Nonrelativistic String Theory and T-Duality

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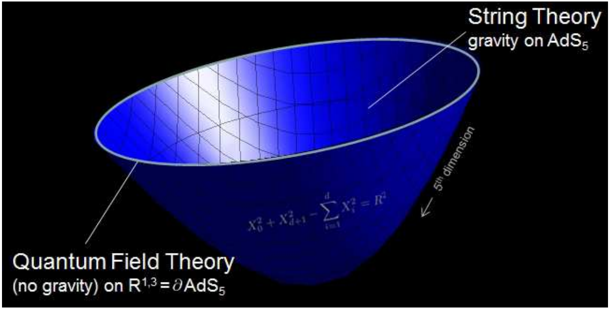
Motivation

Holography



Gravity is not only used to describe the gravitational force!

Non-relativistic Holography

two approaches

- Keep general relativity in the bulk but take background geometry with non-relativistic isometries

Christensen, Hartong, Kiritsis, Obers and Rollier (2013-2015)

- Take non-relativistic gravity in the bulk

Gomis, Ooguri (2001); Gopakumar, Bagchi (2009)

Outline

Newton-Cartan Gravity

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Non-relativistic String Theory

T-duality

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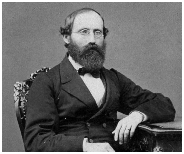
Frames

- Inertial frames: Galilean symmetries

- Constant acceleration: Newtonian gravity/Newton potential $\Phi(x)$

- no frame-independent formulation
  (needs geometry!)



Riemann (1867)

Galilei Symmetries

- time translations: $\delta t = \xi^0$ <u>but not</u> $\delta t = \lambda^i x^i$!

- space translations: $\delta x^i = \xi^i \qquad i = 1, 2, 3$

- spatial rotations: $\delta x^i = \lambda^i{}_j \, x^j$

- Galilean boosts: $\delta x^i = \lambda^i t$

'Gauging' Galilei

symmetry	generators	gauge field	curvatures
time translations	H	τ_μ	$\tau_{\mu\nu} = \partial_{[\mu}\tau_{\nu]}$
space translations	P^A	$E_\mu{}^A$	$R_{\mu\nu}{}^A(P)$
Galilean boosts	G^A	$\omega_\mu{}^A$	$R_{\mu\nu}{}^A(G)$
spatial rotations	J^{AB}	$\omega_\mu{}^{AB}$	$R_{\mu\nu}{}^{AB}(J)$

Imposing Constraints

$R_{\mu\nu}{}^a(P) = 0$: does only solve for part of $\omega_\mu{}^a\,,\omega_\mu{}^{ab}$

Absolute Time

$$\tau_{\mu\nu} \equiv \partial_{[\mu}\tau_{\nu]} = 0 \qquad \to \qquad \tau_\mu = \partial_\mu \rho$$

$$\Delta T = \int_{\mathcal{C}} \mathrm{d}x^\mu \tau_\mu = \int_{\mathcal{C}} \mathrm{d}\rho \ \text{ is path-independent}$$

From Galilei to Bargmann

the zero commutator

$$[G_A, P_B] = 0$$

implies that a massive particle with non-zero spatial momentum P_B cannot by any boost transformation G_A be brought to a rest frame $\Rightarrow$

$$[G_A, P_B] = \delta_{AB} M \qquad \rightarrow \qquad \text{extra gauge field } m_\mu$$

The NC Transformation Rules

The independent NC fields $\{\tau_\mu, e_\mu{}^A, m_\mu\}$ transform as follows:

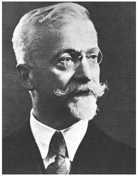
$$\delta\tau_\mu = 0\,,$$
$$\delta E_\mu{}^A = \Lambda^A{}_B\, E_\mu{}^B + \Sigma^A \tau_\mu\,,$$
$$\delta m_\mu = \partial_\mu\sigma + \Sigma_A\, E_\mu{}^A$$

The spin-connection fields $\omega_\mu{}^{AB}$ and $\omega_\mu{}^A$ are functions of τ_μ , $E_\mu{}^A$ and m_μ

What about the dynamics ?

