Signatures of alternative models of gravity in the gravitational waves: the quasi-normal modes of black holes and neutron stars

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JLBS, Daniela D. Doneva, Jutta Kunz, Stoytcho S. Yazadjiev [arXiv:1805.05755]

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Signatures of alternative models of gravity in the gravitational waves: the quasi-normal modes of black holes and neutron stars

1. Introduction

- 2. Black holes in scalar-Einstein-Gauss-Bonnet theory
 - a) Perturbation theory and quasinormal modes
 - b) Spectrum of the dilatonic black holes
 - c) Stability: dilatonic vs scalarized
 - d) Implications for astrophysical black holes
- 3. Neutron stars in scalar-tensor theory
 - a) Matter and equation of state
 - b) Universal relations
- 4. Conclusions and Outlook

Observations of gravitational waves emitted from BH and NS mergers



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Ringdown phase: resonant frequencies and exponential damping



Spectrum of quasinormal modes

GR: Mass, angular momentum, matter composition

Alternative theories: it may depend also on the parameters of the theory // extra charges (scalar fields)

Why modified gravity?

- Dark energy and dark matter
- Quantum gravity

Compatible with experimental tests and theoretical requirements

- Horndeski gravity
- Scalar-tensor theory
- f(R)

- ...

Einstein theory expected to break down at high energies / strong gravity regimes:

Black holes, neutron stars, ...

In this talk:

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- Black holes in scalar-Einstein-Gauss-Bonnet theory

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \Big[R - 2\nabla_\mu \varphi \nabla^\mu \varphi - V(\varphi) + \lambda^2 f(\varphi) \mathcal{R}_{GB}^2 \Big]$$

$$R_{\rm GB}^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

In this talk:

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- Neutron stars in scalar-tensor theory (R²)

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R - 2g \ ^{\mu\nu}\partial_{\mu}\varphi \partial_{\nu}\varphi - V(\varphi) \right] + S_{\text{matter}}$$

$$V(\varphi) = \frac{1}{4a} \left(1 - e^{-\frac{2\varphi}{\sqrt{3}}} \right)^2$$

In this talk:

- Black holes in scalar-Einstein-Gauss-Bonnet theory

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- Neutron stars in nonminimal derivative coupling (Horndeski theory)

$$S = \int d^4x \sqrt{-g} \bigg[R + \eta G^{\mu\nu} \nabla_{\mu} \phi \nabla_{\nu} \phi \bigg] + S_{\text{matter}}$$

Einstein-Gauss-Bonnet-dilaton gravity

- Higher order curvature corrections to Einstein theory are predicted by string theory

- Low energy limit of heterotic string theory Gauss-Bonnet term + scalar field
- Quadratic in curvature

second order differential equations (ghost free)

$$S_{\rm EGBd}(g,\Phi) = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left(R - \frac{1}{2} \partial_\mu \Phi \,\partial^\mu \Phi + \frac{1}{4} \alpha e^{\gamma \Phi} R_{\rm GB}^2 \right)$$

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$$S_{\rm EGBd}(g,\Phi) = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left(R - \frac{1}{2} \partial_\mu \Phi \,\partial^\mu \Phi + \frac{1}{4} \alpha e^{\gamma \Phi} R_{\rm GB}^2 \right)$$

Field equations of Einstein-Gauss-Bonnet-dilaton gravity

$$G_{\mu\nu} = \frac{1}{2} T_{\mu\nu} \qquad \nabla^2 \Phi = \frac{1}{4} \alpha \gamma e^{\gamma \Phi} R_{\text{GB}}^2$$

$$T_{\mu\nu} = T_{\mu\nu}^{(\phi)} + \frac{1}{4} \alpha e^{\gamma \Phi} T_{\mu\nu}^{(\text{GBd})}$$

$$T_{\mu\nu}^{(\phi)} = \partial_\mu \Phi \ \partial_\nu \Phi - \frac{1}{2} g_{\mu\nu} \partial_\rho \Phi \ \partial^\rho \Phi$$

