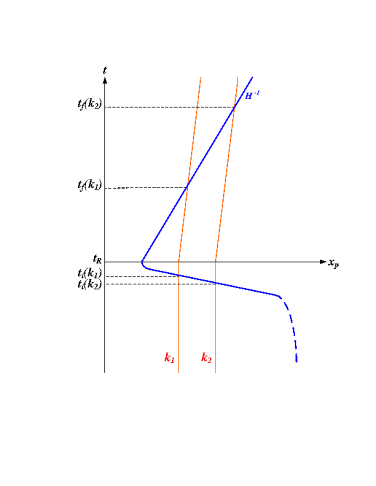
N.B. Perturbations originate as quantum vacuum fluctuations.

Background for string gas cosmology



Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal string gas fluctuations.

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\big((1 + 2\Phi)d\eta^2 - [(1 - 2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big) .$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^0{}_0(k)\delta T^0{}_0(k)\rangle ,$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^i{}_j(k)\delta T^i{}_j(k)\rangle .$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V .$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} .$$

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Power spectrum of cosmological fluctuations

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\begin{aligned}
P_\Phi(k) &= 8G^2k^{-1} < |\delta\rho(k)|^2 > \\
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Key features:

- scale-invariant like for inflation
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Comments

- Evolution for $t > t_i(k)$: $\Phi \simeq \mathrm{const}$ since the equation of state parameter $1 + w$ stays the same order of magnitude unlike in inflationary cosmology.
- Squeezing of the fluctuation modes takes place on super-Hubble scales like in inflationary cosmology $\rightarrow$ acoustic oscillations in the CMB angular power spectrum
- In a dilaton gravity background the dilaton fluctuations dominate $\rightarrow$ different spectrum [R.B. et al, 2006; Kaloper, Kofman, Linde and Mukhanov, 2006]

Prediction: Spectrum of Gravitational Waves

R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

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\begin{aligned}
P_h(k) &= 16\pi^2 G^2 k^{-1} < |T_{ij}(k)|^2 > \\
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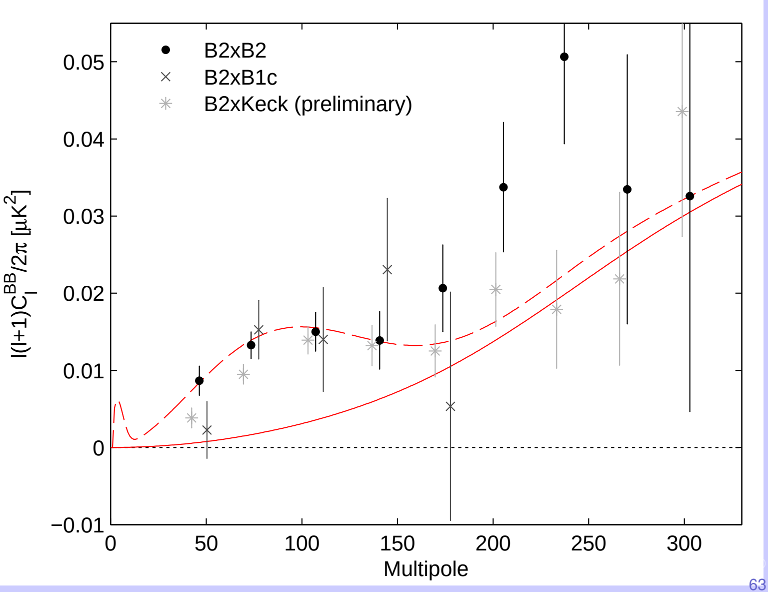
BICEP-2 Results



Requirements

- Static Hagedorn phase (including static dilaton) $\rightarrow$ new physics required.
- $C_V(R) \sim R^2$ obtained from a thermal gas of strings provided there are winding modes which dominate.
- Cosmological fluctuations in the IR are described by Einstein gravity.

Prediction: Running of the Spectrum of Cosmological Perturbations

R.B., G. Franzmann and Q. Liang, arXiv:1708.06793 [hep-th]

Running

$$\alpha_s \equiv \frac{d^2 \ln P_\Phi(k)}{d \ln k^2}\Big|_{k=aH}$$

- For **Inflation**: $\alpha_s \sim (1 - n_s)^2$
- For **String Gas Cosmology**: $\alpha_s \sim (1 - n_s)$

$\rightarrow$ String Gas Cosmology predicts a parametrically larger running.

Plan

Conclusions

- **Cosmology of string theory** must take into account the key symmetries of string theory, in particular the T-duality symmetry.
- Standard effective field theory of supergravity will break down in the very early universe.
- Double Field Theory may provide a better description of the background for string cosmology.
- Cosmological evolution is **nonsingular**.
- Our universe emerges from an early Hagedorn phase.
- Thermal string fluctuations in the Hagedorn phase yield an almost scale-invariant spectrum of cosmological fluctuations.
- **Characteristic signal**: blue tilt in the spectrum of gravitational waves.