The NC Equations of Motion

The NC equations of motion are given by



Élie Cartan 1923

$$\mathcal{R}_{0C}{}^{C}(G) \;=\; \mathcal{R}_{0C}{}^{CA}(J) \;=\; \mathcal{R}^{(A}{}_{C}{}^{CB)}(J) \;=\; 0$$

1 **A** **(AB)**

- there is no known action that gives rise to these equations of motion

  see, however, Hartong, Obers (2018)

- after gauge-fixing $\tau_\mu = \delta_{\mu,0}$, $e_\mu{}^A = \delta_\mu{}^A$ and $m_0 = \Phi$ the 4D NC e.o.m. reduce to $\triangle\Phi = 0$

NC Gravity couples to particles

what about strings?

Outline

String Galilei Symmetries

$$D+1 \text{ flat indices} \rightarrow \begin{cases} 2 \ \text{longitudinal indices } A \\ D-1 \ \text{transverse indices } A' \end{cases}$$

longitudinal translations	H_A
transverse translations	$P_{A'}$
string Galilei boosts	$G_{AB'}$
longitudinal Lorentz rotations	M_{AB}
transverse spatial rotations	$J_{A'B'}$

Two-dimensional Foliation

$$D_{[\mu}\tau_{\nu]}{}^{A} = 0 \quad \text{with} \quad \tau_\mu{}^A \ \text{generalized clock function}$$

$$\partial_{[\mu}\left(\tau_\nu{}^A\tau_{\rho]}{}^B\epsilon_{AB}\right) = 0 \qquad \rightarrow \qquad \tau_\mu{}^A\tau_\nu{}^B\epsilon_{AB} = \partial_{[\mu}\rho_{\nu]}$$

String Newton-Cartan Geometry

The independent string NC fields $\{\tau_\mu{}^A, e_\mu{}^{A'}, m_\mu{}^A\}$ transform as follows:

$$\delta\tau_\mu{}^A = \Lambda^A{}_B\tau_\mu{}^B\,,$$

$$\delta E_\mu{}^{A'} = \Lambda^{A'}{}_{B'}\,E_\mu{}^{B'} - \Sigma_A{}^{A'}\tau_\mu{}^A\,,$$

$$\delta m_\mu{}^A = D_\mu\sigma^A + \Sigma^A{}_{A'}\,E_\mu{}^{A'}$$

The spin-connection fields $\omega_\mu{}^{AB'}$, $\omega_\mu{}^{AB}$ and $\omega_\mu{}^{A'B'}$ are functions of $\tau_\mu{}^A$, $E_\mu{}^{A'}$ and $m_\mu{}^A$

$$H_{\mu\nu} \equiv E_\mu{}^{A'}E_\nu{}^{B'}\delta_{A'B'} + \left(\tau_\mu{}^A m_\nu{}^B + \tau_\nu{}^A m_\mu{}^B\right)\eta_{AB}$$

Poyakov versus Nambu-Goto

Nambu-Goto formulation

$$S_{\mathrm{NG}} = -\frac{T}{2}\int d^2\sigma\,\sqrt{-G} \qquad \text{with} \qquad G \equiv \det \partial_\alpha x^\mu \partial_\beta x^\nu\, G_{\mu\nu}(x)$$

Polyakov formulation

$$S_{\mathrm{Pol.}} = -\frac{T}{2}\int d^2\sigma\,\sqrt{-h}\,h^{\alpha\beta}\,\partial_\alpha x^\mu \partial_\beta x^\nu\, G_{\mu\nu}(x)$$