$$T_{\mu\nu}^{(\text{GBd})} = H_{\mu\nu} + 4 (\partial^\rho \Phi \ \partial^\sigma \Phi + \partial^\rho \partial^\sigma \Phi) P_{\mu\rho\nu\sigma}$$

$$H_{\mu\nu} = 2 (RR_{\mu\nu} - 2R_{\mu\alpha}R_{\nu}^{\alpha} - 2R_{\mu\alpha\beta\nu}R^{\alpha\beta} + R_{\mu\alpha\beta\delta}R_{\nu}^{\alpha\beta\delta}) - \frac{1}{2} g_{\mu\nu}R_{\text{GB}}^2$$

$$P_{\alpha\beta\gamma\delta} = R_{\alpha\beta\delta\gamma} + g_{\alpha\gamma}R_{\delta\beta} - g_{\alpha\delta}R_{\gamma\beta} + g_{\beta\delta}R_{\gamma\alpha} - g_{\beta\gamma}R_{\delta\alpha} + \frac{1}{2}Rg_{\alpha\delta}g_{\gamma\beta} - \frac{1}{2}Rg_{\alpha\gamma}g_{\delta\beta}$$

Static and spherically symmetric black holes with scalar field:

Mignemi, Stewart, PRD47 5259 (1993) Mignemi. PRD51 934 (1995) Kanti et al, PRD54 5049 (1996) Torii, Yajima, Maeda. PRD55 739 (1997)

$$ds^{2} = g^{(0)}_{\mu\nu} dx^{\mu} dx^{\nu} = -F(r)dt^{2} + K(r)dr^{2} + r^{2}(d\theta^{2} + \sin^{2}\theta \, d\varphi^{2})$$

$$\Phi = \Phi_{0}(r)$$

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$$\Phi = \Phi_{0}(r)$$

Near-horizon expansion:

$$m_1 = \frac{\alpha \gamma \Phi_{01}}{2\alpha \gamma \Phi_{01} + 4r_H e^{\gamma \Phi_{00}}}$$

$$K(r) \approx \frac{1}{1 - 2m_1} \cdot \frac{r_H}{r - r_H},$$

$$F(r) \approx f_1(r - r_H)$$

$$\Phi_0(r) \approx \Phi_{00} + \Phi_{01}(r - r_H),$$

Regularity implies:

$$\alpha \gamma r_H^2 \Phi_{01}^2 + 2e^{\gamma \Phi_{00}} r_H^3 \Phi_{01} + 6\alpha \gamma = 0$$

$$e^{2\gamma\Phi_{00}}r_H^4 - 6\alpha^2\gamma^2 > 0$$

Static and spherically symmetric black holes with scalar field:

$$ds^{2} = g_{\mu\nu}^{(0)} dx^{\mu} dx^{\nu} = -F(r) dt^{2} + K(r) dr^{2} + r^{2} (d\theta^{2} + \sin^{2} \theta \, d\varphi^{2})$$

$$\Phi = \Phi_{0}(r)$$

Asymptotically flat:

$$K(r) \approx 1 + 2M/r$$

$$F(r) \approx 1 - 2M/r + f_2/r^2$$

$$\Phi_0(r) = Q/r$$











a) Perturbations theory and quasinormal modes

Non-radial perturbations of the static configuration:

$$g_{\mu\nu} = g^{(0)}_{\mu\nu}(r) + \epsilon h_{\mu\nu}(t, r, \theta, \varphi)$$
$$\Phi = \Phi_0(r) + \epsilon \delta \Phi(t, r, \theta, \varphi)$$

The perturbation depends in principle on all the coordinates (t,r,θ,ϕ)

Simplification:

- Expansion of the angular dependence in tensor harmonics (I,m)

Decoupling into axial
$$Y_{lm}(\theta,\varphi) \to Y_{lm}(\pi-\theta,\pi+\varphi) = (-1)^{l+1}Y_{lm}(\theta,\varphi)$$

- polar $Y_{lm}(\theta,\varphi) \to Y_{lm}(\pi-\theta,\pi+\varphi) = (-1)^{l}Y_{lm}(\theta,\varphi)$

- Laplace transformation of the temporal dependence ($\omega = \omega_{R} + i\omega_{I}$)

Ansatz for axial perturbations:

$$h_{\mu\nu}^{(\text{axial})} = \int d\omega \, e^{-i\omega t} \sum_{l,m} \begin{bmatrix} 0 & 0 & -h_0 \frac{1}{\sin \theta} \frac{\partial}{\partial \varphi} Y_{lm} & h_0 \sin \theta \frac{\partial}{\partial \theta} Y_{lm} \\ 0 & 0 & -h_1 \frac{1}{\sin \theta} \frac{\partial}{\partial \varphi} Y_{lm} & h_1 \sin \theta \frac{\partial}{\partial \theta} Y_{lm} \\ -h_0 \frac{1}{\sin \theta} \frac{\partial}{\partial \varphi} Y_{lm} & -h_1 \frac{1}{\sin \theta} \frac{\partial}{\partial \varphi} Y_{lm} & h_2 \frac{1}{2\sin \theta} X_{lm} & -\frac{1}{2} h_2 \sin \theta W_{lm} \\ h_0 \sin \theta \frac{\partial}{\partial \theta} Y_{lm} & h_1 \sin \theta \frac{\partial}{\partial \theta} Y_{lm} & -\frac{1}{2} h_2 \sin \theta W_{lm} & -\frac{1}{2} h_2 \sin \theta X_{lm} \end{bmatrix}$$
$$W_{lm} = \frac{\partial^2}{\partial \theta^2} Y_{lm} - \cot \theta \frac{\partial}{\partial \theta} Y_{lm} - \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} Y_{lm} , \quad X_{lm} = 2 \frac{\partial^2}{\partial \theta \partial \varphi} Y_{lm} - 2 \cot \theta \frac{\partial}{\partial \varphi} Y_{lm}$$

We use the Regge-Wheler gauge: $h_2 = 0$

Axial perturbations do not couple to dilaton perturbations

Ansatz for polar perturbations:

$$h_{\mu\nu}^{(\text{polar})} = \int d\omega \, e^{-i\omega t} \sum_{l,m} \begin{bmatrix} 2NF(r)Y_{lm} & -H_1Y_{lm} & -h_{0p}\frac{\partial}{\partial\theta}Y_{lm} & -h_{0p}\frac{\partial}{\partial\varphi}Y_{lm} \\ -H_1Y_{lm} & -2K(r)LY_{lm} & h_{1p}\frac{\partial}{\partial\theta}Y_{lm} & h_{1p}\frac{\partial}{\partial\varphi}Y_{lm} \\ -h_{0p}\frac{\partial}{\partial\theta}Y_{lm} & h_{1p}\frac{\partial}{\partial\theta}Y_{lm} & B & -r^2VX_{lm} \\ -h_{0p}\frac{\partial}{\partial\varphi}Y_{lm} & h_{1p}\frac{\partial}{\partial\varphi}Y_{lm} & -r^2VX_{lm} & A \end{bmatrix}$$

$$A = (l(l+1)V - 2T)r^2 \sin^2 \theta Y_{lm} + r^2 V \sin^2 \theta W_{lm}$$
$$B = (l(l+1)V - 2T)r^2 Y_{lm} - r^2 V W_{lm}$$

Gauge fixing:
$$h_{0p} = h_{1p} = V = 0.$$

Scalar polar perturbations:

$$\delta \Phi = \int d\omega \, e^{-i\omega t} \sum_{l,m} \Phi_1 \, Y_{lm}$$

Axial equations:

$$\frac{d}{dr}\Psi_{(i)} + U_{(i)}\Psi_{(i)} = 0$$
$$\Psi_{axial} = (h_0, h_1)$$

Outgoing wave at infinity:

$$h_0(r) \approx e^{i\omega r_*} \cdot [h_{00} + \frac{h_{01}}{r} + O(r^{-2})] ,$$

$$h_1(r) \approx e^{i\omega r_*} \cdot [-h_{00} + \frac{i}{\omega}((2iM\omega - 1)h_{01} + i\omega h_{00})\frac{1}{r} + O(r^{-2})] ,$$