The Nonrelativistic Polyakov Model

$$h_{\alpha\beta} = e_\alpha{}^a e_\beta{}^b \eta_{ab}$$

$$e_\alpha \equiv e_\alpha{}^0 + e_\alpha{}^1 \,, \qquad \bar{e}_\alpha \equiv e_\alpha{}^0 - e_\alpha{}^1$$

$$\tau_\mu \equiv \tau_\mu{}^0 + \tau_\mu{}^1 \,, \qquad \bar{\tau}_\mu \equiv \tau_\mu{}^0 - \tau_\mu{}^1$$

$$S_{\text{Pol.}} = -\frac{T}{2}\int d^2\sigma \left[\sqrt{-h}\, h^{\alpha\beta}\,\partial_\alpha x^\mu \partial_\beta x^\nu H_{\mu\nu} + \epsilon^{\alpha\beta}\left(\lambda\, e_\alpha \tau_\mu + \bar{\lambda}\,\bar{e}_\alpha \bar{\tau}_\mu\right)\partial_\beta x^\mu\right]$$

$$-\frac{T}{2}\int d^2\sigma\, \epsilon^{\alpha\beta}\partial_\alpha x^\mu \partial_\beta x^\nu B_{\mu\nu} + \frac{1}{4\pi}\int d^2\sigma \sqrt{-h}\, R(h)\, \Phi$$

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rewrite $\frac{1}{2}\omega^2 X^2$ as $\lambda X - \frac{1}{2\omega^2}\lambda^2$

The Nonrelativistic Nambu-Goto model

Integrating out Lagrange multipliers λ and $\bar{\lambda}$ $\Rightarrow$ $h_{\alpha\beta} = \tau_{\alpha\beta}$

$$S_{\mathrm{NG}} = -\frac{T}{2}\int d^2\sigma\left(\sqrt{-\tau}\,\tau^{\alpha\beta}\partial_\alpha x^\mu\partial_\beta x^\nu H_{\mu\nu} + \epsilon^{\alpha\beta}\partial_\alpha x^\mu\partial_\beta x^\nu B_{\mu\nu}\right) + \frac{1}{4\pi}\int d^2\sigma\sqrt{-\tau}\,R(\tau)\,\Phi$$

with $\tau_{\alpha\beta} = \partial_\alpha x^\mu\partial_\beta x^\nu\tau_{\mu\nu}(x)$

Outline

Newton-Cartan Gravity

Non-relativistic String Theory

T-duality

Comments

Relativistic Spatial T-duality

Buscher (1987,1988); Roček, Verlinde (1992)

adapted coordinates: $x^\mu = (y, x^i)$ $\quad k^\mu \partial_\mu = \partial_y$

$$S_{\text{parent}} = \underbrace{S_{\text{Pol.}}(\partial_\alpha y \to v_\alpha)}_{\text{quadratic in } v_\alpha!} - T \int d^2\sigma \epsilon^{\alpha\beta} \tilde{y} \partial_\alpha v_\beta$$

$$\frac{\delta S_{\text{parent}}}{\delta v_\alpha} = 0 \quad \to \quad v_\alpha \text{ is solved for in terms of dual coordinate } \tilde{y}$$

$$\tilde{G}_{yy} = \frac{1}{G_{yy}} \quad \Rightarrow \quad R \Leftrightarrow \frac{1}{R}$$

Three Different T-dualities

- Longitudinal Spatial T-duality
- Longitudinal lightlike T-duality
- Transverse T-duality

Longitudinal spatial T-duality

$$\tau_\mu{}^0 k^\mu = 0, \qquad \tau_\mu{}^1 k^\mu \neq 0\,, \qquad E_\mu{}^{A'} k^\mu = 0$$

adapted coordinates: $x^\mu = (y, x^i) \qquad k^\mu \partial_\mu = \partial_y$

$$S_{\text{parent}} = \underbrace{S_{\text{Pol.}}(\partial_\alpha y \to v_\alpha)}_{\text{quadratic in } v_\alpha!} - T \int d^2\sigma \epsilon^{\alpha\beta} \tilde{y} \partial_\alpha v_\beta$$

$$\frac{\delta S_{\text{parent}}}{\delta v_\alpha} = 0 \quad \to \quad v_\alpha \text{ is solved for (in terms of } \lambda \text{ and } \bar{\lambda}\,!)$$