$$h_{01} = \frac{ih_{00}}{8\omega}(\omega^2(4f_2 + Q^2) + 4(l+2)(l-1))$$

Axial equations:

$$\frac{d}{dr}\Psi_{(i)} + U_{(i)}\Psi_{(i)} = 0$$
$$\Psi_{axial} = (h_0, h_1)$$

Ingoing wave at the horizon:

$$h_{0}(r) \approx \frac{e^{-i\omega r_{*}}}{r - r_{H}} \cdot [\hat{h}_{00} + O(r - r_{H})] ,$$

$$h_{1}(r) \approx -\frac{e^{-i\omega r_{*}}}{r_{H}} \cdot [\hat{h}_{10} + O(r - r_{H})] .$$

$$\hat{h}_{10} = -\hat{h}_{00} \sqrt{\frac{\alpha \gamma \Phi_{01} + 2e^{\gamma \Phi_{00}} r_{H}}{2r_{H}^{2} f_{1} e^{\gamma \Phi_{00}}}}$$

Polar equations:

$$\frac{d}{dr}\Psi_{(i)} + U_{(i)}\Psi_{(i)} = 0$$
$$\Psi_{polar} = (H_1, T, \Phi_1, \frac{d}{dr}\Phi_1)$$

Outgoing wave at infinity:

$$T(r) \approx e^{i\omega r_*} \cdot [T_0 + O(r^{-2})] ,$$

$$H_1(r) \approx \omega r e^{i\omega r_*} \cdot [-T_0 - \frac{1}{2\omega r} (4M\omega + i(l^2 + l - 2))T_0 + O(r^{-2})] ,$$

$$\Phi_1(r) \approx e^{i\omega r_*} \cdot \frac{1}{r} \cdot [\Phi_{10} - \frac{i}{2\omega r} (2iM\omega QT_0 - l(l+1)\Phi_{10}) + O(r^{-2})]$$

Polar equations:

$$\frac{d}{dr}\Psi_{(i)} + U_{(i)}\Psi_{(i)} = 0$$
$$\Psi_{polar} = (H_1, T, \Phi_1, \frac{d}{dr}\Phi_1)$$

Ingoing wave at the horizon:

$$\begin{split} T(r) &\approx e^{-i\omega r_*} \cdot \left[\hat{T}_0 + O(r - r_H)\right] ,\\ H_1(r) &\approx e^{-i\omega r_*} \cdot \frac{\omega}{r - r_H} \cdot \left[\hat{H}_{10} + O(r - r_H)\right] ,\\ \Phi_1(r) &\approx e^{-i\omega r_*} \cdot \left[\hat{\Phi}_{10} + O(r - r_H)\right] .\\ \hat{H}_{10} &= \frac{D_2}{D_1} \left(e^{-\gamma \Phi_{00}} \alpha \gamma \hat{\Phi}_{10} - r_H^3 \hat{T}_0\right) \stackrel{D_2 := (4e^{\gamma \Phi_{00}} r_H + 2\alpha \gamma \Phi_{01})\omega^2 + e^{\gamma \Phi_{00}} f_1 ,\\ i\omega r_H e^{\gamma \Phi_{00}} (l^2 + l + 1)\sqrt{2\alpha \gamma e^{-\gamma \Phi_{00}} \Phi_{01} f_1 + 4r_H f_1} . \end{split}$$

Numerical implementation:

Static back ground solution for a fixed mass, coupling constant, etc



Numerical implementation:

Divide background solution in two pieces



Numerical implementation:

Asymptotic behavior of the perturbation



Numerical implementation:

Asymptotic behavior of the perturbation



Numerical implementation:

Approximate perturbation close to the boundaries (analytical)



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Approximate perturbation close to the boundaries (analytical)



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Numerical implementation:

Approximate perturbation close to the boundaries (analytical)



Numerical implementation:

Approximate perturbation close to the boundaries (analytical)



Numerical implementation:

Generate perturbations at each piece (numerical)



Numerical implementation:

Generate perturbations at each piece (numerical)



Numerical implementation:

In general the perturbations are not continuous



Numerical implementation:

Repeat until the ω that satisfies junction conditions are satisfied



2. Black holes sEGB theory

















2. Black holes in sEGB theory







$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \Big[R - 2\nabla_\mu \varphi \nabla^\mu \varphi - V(\varphi) + \lambda^2 f(\varphi) \mathcal{R}_{GB}^2 \Big]$$

Dilatonic:

$$f(\varphi) = e^{2\gamma\varphi} \quad \lambda^2 = \frac{\alpha}{4}$$

New class of scalarized black holes:

Doneva & Yazadjiev, PRL 120 131103 (2018) [arXiv:1711.01187] Silva, Sakstein, Gualtieri, Sotiriou, Berti, PRL 120 131104 (2018) [arXiv:1711.02080] Antoniou, Bakopoulos, Kanti, PRL 120, 131102 (2018) [arXiv:1711.03390] Antoniou, Bakopoulos, Kanti, PRD 97, 084037 (2018) [arXiv:1711.07431]

$$f(\varphi) = \frac{1}{12} \left(1 - e^{-6\varphi^2} \right)$$

$$f(\varphi) = \frac{1}{2}\varphi^2$$









 $\frac{dR}{dr} = g$



 $M < 0.171 \lambda$







$$f(\varphi) = \frac{1}{2}\varphi^2$$

2. Black holes in sEGB theory

d) Implications for astrophysical black holes

2. d) Implications for astrophysical black holes



2. d) Implications for astrophysical black holes

Dilatonic EGB black holes I=2 polar grav-led mode



2. d) Implications for astrophysical black holes



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$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R - 2g \,^{\mu\nu} \partial_{\mu}\varphi \partial_{\nu}\varphi - V(\varphi) \right] + S_{\text{matter}}$$

$$V(\varphi) = \frac{1}{4a} \left(1 - e^{-\frac{2\varphi}{\sqrt{3}}} \right)^2$$

Interior of a neutron star:



Beam of X-rays coming from the neutron star's poles, which sweeps around as the star rotates.

onature

Scalarized neutron stars with realistic equations of state

Yazadjiev et al. JCAP 1406 (2014) 003



Scalarized neutron stars with realistic equations of state

Yazadjiev et al. JCAP 1406 (2014) 003





Scalarized neutron stars with realistic equations of state

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Fundamental I=2 axial mode






 $\omega[kHz] \cdot R_s[Km]$

 $\frac{M[M_{\odot}]}{\tau[\mu s]}$



 $\omega_{R}[\rm kHz]\cdot M[\rm M_{\odot}]$

 $\tilde{\omega} = \omega / \sqrt{\hat{p}_0}$

$$S = \int d^4x \sqrt{-g} \bigg[R + \eta G^{\mu\nu} \nabla_{\mu} \phi \nabla_{\nu} \phi \bigg] + S_{\text{matter}}$$

Scalarized neutron stars with realistic equations of state

Cisterna et al. PRD92 (2015) 044050



$$S = \int d^4x \sqrt{-g} \bigg[R + \eta G^{\mu\nu} \nabla_{\mu} \phi \nabla_{\nu} \phi \bigg] + S_{\text{matter}}$$

Scalarized neutron stars with realistic equations of state

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Scalarized neutron stars with realistic equations of state

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4. Conclusions and Outlook

First steps in the calculation of QNM spectrum of compact objects in alternative theories of gravity

New channel of emission (scalar-led modes), resulting in a richer spectrum (breaking of isospectrality)

Instabilities of black holes with scalar hair (radial unstable mode)

Minimum black hole mass in scalar-EGB theory, related with the appearance of the unstable mode in the strong coupling regime

Precise measurement of the ringdown phase could be used to constrain the theory coupling parameters

In the case of neutron stars, deviations in the matter independent universal relations can be used to constrain the scalar charge Outlook:

- Polar QNM of neutron stars with scalar hair

- QNM of rotating configurations

Realistic models of the ring-down phase should include the effect of rotation

- QNM of other compact objects Wormholes [arXiv:1806.03282] Boson stars...

- Other theories: more general scalar-tensor theories, ...

Thank you very much for your attention!

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