Intermediate Dual Action

substituting the solution for v_α back into S_{parent} leads to an intermediate dual action S'_{long} with

$$H'_{yy} = \frac{1}{H_{yy}}\,, \qquad \Phi' = \Phi - \frac{1}{2}\log H_{yy}\,,$$

$$H'_{yi} = \frac{B_{yi}}{H_{yy}}\,, \qquad B'_{yi} = \frac{H_{yi}}{H_{yy}}\,,$$

$$H'_{ij} = H_{ij} + \frac{B_{yi}B_{yj} - H_{yi}H_{yj}}{H_{yy}}\,, \qquad B'_{ij} = B_{ij} + \frac{B_{yi}H_{yj} - B_{yj}H_{yi}}{H_{yy}}\,.$$

we can integrate out the Lagrange multipliers λ and $\bar{\lambda}$ and substitute back into S'_{long} $\Rightarrow$

Final Dual Action

$$\tilde{S}_{\text{long.}} = -\frac{T}{2}\int d^2\sigma\left(\sqrt{-h}\,h^{\alpha\beta}\,\partial_\alpha\tilde{x}^\mu\partial_\beta\tilde{x}^\nu\,\tilde{G}_{\mu\nu} + \epsilon^{\alpha\beta}\,\partial_\alpha\tilde{x}^\mu\partial_\beta\tilde{x}^\nu\,\tilde{B}_{\mu\nu}\right) + \frac{1}{4\pi}\int d^2\sigma\sqrt{-h}\,R\,\tilde{\Phi} \qquad \text{with} \quad \tilde{x}^\mu = (\tilde{y}, x^i): \quad \text{Polyakov string!}$$

$$\tilde{G}_{yy} = 0: \ \text{lightlike direction}, \qquad \tilde{\Phi} = \Phi - \frac{1}{2}\log\tau_{yy},$$

$$\tilde{G}_{yi} = \frac{{\tau_i}^A{\tau_y}^B\epsilon_{AB}}{\tau_{yy}}, \qquad \tilde{B}_{yi} = \frac{\tau_{yi}}{\tau_{yy}},$$

$$\tilde{G}_{ij} = H_{ij} + \frac{\left(B_{yi}{\tau_j}^A + B_{yj}{\tau_i}^A\right){\tau_y}^B\epsilon_{AB} + H_{yy}\tau_{ij} - H_{yi}\tau_{yj} - H_{yj}\tau_{yi}}{\tau_{yy}},$$

$$\tilde{B}_{ij} = B_{ij} + \frac{B_{yi}\tau_{yj} - B_{yj}\tau_{yi} - \left(H_{yy}{\tau_i}^A{\tau_j}^B - H_{yi}{\tau_y}^A{\tau_j}^B + H_{yj}{\tau_y}^A{\tau_i}^B\right)\epsilon_{AB}}{\tau_{yy}}$$

Summary

- The longitudinal spatial T-dual of the NR string is the Polyakov string moving in a GR background with a lightlike direction

- The longitudinal lightlike T-dual of the NR string is again a NR string with a longitudinal lightlike direction

- The transverse spatial T-dual of the NR string is again a NR string with a transverse spatial isometry direction à la Buscher

Buscher (1987,1988); Roček, Verlinde (1992)

Outline

Nonrelativitic String Theory

- Nonrelativistic String Theory can be defined independent of any limit of relativistic string theory

- String NC Geometry is to NR string theory what Riemannian geometry is to relativistic string theory

- nonrelativistic string provides first principles definition of Discrete Lightcone Quantization of relativistic string?

Open Issues

- Does β-function calculation leads to consistent backgrounds?
- Nonrelativistic holography?

Gopakumer, Bagchi (2009)

- Double Field Theory?

S. M. Ko, C. Melby-Thompson, R. Meyer and J.-H. Park (2005)

- NR superstrings

Take Home Message

Nonrelativistic String Theory can be studied!

Gomis, Ooguri (2001)